Exercise 1: Prestressed cross-section; pretensioned beam with bonded strands

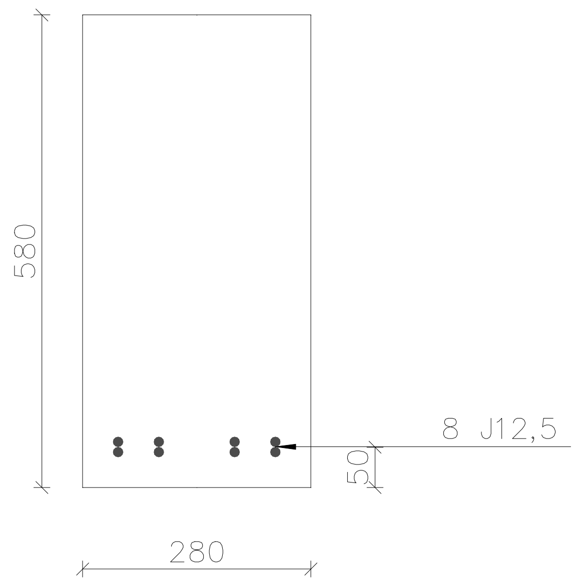
Calculate stresses due to prestressing and self weight at transfer for the beam in the figure

Beam: depth h=580 mmm
width b=280 mm

Strands: 8 ϕ_p 12,5 mm strands (A_p = 93 mm^2 /strand) grade 1600/1800
Elastic modulus of the strands E_p = 195000 MPa
prestress just before the transfer σ_{po} = 1317 MPa
Distance of the strands from the bottom fibre c=50 mm

Concrete: C40/50; mean concrete strengtn at transfer f_{cmi} = 0,75 f_{cm}
Elastic modulus of concrete at transfer E_{cmi} = 32300 MPa

Length (span) of the beam L=10 m



DIMENSIONING OF THE PRESTRESSED STRUCTURE

1) Choosing the cross-section dimensions and the required amount of reinforcement (ultimate limit state, $t = \infty$)

2) Choosing the required prestress force (service limit state, $t = \infty$)

3) Prestress loss calculation

4) Stress and cracking analysis (service limit state, $t = \infty$)

5) Ultimate limit state , $t = \infty$
- flexural resistance
- shear resistance
- torsion + combined actions

6) Lifting for precast unit

7) Deflection (service limit state, assembling and final stage $t = \infty$)

8) Anchorage and spalling at the end of the structure

9) Fire resistance

CHOOSING THE INITIAL PRESTRESS

1) Choosing the degree of prestress

- Fully prestressed structure
  The structure is fully compressed, no tension stresses
  water tight structures
  explosure class

- Partially prestressed structure
  Bridges ; no tension

  Buildings: usually tension stresses are allowed
  under quasi-permanent load cracking is allowed,
  the crack width is limited

EC 2 table 7.1 N(FI):
Requirements at the cracking limit state depending on the exposure class

2) Choosing the initial prestress
- Code limitations:
  EC2: $\sigma_{p0} \leq 0{,}8\ f_{puk}$ or $0{,}9\ f_{p,0,1k}$

- The required balanced forces

- Cracking (or tension stress) in the tension side during prestressing stage, transfer (release) stage or lifting stage before any imposed loads

- Resistance of compressed area during prestressing stage, transfer (release) stage or lifting stage before any imposed loads

- Typically the initial prestress is σ_{p0} = 1000 ... 1300 MPa

- Precast prestressed structure with bonded strands:
  usually the whole main tensile reinforcent is prestressed strands

Low prestress - disadvantages:
- cracking
- problem with strain about flexure
- prestressing steel is not optimically used
  Yield force of ϕ 12,7 (99 mm^2) corresponds reinforcing bar T 20 (A500HW)

High prestress – disadvantages:
- upward deflection (camber) is great
- much debonding is required
- much spalling reinforcement at the end of the structure
- too high prestress may cause prestressing work more difficult

