THE HERITAGE OF ARCHIMEDES IN SHIP HYDROSTATICS: 2000 YEARS FROM THEORIES TO APPLICATIONS

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ABSTRACT Archimedes left to posterity his famous treatise "On Floating Bodies", which establishes the physical foundations for the floatability and stability of ships and other maritime objects. Yet since this treatise was long lost and also simply ignored by practitioners, it took many centuries before Archimedes' brilliant insights were actually applied in ship design and ship safety assessment. This article traces the tedious acceptance of Archimedes' principles of hydrostatics and stability in practical applications. It will document important milestones and explain how this knowledge was passed down through the centuries and ultimately spread into ship design practice.

1. INTRODUCTION

Archimedes (ca. 287–212 B.C.) in his famous treatise "On Floating Bodies" [1, 2] laid the foundations of hydrostatics, especially for the equilibrium and stability of objects floating on the surface of a liquid or immersed in a liquid medium. Evidently his principles and brilliant theories are immediately applicable to ships and can thus form the basis of ship hydrostatics. These fundamental principles are apt to play a crucial role in ship design and ship safety assessment. Yet this knowledge from his treatise did not spread very far in Archimedes' lifetime and was lost or ignored by practitioners for more than a millennium until it was rediscovered many centuries later during the late Middle Ages. It then still took until the 18^{th} c. before the theoretical principles established by Archimedes were actually applied in ship design and stability assessment. Why did this long delay occur?

This article will examine the long history of ship hydrostatics from Archimedes to the modern era and will document the most important milestones in this development. It will follow the route of knowledge

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transfer from its classical origins to current practical applications in ship design and operations. It will also discuss other prerequisites for making the physical principles of hydrostatics applicable to practical applications in ship design and stability. Overall a critical mass of knowledge has to be brought together in order to raise the theoretical knowledge to maturity for applications in practice. The following subjects are essential elements of knowledge for an adequate solution:

- Physical principles of hydrostatics for the equilibrium and stability of floating objects.
- Ship hull geometry definition and representation in some reliable medium, preferably at the design stage.
- Evaluation of ship geometry data by numerical calculation.
- Stability criteria and risk evaluation.

Archimedes firmly established the physical principles and took the opening moves in the other related topics. But it still took a long time before all other ingredients had reached sufficient maturity for actual application. We will describe this arduous road.

This article does not claim to present a complete history of ship hydrostatics. Rather it focusses on how the ingenious ideas of Archimedes were passed down to posterity, were lost and resurrected again, and then supplemented by other fundamental knowledge until they found their application in ship design, which Archimedes perhaps foresaw and which we take for granted today.

2. THEORIES

Archimedes

Precursors: Greek mathematics and Mediterranean shipbuilding, especially also in classical Greece, had reached an advanced level before Archimedes, on which he based his original achievements in the 3rd c. B.C. This background material which should be studied to appreciate the magnitude of his creative contributions can be found in the literature (e.g., Heath [3]), Nowacki [4]).

Force Equilibrium: Buoyancy and Displacement: Archimedes in his famous treatise "On Floating Bodies" (OFB) pronounced the fundamental laws of hydrostatics, i.e., the physical laws of equilibrium for bodies floating in a liquid at rest. Book I deals with the force equilibrium between buoyancy and displacement forces and contains the Principle of Archimedes, which holds for bodies of any shape. Book II treats the moment equilibrium and pertains to the stability of the floating condition, derived for the special case of a paraboloid of revolution. Indirectly hereby Archimedes also laid the foundations for ship hydrostatics since his approach is immediately applicable to ships, even if he did not mention ships anywhere in his treatise.

How did Archimedes arrive at his Principle of Hydrostatics? This is described in his own words in this treatise [1, 2]. He makes two essential axiomatic assumptions.

1. In Book I, preamble he states the properties of the liquid (Heath [1]):

"Let it be supposed that a liquid is of such character that its parts lying evenly and being continuous, that part which is thrust the less is driven along by that part which is thrust the more; and that each of its parts is thrust by the liquid which is perpendicularly above it…"

These lines infer a homogeneous, isotropic liquid whose parts are at rest when in equilibrium. Although the Greeks did not know the concept of pressure, the idea of a hydrostatic pressure distribution is implied here between the lines.

2. In Book I, §5 Archimedes postulates his Principle as follows (Heath [1]):

"Any solid lighter than the liquid will, if placed in the liquid, be so far immersed that the weight of the solid will be equal to the weight of the liquid displaced".

The proof is illustrated in Fig. 1 and described in more detail in Nowacki [4]. The surface of any liquid at rest is a spherical surface whose center point is at the center of the earth (section ALMND). The body EZTH be specifically lighter than the liquid. In two equal adjacent sectors of the liquid at rest the body EZTH floats in equilibrium on the surface such that its submerged volume BCTH is equal to volume RYCS in the neighboring sector. Since in equilibrium the total weight of the masses in each sector must be equal, the weight of the floating body EZTH must be the same as that of the volume of RYCS, hence must also be equal to the weight of the liquid volume it displaces.

Note that this elegant proof of the Principle of Archimedes (buoyancy force is equal and opposite to gravity force or displacement) is based entirely on an experiment of thought. The proof is entirely deductive from a few axioms regarding the liquid properties, no observations are required. It holds for floating bodies of arbitrary shape in an arbitrary type of liquid and was derived for liquids at rest without explicit knowledge of local pressure anywhere. Buoyancy and displacement are force resultants, which in equilibrium are equal in magnitude and opposite in direction. This proof is an outstanding example of Greek logical thought and of the brilliance of Archimedes.

Archimedes has also shown that this Principle holds for a fully immersed object of equal specific weight as the liquid (neutral force equilibrium), but does not apply when the solid is heavier than the liquid because the object then is grounded and loses as much weight as the displaced volume weighs, the rest of the weight is taken up by the grounding support force (Book I, §7).

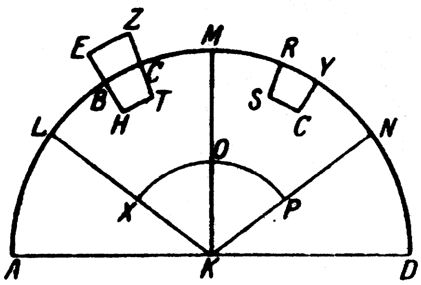

Fig. 1. Proof of Archimedes' Principle (from [2]).

The Eureka Legend: This evidence sheds some special light on the famous "Eureka" legend, as reported by Vitruvius [5], Book IX.3. According to this account Archimedes was challenged by king Hieron of Syracuse to determine whether a wreath, made for the king by a goldsmith for a sacrificial offering, was of pure gold or fraudulently made of gold mixed with silver. Archimedes is said to have sat in a brimful bathtub when he discovered a method to measure the volume his body displaced in the water: After leaving the tub he could fill up the water to the brim again with a measured volume of water. He was elated at this discovery and spontaneously ran through the streets of Syracuse nakedly shouting "Eureka" because he had found a method to prove the fraud. Archimedes went on to sink the wreath and two equally heavy pieces of pure silver and gold each in a bowl full of liquid to the brim, then after removing each object to refill the bowls with a measured volume of liquid. Then since the weights were known, the different volumes gave an indication of the different densities of the objects and the fraud was revealed.

Thus Archimedes thereby discovered a method for measuring the volumes of solid objects and, if their weights are known, their relative densities. But in this bathtub experiment he did not discover the law of equality of buoyancy and displacement, hence the principle of hydrostatics, as is sometimes falsely claimed. This law does not hold there because the human body in the tub will usually touch the ground and the ground force must be taken into account (Fig. 2).

<u>Moment Equilibrium: Hydrostatic Stability:</u> In Book II of OFB Archimedes deals with the moment equilibrium of a floating solid paraboloid of revolution when inclined from an initially upright position. Thereby he derives the righting moments of the inclined solid which he uses as a stability criterion: The equilibrium is stable, if - in the absence of any heeling moments – the inclined object restores itself to its upright position. How does Archimedes determine the righting moment in this case?

Fig. 2. Archimedes in the Bathtub.

First he makes the same assumptions regarding the properties of the liquid as in Book I. The liquid domain is again unbounded, the liquid is at rest. Then the floating object, initially at rest in an upright position, is inclined by a certain, finite angle, but so that the base of the paraboloid is not wetted (Fig. 3). The homogeneous paraboloid segment is cut off perpendicularly to its axis, the paraboloid segment axis length is not greater than 1.5 times its half-parameter. For this case Archimedes demonstrates that the righting moments are positive.

The actual proof applies several mechanical and geometrical principles, deduced in this treatise or derived by Archimedes in his earlier work (for details see Nowacki [4]): For the inclined paraboloid he disregards the underwater part under the water surface JS because its buoyancy and gravity forces are equal and opposite for the homogeneous solid and thus produce no moments. For the abovewater part he proves (by means of his centroid shift theorem [4]) that the vertical gravity force through its centroid C is equal to the incremental buoyancy force, due to the inclination, through B, but opposite in direction so that they form a couple or righting moment, tending to bring the body back to the upright position. Thus this shape for a solid of this specific weight is in stable equilibrium.

Although this derivation holds only for the homogeneous solid paraboloid and is limited to the incremental righting moments contributed by the immersed and abovewater parts, respectively, it can be shown that a similar reasoning can be developed for a solid of any shape and with non-homogeneous mass distribution, hence also for ships. The lever arm between the buoyancy and displacement forces in the paraboloid is thus the ancestor of the "righting arm", which today is conventionally used for the same couple of forces in modern ship stability analysis. Positive righting arms are a necessary condition of upright stability.

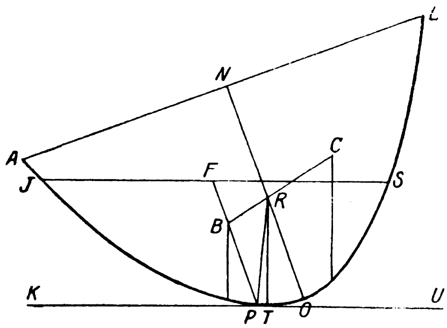

Fig. 3. Inclined Paraboloid (from [6]).