Estimation of the prestress losses

Influencing factor	**Influence**	High losses	Small losses
$M_{quasi\text{-}perm}/M_{total}$	creep	$M_{quasi\text{-}perm}/M_{total}$ small	$M_{quasi\text{-}perm}/M_{total}$ high
Conrete stress σ_{cp} at the point of prestress tendons	creep	σ_{cp} high	σ_{cp} small
Cross-section	shrinkage creep	light, slender, thin	thick, massive
Initial prestress σ_{po}	relaxation	σ_{po} high	σ_{po} small

Typically the prestress losses are:

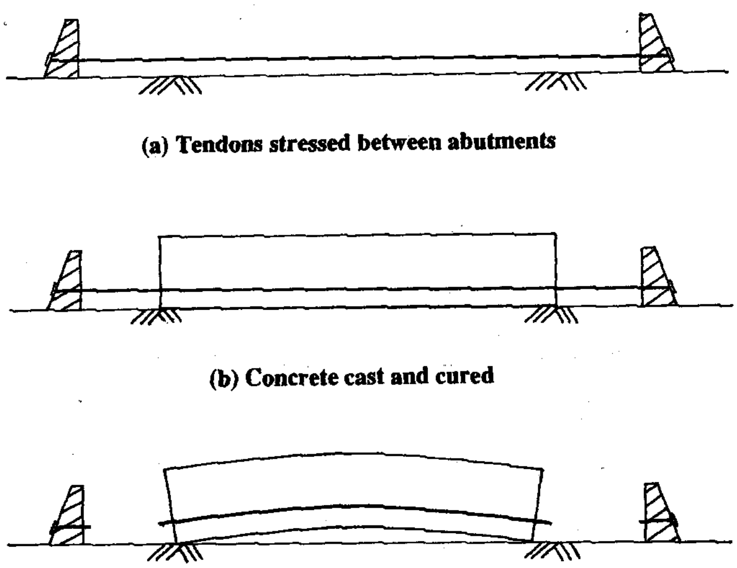
pre-tensioned structure: 15 ... 25 %, normally about 20 %

post-tensioned structure: 10 ... 20 %, normally about 15 %

ANALYSIS OF PRESTRESSED CROSS-SECTION

Presstressing means that the prestressing reinforcement has stresses before loading of the structure.

1. PRETENSIONED STRUCTURE WITH BONDED STRANDS



(a) Tendons stressed between abutments

(b) Concrete cast and cured

(c) Tendons released and prestress transferred

Pretensioning procedure.

- Strand are tensioned against the anchors of the bed before casting the concrete
  Usually strands are straight

- When the strength of concrete is developed to the required strength (transfer strength, normally about 65 … 75 % of nominal strength of concrete) the strands are released from the anchors by cutting (sudden release) or by hydraulic jack (gradual release)

- Because of the transfer strength there is bond between the strands and concrete; so the deformation (strain) of the strand is the same as the deformation of concrete at the same point.

- When the strand is released it tends to shorten but the shortening is mostly eliminated by the surrounding concrete due to bond between the stand and concrete
  ⇨ this means that the counterforce of the pre-tensioned force of the strand transmits from the anchor to the concrete as a compressive force.

- Due to bond between the strand and concrete the structure function for a normal imposed compressive force like a reinforced concrete section where the strand function as a reinforcement.

The compressive counterforce of the prestress force is distributed between concrete cross section and the strand so that concrete (at the level of the strand) and strand get the same compressive strain due to the normal force.
The force of the strand decrease due to this elastic shortening strain.

Initial before the transfer: The counterforce of the prestressing force of the strands acts as a compressive normal force to the cross-section composed by concrete and the strands (transformed cross-section)

Prestressed strand has tensile prestressing force P and the stress

$$\sigma_{po} = \frac{P}{A_p}$$

Counterforce of the strand is affected to the anchor blocks.

After the transfer: Concrete cross section gets compressive force which is the counterforce of the prestressing force of the strand
Compressive force transfers to concrete by bond between the strand and concrete
Due to bond the strand gets the same compressive strain as concrete at the level of the strand =>
Prestressed strand gets compressive force which reduce the initial force of the strand
Equilibrium: Resultant of the concrete stresses + reduced force of strands = 0

For imposed loads: Structure function as an uncracked reinforced cross section.
Stresses are distributed between concrete cross section and the strands so that they have the same change of strain at the same level.
Strands act like a normal reinforcement getting tensile stresses which eliminated the effect of the elastic shortening at the transfer.
Tensile stress of the strand will reduce the tensile stresses of concrete.

So the effect of the elastic shortening is partly reversible.

Due to the bond between strand and concrete the stress analysis is obtained using so called transformed cross-section where the strands and other reinforcement are taken into account with respect of the elastic modulies; it means the area of the strands are multiplied by the factor Ep/Ec-1, where Ep is the elastic modulus of the strands and Ec is the elastic modulus of concrete.

Ratio of the elastic modulies of strands and conrete $n_e := \frac{E_p}{E_{cmi}}$

Cross-section area $A_m := b \cdot h + (n_e - 1) \cdot A_p$

First moment about the bottom fibre $S_m := b \cdot h \cdot \frac{h}{2} + (n_e - 1) \cdot A_p \cdot c_p$

Distance of the centroid from the bottom $pp := \frac{S_m}{A_m}$

Second moment $I_m := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - pp\right)^2 + (n_e - 1) \cdot A_p \cdot (c_p - pp)^2$

Modulus about the bottom fibre $W_{ma} := \frac{I_m}{pp}$

Modulus about the top fibre $W_{my} := \frac{I_m}{pp - h}$

For the transfer situation the elastic module of concrete E_{mi} represents the transfer strength f_{ci}
=> A_{mi}, I_{mi} , W_{mai}, W_{myi}

P is the prestressing force just before the transfer
Prestressing load is treated in say manner as the normal compressive imposed load -P

Stresses:

Concrete:

$$\sigma_{cp} = 0 + \frac{-P}{A_{mi}} + \frac{-P \cdot y_p}{I_m i} \cdot y = 0 + \frac{-P}{A_{mi}} + \frac{-M_p}{W_{mi}}$$ (initial concrte stress before the transfer is 0)

Prestressing strand:

$$\sigma_{p,P} = \sigma_{po} + n_e \cdot \left(\frac{-P}{A_{mi}} + \frac{-P \cdot y_p}{I_m i} \cdot y_p \right)$$ (initial stress is σ_{po})

y_p is the distance of the prestressing strand from the centroid

The negative bending moment due to the eccentricity y_p of the prestressing force $M_p = -P \cdot y_p$

For imposed loads:
The elastic module of concrete E_{cm} represents the final strength of concrete f_c
=> the cross-section values should be calculated newly

Change of the stresses due to the imposed load N_{imp}, M_{imp}:

Concrete:

$$\Delta\sigma_c = \frac{N_{imp}}{A_m} + \frac{M_{imp}}{I_m} \cdot y = \frac{-P}{A_m} + \frac{-M_p}{W_m}$$

(initial concrete stress before the transfer is 0)

Total stress $\sigma_c = \sigma_{cp} + \Delta\sigma_c$
+

Prestressing strand:

$$\Delta\sigma_p = n_e \cdot \left(\frac{N_{imp}}{A_m} + \frac{M_{imp}}{I_m} \cdot y_p \right)$$

Total stress in the strand $\sigma_p = \sigma_{p,P} + \Delta\sigma_p$

BASIC ASSUMPTIONS

1. Compatibility condition
   => same strain in the reinforcement and concrete at the same level ($\varepsilon_s = \varepsilon_c$)
   => no slip between reinforcement and concrete
   => plane sections remain plain section under the loading
   => Bernoulli's assumption
   => strain ε at the certain distance y from the centroid can be obtainde by linear equation

$$\varepsilon = \varepsilon_0 + \psi \cdot y$$

where ε_o is the strain at the centroid due to a normal force
ψ is the curvature due to a bending moment
y is the distance from the centroid

2. Constitutive equations of concrete and reinforcement steel
   => stress-strain-relation
   => Hooke's law: $\sigma = E \cdot \varepsilon$ Concrete: $\sigma_c = E_c \cdot \varepsilon_c$
   Prestressing reinforcement: $\sigma_p = E_p \cdot \varepsilon_c$ ($\varepsilon_s = \varepsilon_c$)