Achievements and Deficits: Archimedes thus laid the physical foundations for ship hydrostatics. He defined the resultants of buoyancy and displacement and pronounced the equilibrium principle of their equality in the same line of action and in opposite directions. From moment equilibrium he deduced a measure of hydrostatic stability by introducing the concept of righting moments based on the couple of buoyancy and displacement. This has remained the physical basis for judging the floating ability and stability of ships in design and operations. To evaluate ship properties at the design stage and during ship operations some further information is required:

– A reliable, complete *hull form definition*, in whatever medium (mould, model, drawing etc.).
– A method to calculate the volume and volume centroid of the underwater hull (center of buoyancy), for both the upright and inclined positions.
– A practical scheme to determine the centers of gravity of the ship's parts and therefrom the aggregate center of gravity of the entire ship.

– Criteria to assess the required safety margins in ship stability for different operating conditions and environments.

According to the available historical evidence Archimedes was not yet able to meet these additional requirements. Thus in the practice of ship design for many centuries the estimate of the ship's floating condition (draft and trim) and stability remained a matter of empirical judgment and approximation.

As for the calculation of volumes and their centroids Archimedes frequently applied the method of Eudoxus (410–356 B.C.), later known as the method of exhaustion (Boyer [7]), to simple shapes. In this scheme a polygonal approximant to the curved surface (or curve) is constructed and successively refined until the error drops below a given bound. The approximation becomes as close as desired, but falls short of calculus for lack of a limiting process to the infinitesimal [7]. Despite that, a numerical approximation for ship geometries might have been constructed on similar grounds, even in antiquity. But a continuous, arbitrarily refinable hull form definition was not available to Archimedes and his generation.

Some claim that Archimedes may have been involved in the conception of the famous *Syrakosia,* the giant ship ordered by Hieron, the ruler of Syracuse, or may have helped with engineering calculations, as e.g., suggested by Pomey and Tchernia in [8]. Bonino [9] has performed a thorough reconstruction or redesign of the vessel, based on the limited data, and arrives at a size of ca. 3000 tons of displacement and principal dimensions of length x beam x draft = 80x15.5x3.9 m. He has also built a realistic model replica illustrating the feasibility of this design. He concludes from the overall context of the shipbuilding methodologies of that period that Archimedes was not directly involved in any responsible design decisions although he may have acted as a consultant and advisor to Archias and Hieron. I share this cautious opinion.

History of Archimedes' Manuscripts

Today only 12 of Archimedes' treatises are preserved, several more existed in antiquity. They stem from Greek copies of his manuscripts and Latin translations. The adventurous history of these texts has been thoroughly researched by Heiberg [10] and updated by Dijksterhuis [11]. Clagett [12] has carefully examined the mediaeval reception of Archimedes. The story of a recent rediscovery of a palimpsest with Archimedes' texts is told by Netz and Noel [13]. This short survey will concentrate on events relevant to the history of "On Floating Bodies" (OFB). Much more detail is given by Nowacki [4].

The path of the preserved manuscript copies has followed a circuitous route. In essence only three master copies in Greek have existed, all stemming from mediaeval Byzantine sources in Constantinople, where Greek clerics collected the remainders of Archimedes' dispersed works in the 9th c. and later took some along into exile to Sicily under Norman and Hohenstaufen rule. They were numbered Codices A, B and C by Heiberg [10].

Codex A: In the papal libraries after 1266 with seven treatises by Archimedes, *but not OFB.* Copied several times, but the master irretrievably lost by 1564.

Codex B: A Latin translation of 1269 prepared by Willem van Moerbeke, a Flemish Dominican monk and papal translator. This translation was based in part on Codex A, in part on another Greek master *with OFB*, then existing in the papal archives, but soon lost. Thus Codex B contains OFB in Latin. This text formed the basis of several Renaissance humanist reprints after 1500, above all a brilliant reconstruction by Commandino [14] (1565) with both books of OFB. Commandino purged the text of apparent errors, removed some lacunae and completed missing arguments in proofs. This version became the most respected reference after the Renaissance (Clagett [12]). After 1600 many other editions followed in Greek, Latin and modern languages (Dijksterhuis [11]).

Codex C: Incredibly, a third Greek master copy was discovered in a Greek monastery in Constantinople in 1899 in a palimpsest, which contained the rinsed off and scraped off Archimedean text under a 13th c. prayer book, but still barely legible under a magnifying glass. This document was inspected, photographed, transcribed and immediately translated by Heiberg [15]. It did contain the only preserved Greek versions of both books of OFB. Codex C was lost during the Greek-Turkish wars in 1920–22, but resurfaced at an auction in New York in 1998, where the anonymous bidder who acquired it gave it to the Walters Art Museum in Baltimore for scientific reevaluation [13].

Thus OFB was accessible to scientists in Latin and increasingly in modern language translations since about 1600 and in the original Greek transcription by Heiberg [15] since 1907.