3. Equilibrium equations
   => Stress resultant of concrete + stress resultant of reinforcement = imposed load

Concrete stress resultant $N_c = \int_A \sigma_c \cdot dA$

Moment of stress resultant about the centroid $M_c = \int_A \sigma_c \cdot y \cdot dA$

Reinforcement stress resultant $N_s = \sum \sigma_p \cdot A_p$

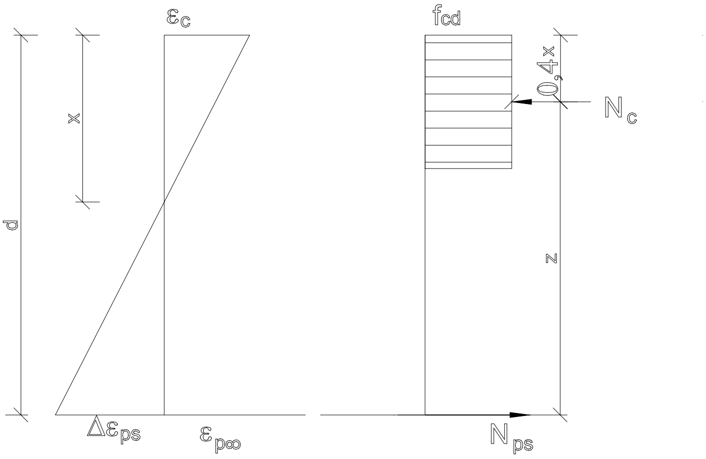
Moment of the reinforcement forces about the centroid $M_s = \sum \sigma_p \cdot A_p \cdot y_s$

A_p is the area of one prestressing strand
y_s is the distance of a strand from the centroid

Equilibrium: $N_c + N_s = N_{imp}$
$M_c + M_s = M_{imp}$

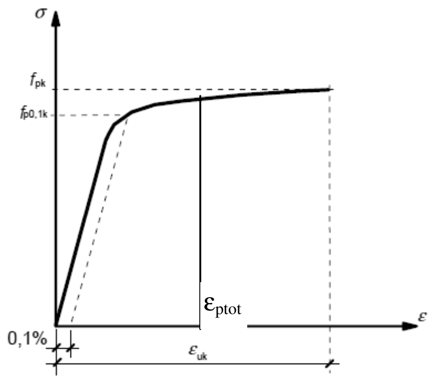
Ultimate limit state.

Due to bond between concrete and strands the strands get due to loading the same strain $\Delta\varepsilon_{\pi}$ as concrete at the level of the strands. The total strain int the strands is $\varepsilon_{ptot} = \varepsilon_{p\propto} + \Delta\varepsilon_{p}$.

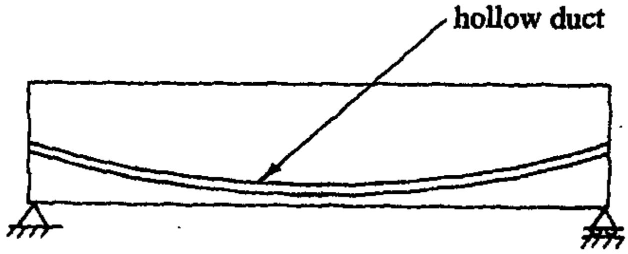


The strain due to prestressing $\varepsilon_{p\propto} = P_{\propto}/A_p$ where $P_{\propto}$ is the effective prestress after losses (loss due to elastic shortening excluded).

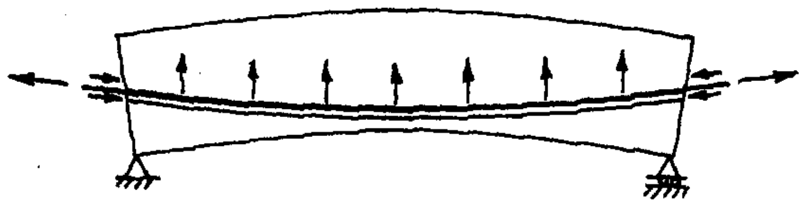
The stress of the strands is calulated form the stress-strain diagram of the strand with the totla strain ε_p



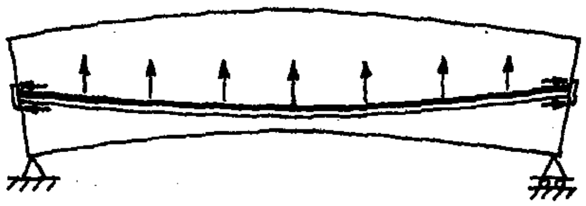
2. POST-TENSIONED STRUCTURE



(a) Concrete cast and cured



(b) Tendons stressed and prestress transferred



(c) Tendons anchored and subsequently grouted

Post-tensioning procedure.