3. TOWARD APPLICATIONS

Late Antiquity and the Middle Ages

While the knowledge of Archimedes in hydrostatics lay dormant for at least a millennium in late antiquity and the early Middle Ages, shipbuilding technology did advance and underwent significant changes. Little specific is known about ship design methodology in antiquity, though it is evident in view of the complexity of some major shipbuilding projects that methods of advance planning and design must have existed (Pomey [16]). Archaeological sources from late antiquity show traces of prevailing practices, and later excavations of ship wrecks give indications of some basic reorientation in shipbuilding and ship design in the transition to the Middle Ages. This includes:

Marking: Bockius [18] excavated several Roman shipwrecks dating from the 4th c. A.D., which lay buried in the silt of the harbour and river-bed of the Rhine at Mainz, evidently river patrol boats of the Roman occupation period. He searched for traces of the shipbuildung process and found several transverse grooves on the inside of the keel planks, each sawed about 3 mm wide and arranged in uniform distances, as well as the remains of treenails or wooden pegs in the keel and side planking, carefully aligned in the same transverse planes. Since the hulls were built planks-first, he interpreted these findings as evidence of an assembly process, i.e., as layout markings and attachment points for template fixtures to hold the planks in place during assembly, but later to be removed to make room for transverse ribs as passive frame reinforcements in the same planes. This suggests that the idea of shape predefinition in transverse planes may have already existed in plank-first shipbuilding.

Skeleton-first assembly: Rieth [19] carefully describes the archaeological evidence for the important transition from plank-first to skeleton-first shipbuilding in the Mediterranean countries occurring during the 7th c. Here a skeleton of structurally active frames was erected in numerous transverse planes before the planking of the outside shell was attached to it. This necessitated a reorientation of the hull shape design process defining the desired shape in terms of planar transverse sections.

Moulding and lofting: While the use of templates or moulds for defining the shape of individual planar ship parts may be ancient, the use of unique master moulds, say, for the midship frame, from which all other transverse section shapes can be derived by a lofting process, was a new idea, apparently introduced in France just before 1300 (Rieth [20]). The individual section shapes at any longitudinal station of the ship can thus be deduced from the master mould (in French: Maître gabarit) by a transformation consisting of translation, rotation and clipping of shape elements (Nowacki [21]). Thus the shape of a single curve is sufficient to define the hull surface continuously at any desired point (except for the ship ends). This opens the door to the required volume and centroid calculations for ship hydrostatics.

Venice and the Italian Renaissance

During the Middle Ages and Renaissance Venice was a leading sea power in the Mediterranean and also a productive shipbuilding center, well-known for its Arsenal where many famous galleys were built. The contemporary written records on this shipbuilding activity are scarce. The earliest preserved documents stem from Michael of Rhodes (ca. 1435, McGee [22]) and Trombetta de Modon (ca. 1445). These are technical notebooks, written chiefly for specialists, with many illustrations, but little text. It was only much later by Drachio (1598, [23]) that explanations and commentaries were added that helped to understand this technology. In essence the Venetians had their own moulding and lofting techniques, based on a master mould (sesto) and rules for deducing section shapes at any desired longitudinal station (Alertz [24]).

The methods of lofting for ship parts were similar to those applied in other Mediterranean yards in Italy, France and Spain. They ensured a unique definition of hull shape and efficient fabrication of ship parts, allowing room for shape variation. The written sources deal chiefly with ship geometry, but do not make reference to design calculations, let alone to any thoughts from Archimedes in OFB.

This is disappointing since it was essentially during the same period that Italian humanists rediscovered OFB and made access to this classical knowledge feasible again. Van Moerbeke's translation (1269) and Commandino's brilliant revision [14] (Venice, 1565) were already mentioned. There are other indications that the ideas of buoyancy and displacement were at least intuitively known. Alberti, e.g., the famous Renaissance architect and writer, in one of his main works "De re aedificatoria" (ca. 1450), Book V, Ch. 12, alludes to his knowledge of the equality of buoyancy and displacement, at least for the cargo carrying capacity as an increment, though without giving any source. Leonardo likewise knew certain fragments of Archimedean thought. But in both cases they may also have run across some popularized pseudo-Archimedean text that was around since the 13th c. (Clagett [12]).

The Treatisers

Toward the end of the 16th and throughout the 17th c. a tradition developed in all major European seafaring nations to document the existing and evolving shipbuilding knowledge, whether practical or more theoretical, in more or less learned treatises for diverse purposes. The authors are often called treatisers. The treatises served as technical notebooks for insiders, as basic introductory texts for the general public or for the shipping community or even just as an opportunity to display scientific and technical excellence. The authors came from shipbuilding practice or from some scientific background or were knowledgeable in both aspects. In view of the rapid transitions in Europe to new technologies and methodologies during this very period the treatises are most valuable as contemporary sources on the intensive changes in practical and scientific knowhow. We will take a short survey of the major sources in our search for traces of Archimedean heritage. See Barker [25] for a more detailed overview. We will note the dates of treatise appearance in parentheses.

Portugal, riding on the wave of success of the age of exploration and possessing a strong position in shipbuilding, was also among the first to produce naval treatises. Pedro Nunes, a scientist, studied the theory of rowing ("O Comentario de Pedro Nunes à Navegação a Remos", 1566) and criticized the errors in Aristotle's approach in Problemata Mechanika (wrong conclusions from the law of the lever). Such publications challenged the scholastic dominance of Aristotle and paved the way for Archimedean thought.

On the practical side the shipbuilding treatises by Oliveira ("Ars nautica", 1570, "Livro da Fabrica das Naos", 1580), Lavanha ("Livro primeiro da architectura naval" [26], 1614–1616) and Fernandes ("Livro da Traças de Carpinteria", 1616) deserve to be noted. They deal essentially with ship geometry, moulding rules and ship construction. Lavanha cites Vitruvius and Alberti as precursors and raises the naval architect to comparable rank as the famous architects. He develops precise ship drawings and sketches. Fernandes already presents a rudimentary ship lines plan. These Portuguese sources contain no hydrostatic calculations or references to Archimedes.

In England William Bourne ("Treasure for Travaylers", 1578), one of the first treatisers there, already explains how to obtain a ship's volume estimate by taking its offsets when on dry ground by means of measuring rods relative to some suitable reference plane on the outside of the hull and up to the desired waterline. The offsets are then connected by linear approximants for estimating cross-sectional areas and likewise linearly volumes of ship segments between measured stations. In the end a reasonably rough volume estimate is obtained to which the Principle of Archimedes is applied to derive the ship's weight (or displacement) on that draft.

Other famous early treatisers (Mathew Baker/ John Wells: "Fragments of Early English Shipwrightry", 1570–1627, see Barker [27]; R. Dudley: "Arcano del Mare", 1646; E. Bushnell: "The Compleat Ship-Wright", 1664) deal chiefly with ship geometry, moulding methods and ship drawings up to first lines plans on paper. But they did not yet enter into Archimedean style calculations. However Anthony Deane ("Deane's Doctrine of Naval Architecture", 1670) resumed the subject of volume estimates by approximate planimetry of section areas, using circular arc or triangular approximants, and segment volume calculation. He thus obtained the ship's buoyancy force (= displacement) according to Archimedes for any desired draft. One motivation apparently was to provide enough freeboard to keep the gunports above water.

In Germany Joseph Furttenbach, a well-known architect and writer from Ulm, had traveled much in Italy as a young man and had picked up the basic naval architecture knowledge there. His treatise "Architectura navalis" (1629) concentrates on matters of ship geometry, ship construction and ship types, showing strong Italian influence, but not on hydrostatic calculations. His work remained rather solitary in Germany.

Early French treatisers (Fournier: "Hydrographie", 1643, Pardies: "La Statique ou la Science des Forces Mouvantes", 1673) were interested in nautical matters for textbooks in seamanship. It was actually the Jesuit Père Paul Hoste who first took on the challenge of calculating the displacement from lines plans (or offset measurement not unlike Bourne) and of defining a measure of ship stability on hydrostatic grounds, based on Aristotle and Archimedes. Unfortunately his stability analysis failed because he misinterpreted Archimedes' derivation and missed the effects of the shift of volume centroids by heeling inclination.

In the Netherlands, based on the pioneering work by Simon Stevin (1548–1620) to be discussed later, there existed an early understanding among practitioners for the principles of hydrostatics stemming from Archimedes. The Dutch mathematician Johannes Hudde (1628–1704) had proposed a method (1652), later called the difference-in-drafts method, for measuring the cargo payload (or tonnage) by taking the difference between the ship's displacement fully loaded minus empty. Offsets were taken in both floating conditions and the volume of the layer between the two waterlines was estimated numerically by means of trapezoids and triangles. The volume of this layer was converted to weight by Archimedes' Principle. In Britain Bushnell (1664) devised a similar technique.

Nicolaes Witsen (1641–1717) in his treatise [28] worked out a similar method (1671) in more detail, but also extended it to estimating the displacement for the whole hull. Certain details cast doubt on whether this method was ever practiced. Witsen also explicitly gives credit to Archimedes. For ship stability he follows Stevin, whose criterion was flawed (see below).

Cornelis van Yk in his treatise "De Nederlandsche Scheeps-Bouw-Konst Open Gestelt" (1697) cites Witsen , but as a practitioner has a more practical orientation. He pursues the method of difference–in-drafts for applications in tonnage measurement.

This short survey has been confined to traces of a growing understanding in Archimedean ship hydrostatics. Much more detail on the treatisers and their work is found in Barker [25] and Ferreiro [29].

In summary it is fair to state that by 1700 Archimedes' texts were known to scientists, but very little of his knowledge had found its way into ship applications. As for ship stability his criterion was not yet properly understood, let alone practiced in ship design or operations.

The Rebirth of Hydrostatics: Stevin, Galileo, Huygens

During the 17th c. the scientific discipline of hydrostatics was virtually reborn in a modern reincarnation. Although access to Archimedes' texts had much improved by 1600 so that scientists were able to study him literally, it took a number of very creative thinkers and physicists to reinvent hydrostatics and hydrostatic stability on new fundamental grounds and to apply it to their own new applications. Such prominent scientists as Stevin, Galileo, Huygens and Pascal made important contributions to this rebirth.

Simon Stevin (1548–1620), the famous Flemish/Dutch mechanician, astronomer and hydraulic engineer, worked on several fundamental problems of mechanics and also reestablished hydrostatics. He introduced the concept of hydrostatic pressure, which the Greeks had not known, and thus was able to determine hydraulic loads acting on submerged surfaces. He axiomatically developed a body of propositions embracing the whole of hydrostatics in his treatise "The Elements of Hydrostatics" [30] (1586 in Dutch, 1608 in Latin translation). His premises are tantamount to the Archimedean properties of the fluid. In a fluid at rest the hydrostatic pressure increases linearly with depth in proportion to the specific weight of the fluid. This was a brilliant breakthrough. Stevin also dealt with the stability of ships in his supplement "On the Floating Top-Heaviness", attached to [31], 1608. He had read Archimedes and praised him. But he had not fully understood the implications of the hydrostatic stability criterion so that he missed the influence upon stability of the volume shift from the emerging to the immersed side of the heeling ship, a stabilizing effect. Consequently he came to the erroneous conclusion that the ship's center of gravity *must always* lie *below* the center of buoyancy for a stable ship. Actually this is a sufficient, but not a necessary condition for ship stability.