- Ducts and anchors are assembled in the mould;
- Tendons composed by several strands are assembled is the ducts
- Tendons have usually parabolic curve form

- Concrete structure with the ordinary reinforcement is casted

- Tendons are tensioned when the strength of the concrete has been developed to the required value => post-tensioning;
In this situation the ducts are not yet injected => there is no bond between the tendons and concrete.
Tendons are tensioned against the anchors => Prestressing force goes to the concrete at the ends of the structure by the anchors

- Ducts are injected by mortar => when the mortar has hardened there exist bond between the tendon and concrete.

Tensioning: No bond between the tendon and concrete

The compressive counterforce of the prestressing force goes to the concrete only by the anchors

Cross-section is the net concrete cross section where the ducts forms the holes

$A_{net} = A_c - n \cdot A_{duct}$

Where A_{net} is the net concrete cross-section

A_{duct} is the area on one duct

n is the number of the ducts

Compressive force causes the elastic shortening of the structure and the distance between the anchors => the strain of the tendons decrease => force of the tendon decrease => prestressing loss due to the elastic shortening

Stresses:

Concrete:

$$\sigma_{cp} = 0 + \frac{-P}{A_{net}} + \frac{-P \cdot y_p}{I_{net}} \cdot y = 0 + \frac{-P}{A_{net}} + \frac{-M_p}{W_{net}}$$ (initial concrete stress before the transfer is 0)

Prestressing tendon:

$$\sigma_{p,P} = \sigma_{po} + \Delta\sigma_{p,el}$$ (initial stress is σ_{po})

y_p is the distance of the prestressing strand from the centroid

$\Delta\sigma_{p,el}$ is the loss of elastic shortening of the distance between the anchors

$$\Delta\sigma_{p,el} = E_p \cdot \frac{\int_0^L \sum \frac{n-1}{2n} \cdot \frac{\sigma_{cp}}{E_{cmi}} \cdot dL}{L}$$

The negative bending moment due to the eccentricity y_p of the prestressing force $M_p = -P \cdot y_p$

n is the number of the tendons

The average value can be used because there is no bond between the tendons and concrete at the time of prestressing.

For imposed loads: Ducts are injected

=> bond between the tendon and concrete

=> concrete gets tensile strain due to imposed loads

=> due to the bond the tendons gets the same tensile strain as concrete at the level of the tendon

=> transformed cross-section can be used for analyzing stresses from the loads affecting after the injecting the ducts.

The elastic module of concrete E_{cm} represents the final strength of concrete f_c
=> the cross-section values should be calculated newly without duct holes (ducts have been injected with mortar)

Change of the stresses due to the imposed load N_{imp}, M_{imp}:

Concrete:

$$\Delta\sigma_c = \frac{N_{imp}}{A_m} + \frac{M_{imp}}{I_m} \cdot y = \frac{-P}{A_m} + \frac{-M_p}{W_m}$$ (initial concrete stress before the transfer is 0)

Total stress $\sigma_c = \sigma_{cp} + \Delta\sigma_c$
+

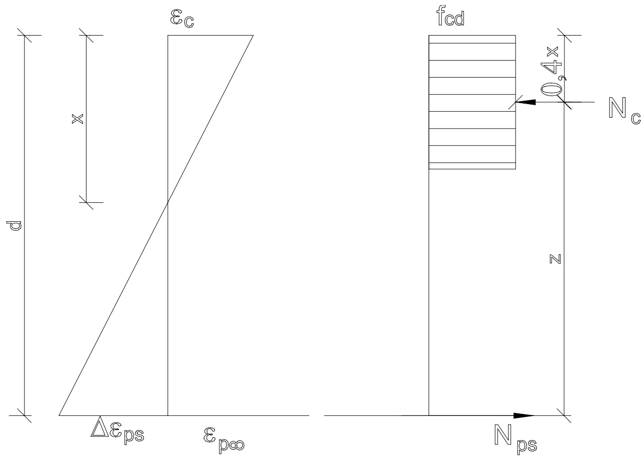
Prestressing strand:

$$\Delta\sigma_p = n_e \cdot \left(\frac{N_{imp}}{A_m} + \frac{M_{imp}}{I_m} \cdot y_p \right)$$

Total stress in the strand $\sigma_p = \sigma_{p,P} + \Delta\sigma_p$

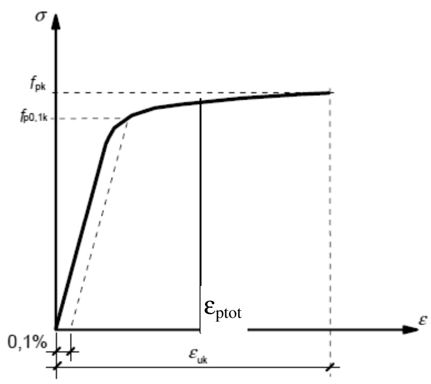
Ultimate limit state.

Due to bond between concrete and tendons after grouting the ducts the strands get due to loading the same strain $\Delta\varepsilon_\pi$ as concrete at the level of the strands. The total strain int the strands is $\varepsilon_{ptot} = \varepsilon_{p\propto} + \Delta\varepsilon_p$.



The strain due to prestressing $\varepsilon_{p\propto} = P_\propto/A_p$ where $P_\propto$ is the effective prestress after losses (loss due to elastic shortening included because elastic shortening happens before grouting the ducts).

The stress of the strands is calulated form the stress-strain diagram of the strand with the totla strain ε_p



3. PERMANENTLY UNBONDED TENDONS

- **Tendon is in the duct which is filled with grease**. The duct does not injected with mortar.

=> there is no bond between the tendon and concrete

Ducts and anchors are assembled in the mould;
- Tendons composed by several strands are assembled is the ducts
- Tendons have usually parabolic curve form

- Concrete structure with the ordinary reinforcement is casted

- Tendons are tensioned when the strength of the concrete has been developed to the required value => post-tensioning;

Tendons are tensioned against the anchors => Prestressing force goes to the concrete at the ends of the structure by the anchors

Tensioning: No bond between the tendon and concrete
The compressive counterforce of the prestressing force goes to the concrete only by the anchors
Cross-section is the net concrete cross section where the ducts forms the holes

$$A_{net} = A_c - n \cdot A_{duct}$$

Where Anet is the net concrete cross-section
Aduct is the area on one duct
n is the number of the ducts

Compressive force causes the elastic shortening of the structure and the distance between the anchors => the strain of the tendons decrease => force of the tendon decrease => prestressing loss due to the elastic shortening

For imposed loads: Ducts are not injected => no bond between the tendon and concrete
The net concrete area is used

Ultimate limit state:
There is no bond between copncrete and the tendons.
Length of the tendon increased due to curvature of the structure under the load in the ultimate limit state.
The increase of the stress from thje effective prestress to the stress in the ultimate limit state is $\Delta\sigma_{p.ULS}$= 100 MPa (according to the Finnish National Annex $\Delta\sigma_{p.ULS}$= 50 MPa).

Prestressed concrete section (with bonded pretensioned strands)

Transfer stage – elastic shortening

Beam with centric prestressing, cross-section A=h*b
The amount of the strands A_p, prestressing just before the transfer σ_{pi}

Strain in the strands just before the transfer $\varepsilon_{pi} = \sigma_{pi}/E_p$

Prestressing force (tension force) just before the transfer $P_i = \sigma_{pi}A_p=\varepsilon_{pi}E_pA_p$