Galileo Galilei (1564–1642), famous as an astronomer and physicist, especially in mechanics and strength of materials, also occupied himself with hydrostatics and its applications, which is less widely known. In fact, in 1612 he published a treatise "Discourse on bodies in water" [32], which is explicitly founded on Archimedean thought and deals with the floating of bodies on the surface of the water. This work originated from a dispute with Aristotelian opponents in Florence about the causes of buoyancy of floating objects [33]. Galileo accepted the Archimedean axiom that a body floats on the water surface if it is specifically lighter than water, but denied the Archimedean Principle for the equilibrium position. Rather he soon drifted off beyond Archimedean hydrostatics by recurring to kinematic principles from his theory of motion. His main contribution from this dispute therefore must be considered to lie in refuting the false Aristotelian theory of buoyancy.

Christiaan Huygens (1629–1695), the famous physicist, is little known for his excursion into hydrostatic stability. He never published his three volume treatise "De iis quae liquido supernatant" [34], which he wrote in 1650 at the youthful age of 21, because he regarded it as incomplete, and later (1679?) as "of small usefulness, if any, although Archimedes in Book II of 'On Floating Bodies' spent work on not dissimilar topics". Incidentally it is reported that Huygens used Commandino's version of the Latin translation of Archimedes, based on codex B. He wanted most of this work of his to be burnt. The manuscript was found in his legacy and was first published in 1908 [34]. The modern reader is bound to admire Huygens' deep insights into Archimedes' work as much as his own creative extensions. Huygens rederived Archimedes' results for the stability of the sphere and the paraboloid using his own method and he provided original solutions for floating cones, parallelepipeds and cylinders. He studied some of these solids through a full cycle of rotation. He recognized that for homogeneous solids their specific weight and their aspect ratio are the essential parameters of hydrostatic stability. In conclusion Huygens was the first modern physicist who understood and was able to apply and extend Archimedes' theory of hydrostatic stability. He did not proceed to apply his theory to ships or similar floating objects because he did not possess a suitable definition of ship geometry, a final obstacle.

Calculus

Archimedes skilfully and routinely used a method of geometrical proof in the derivation of areas, volumes and centroids of figures of simple given shapes, which later became known as the "method of exhaustion", usually attributed to Eudoxus, a pupil of Plato (cf. Boyer [7]). Here a known curve is approximated by a regular polygon whose edges are subdivided successively, doubling the number of edges in each step, until the error between the curve and the polygon becomes as small as desired. After a finite number of steps the remaining error is estimated and the sum of the finite series with truncation error is taken. This proposed result is then confirmed by reductio ad absurdum of any differing assertions.

However the method of exhaustion is not equivalent to integration by infinitesimal calculus (cf. Boyer [7]). It is limited to a finite sequence of steps and relies on geometrical constructions for the proof. It cannot easily be extended to objects of arbitrary shape like ships. It does not comprise the limiting process of calculus. Calculus is based on the concept of an infinite series and derives its results analytically. Thus calculus can be applied to any analytically defined shape, hence also to ships of given arbitrary shape.

The invention of calculus had many precursors and contributors (cf. Boyer [7]). But it was the achievement of Newton and Leibniz to lay the foundations for consistent and procedurally well defined methods of calculus. These methods spread in Europe during the first few decades of the 18th c. Thus when the problems of ship hydrostatics after 1730 were revisited by two leading scientists, Bouguer and Euler, they had the mathematical tools at their disposal to reformulate the auxiliary quantities of areas, volumes and centroids in Archimedes' approach in terms of the elegant and definitive notation of calculus. Developments had now reached the stage where a reformulation of Archimedean hydrostatics as an application of continuum mechanics had become feasible and was at the threshhold of its application to ships.

Bouguer and Euler

After the advent of calculus and with the new concepts of analysis and of functions of one or several variables it became possible to review and restate many classical problems of mathematics and mechanics in new, original ways. For ship hydrostatics and stability the credit for a new, completely modernized approach, based for the first time on calculus, goes to two contemporary scientists, Pierre Bouguer (1698–1758) and Leonhard Euler (1707–1783), who worked on these problems separately, independently and without knowing of the other's work before their own large treatises were completed and ready to be published. Their original work can be well dated because they both participated in a prize contest held by the Parisian Academy of Sciences in 1727 on the optimum masting of sailing ships, where hydrostatics might have played a useful role to determine the equilibrium position of ships under sail. But they both failed to display any knowledge of Archimedean hydrostatics. However they continued to work on this issue during the 1730s, Bouguer essentially during a scientific expedition to the Andes in Peru from 1735 to 1744, Euler as a member of the Russian Academy of Sciences in St. Petersburg from 1737 to 1741.

Bouguer's famous "Traité du Navire" [35] appeared in 1746 soon after his return to France, Euler's fundamental "Scientia Navalis" [36] was published in 1749 after a major delay. But it is undisputed, also by both authors, that they had achieved their results independently and unaware of the other's parallel work (cf. Nowacki and Ferreiro [37]). Their results are essentially equivalent, though expressed in uniquely distinct ways, and have remained a valid basis for ship stability until today.

Bouguer in his pioneering treatise nowhere mentions Archimedes by name, but the spirit of his formulations leaves no doubt that he was familiar with Archimedes' work. e.g., he began his introduction of hydrostatics in Book II, section I, chapter I with this explanation of the buoyancy force:

"The principle of hydrostatics, which must serve as a rule in this whole matter and which one must always have in mind, is that a body that floats on top of a liquid is pushed upward by a force equal to the weight of the water or liquid whose space it occupies".

This is tantamount to the Principle of Archimedes, only slightly rephrased. In the following chapter the same result is also derived by integration of the hydrostatic pressure distribution over the submerged part of the hull surface. The pressure resultant or buoyancy force is then shown to be acting upward through the volume centroid of the submerged hull (or center of buoyancy), equal and opposite to the downward weight force (displacement) through the center of gravity of the hull.

For ship stability for infinitesimally small angles of heel (initial stability) Bouguer invented the metacenter as a stability criterion, i.e., the point of intersection of two infinitesimally adjacent buoyancy directions for a small angle of heel, the point g in Fig. 4. For a stable ship the center of gravity of the ship must not lie above the metacenter. This is a brilliant reinterpretation of Archimedes' stability measure for small angles of heel in terms of a geometric bound. Bouguer evaluated volumes and centroids for this measure by calculus and numerical approximation, also relying on Archimedes' centroid shift theorem.

Euler in the introduction to his "Scientia Navalis" pays full tribute to Archimedes. He begins his axiomatic foundation of hydrostatics with the statement:

"The pressure which the water exerts on a submerged body in specific points is normal to the body surface; and the force which any surface element sustains is equal to the weight of a vertical water column whose basis is equal to this element, whose height however equals the submergence of the element under the water surface".

All other results in ship hydrostatics can be derived from this axiom. e.g., the buoyancy force in the Principle of Archimedes is deduced by pressure integration by means of calculus over the hull of an arbitrary body shape. Euler also applies Archimedean criteria to the hydrostatic stability of ships for infinitesimal angles of heel when he says:

"The stability, which a body floating in water in an equilibrium position maintains, is measured by the moment of the restoring force if the body is inclined from its equilibrium position by a given infinitely small angle".

This stability criterion is formulated in terms of righting moments as by Archimedes, but unlike Bouguer. Physically the two formulations are equivalent. Euler calculates the righting moment taking into account the volume shift from the emerging to the immersed side and using Archimedes' centroid shift theorem. Figure 5 for an inclined cross section shows the stabilizing effect of this volume shift, caused by the couple of gravity force through G and buoyancy force through the new shifted center of buoyancy.

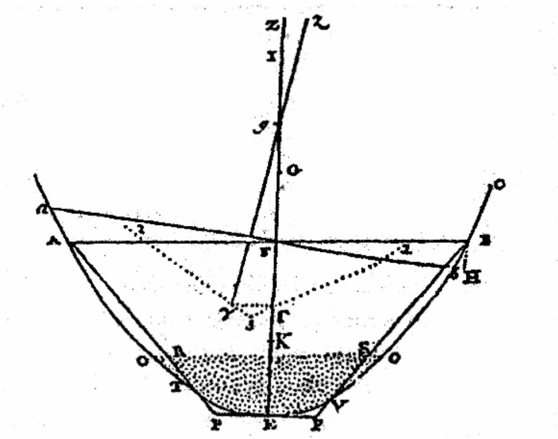

Fig. 4. Bouguer's Figure for the Derivation of the Metacenter (from [35]).

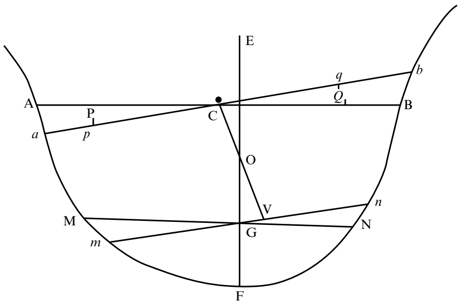

Fig. 5. Euler's Figure for Centroid Shift in An Inclined Cross Section (from [36]).

Note that both authors, Bouguer and Euler, have also addressed stability measures for finite angles of heel. Furthermore both also treated numerous other applications of ship stability in ship design and operations, e.g., ship loading and unloading, seaway motions, maneuvering under sail etc. which become amenable once the hydrostatic restoring reactions of the ship are known. Many more details about their achievements are presented by Ferreiro [29], especially on Bouguer, and by Nowacki [38], mainly on Euler.

Thus Bouguer and Euler have shown that the practical application of sability criteria to arbitrary ship shapes has been made possible by means of calculus formulations and their numerical evaluation.

After the publication of these fundamental treatises in ship theory Bouguer's results soon were widely distributed, his French text, augmented by numerical examples, was readily understood, textbooks with his methods were soon prepared for colleges in France, and the French Navy soon made stability assessments by the metacenter criterion an official requirement. Euler's Scientia was written in Latin, it was not widely circulated to practitioners, lacked numerical examples, and therefore remained relatively unknown in shipbuilding practice, though it made its mark on future scientific developments.

Chapman and Atwood

The Swedish naval constructor and scientist Frederik Henrik af Chapman (1721–1808) is the first and best witness for the actual application of the knowledge created by Archimedes, Bouguer and Euler being applied in actual ship design, construction and operation. Chapman, son of an English shipbuilder and immigrant to Sweden, grew up in an environment of practical shipbuilding and scientific openness. As a young man, practically trained and mathematically inclined, he spent a few years in England, Holland and France in a sort of "apprenticeship", picking up not only some practical trade skills, but also the scientific knowhow then available in those leading shipbuilding nations. He became familiar with the work of the Bernoullis, Bouguer and Euler, and hence with the Archimedean tradition. After his return to Sweden in 1757 he soon acquired much responsibility in Swedish naval and merchant ship design, rose to high rank and remained in a leading position throughout his lifetime. At the same time he not only practiced his scientific insights in his own actual designs, but also developed an ambition to publish his fundamental assumptions and conclusions in scientific treatises, foremost in his "Treatise on Shipbuilding" [39]. This gives us an intimate insider view of his use of scientific knowledge in practical design. Chapman made it a routine matter to calculate the displacement and a stability measure, the metacenter, at the design stage for every ship. For numerical integration he used the efficient quadrature rules acquired in private lessons from the mathematician Thomas Simpson in London. Chapman was also an excellent hull shape designer and lines plan draftsman. Thus he knew and discussed in his treatise how to influence hull shape design so as to achieve appropriate centroid locations and metacentric height. He also gave recommendations for the placement of ballast and cargo in ship operations in order to secure sufficient stability, but not too much metacentric height to avoid rough motions at sea. Further he had certain techniques for estimating the heeling moments by wind in sails, as already proposed by Bouguer and Euler, to derive the required safe margin for righting moments. Thus he rounded off the available physical knowledge, based on Archimedean thought, by further elements needed for safety assessment in practical design. This completed a cycle of 2000 years from the basic theoretical insights to practical applications.

Thomas Atwood (1745–1807), an English physicist and mathematician, assisted by the French naval constructor Vial du Clairbois, just before the end of the 18th c. added another missing piece to the puzzle of ship stability: They recognized that the initial stability for small angles of heel was not sufficient to ensure the ship's safety [40], as of course Bouguer and Euler had also already suggested, but they also proceeded to investigate the ship's righting moments at finite angles of heel, as Archimedes had done for the paraboloid. They used numerical quadrature rules again to calculate the "righting arm" of the vessel for a given draft, center of gravity and angle of inclination. Atwood also pointed out the nonlinear character of this function of heeling angle, which makes the rolling ship a nonlinear system. Thus by 1800 all prerequisites for hydrostatic displacement and stability calculations were available in practice when entering into the age of steam-driven steel ships.

4. CONCLUSIONS

It took about two millennia before the fundamental theories of Archimedes in hydrostatics were actually applied in the practice of ship design and operations. Archimedes laid the physical foundations for this technical purpose, but a number of other knowledge elements were still missing before this crucial assessment of ship safety could be performed on sound theoretical and practical grounds. Moreover access to Archimedes' manuscripts was interrupted for many centuries. The solution required further insights in hull geometry definition, mathematical analysis and data for physical criteria.

Geometry: While Archimedes still adhered to simple geometrical shape definitions, practical shipbuilding technology made many steps forward in hull geometry definition during subsequent centuries. After the skeleton-first construction principle was introduced (around A.D. 700), the use of moulds and in fact single master moulds for the whole ship became feasible (around 1300). Lofting ship parts only in the mould loft was then followed by drafting ship lines plans on paper (since about 1600) with more flexible construction rules [21]. In the 18th c. in addition analytical representations of hull geometry evolved, derived from offset data or form parameters. This much facilitated the numerical calculation of hull form features, especially areas, volumes and centroids. Thereby arbitrary hull shapes could be evaluated hydrostatically.

Analysis: Simple shapes in antiquity were often treated by the method of exhaustion to obtain area, volume and centroid information. But this approach had its limits when dealing with arbitrary practical hull shapes. Although numerical quadature rules were known and used to estimate tonnage since the Middle Ages and later applied to displacement calculations by some treatisers (by 1700), it is owed to the advent of calculus (by 1700) that first analytical and then numerical evaluations of all integral properties of ships could be performed for arbitrary hull shapes (following Bouguer and Euler after ca. 1750). Stability analysis for the metacenter also benefitted from the analytical concept of the center of curvature of a curve (Bouguer: Metacentric curve as an evolute).

Physics: Archimedes had elegantly derived the resultant hydrostatic force of buoyancy without resorting to effects in the liquid. An important alternative, the hydrostatic pressure, was introduced by Stevin (by 1600) so that hydrostatics could be newly developed from the viewpoint of continuum mechanics. This became the dominant basis of modern developments, also by Bouguer and Euler. This facilitated the expression of hydrostatic effects by calculus. Stability criteria for small angles of heel were thus stated in terms of infinitesimals (by 1750), while righting arms for finite angles of heel were formulated by means of calculus and evaluated numerically (by 1800).

Many further developments and insights were added to stability analysis during the 19th and 20th centuries before the current advanced level of risk-based ship design was reached [41]. But the foundations of our safety assessments of ships until today still rest on the principles and theories first pronounced by Archimedes.

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