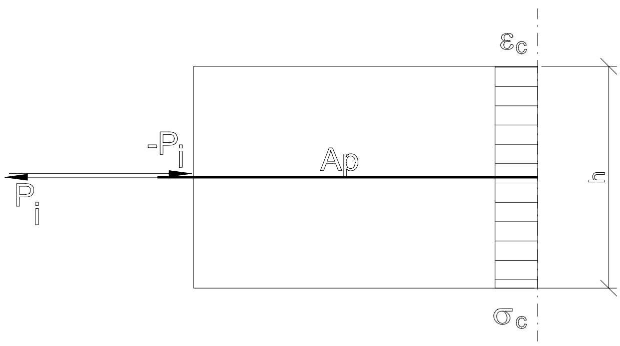
When the compressive counter force $-P_i$ of this prestressing force acting to the structure composed by the concrete and the strands, the concrete gets the compressive strain ε_c (< 0)

Concrete stress $\sigma_c = E_c\varepsilon_c$ (<0)
The stress resultant of concrete $N_c = A_c\sigma_c = \varepsilon_cE_cA_c$ (<0)
The net cross section of concrete $A_c = A - A_p$

Bond between the concrete and reinforcement (strands) => Strands gets the same strain as the concrete at the location of the strands, so $\boldsymbol{\Delta\varepsilon_p = \varepsilon_c}$

Strain of strands just after the transfer $\varepsilon_p = \varepsilon_{pi} + \Delta\varepsilon_p = \varepsilon_{pi} + \varepsilon_c$
Stress in the strands just after the transfer $\sigma_{pm} = \varepsilon_pE_p = (\varepsilon_{pi} + \varepsilon_c) E_p$
The stress resultant just after the transfer $P = \sigma_{pm}A_p = (\varepsilon_{pi} + \varepsilon_c) E_p A_p$

Equilibrium:



$P + N_c = 0$

$(\varepsilon_{pi} + \varepsilon_c) E_p A_p + \varepsilon_cE_cA_c = 0$

$\varepsilon_c(E_pA_p+E_cA_c) = -\varepsilon_{pi}E_p$

$$\varepsilon_c = \frac{-\varepsilon_{pi}E_pA_p}{E_pA_p + E_cA_c} = \frac{-P_i}{E_pA_p + E_c(A - A_p)} = \frac{-P_i}{E_c\left[A + \left(\frac{E_p}{E_c} - 1\right)A_p\right]} = \frac{-P_i}{E_cA_m}$$

The transformed cross-section composed by the concrete and the strands

$$A_m = A + \left(\frac{E_p}{E_c} - 1\right) \cdot A_p$$

Stress in the concrete $\sigma_c = \varepsilon_c E_c = \dfrac{-P_i}{A_m}$

The stress resultant of the concrete $N_c = \sigma_c\ A_c = \dfrac{-P_i}{A_m} A_c$

Elastic shortening of the strands $\Delta\varepsilon_p = \varepsilon_c = \dfrac{-P_i}{E_c A_m}$

The change of the stress of the strands due to the elastic shortening

$$\Delta\sigma_{pe} = \Delta\varepsilon_p E_p = \frac{-P_i}{A_m}\frac{E_p}{E_c}$$

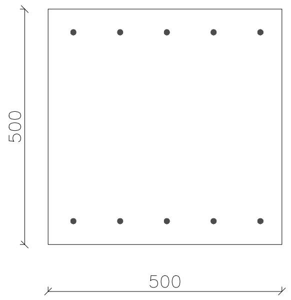
The stress in the strands just after the transfer

$$\sigma_p = \sigma_{pi} + \Delta\sigma_{pe} = \frac{P_i}{A_p}\left(1 - \frac{A_p}{A_m}\frac{E_p}{E_c}\right) = \frac{P_i}{A_p}\left(1 - \frac{A_p}{A + \left(\frac{E_p}{E_c} - 1\right)A_p}\frac{E_p}{E_c}\right)$$

Prestressed concrete structure
Transfer stage - elastic shortening

Example

Beam b*h with **centric prestessing**; area of the prestressing strands Ap



$b := 500 \cdot mm$ $\qquad h := 500 \cdot mm$

Bruttoarea $\qquad A := b \cdot h \qquad A = 0.25\,m^2$

Area of prestressing strands $\qquad A_p := 1000 \cdot mm^2$

Initial stress just before the transfer $\qquad \sigma_{pi} := 1250 \cdot MPa$

Elasic modulus of the strands $\qquad E_p := 195000 \cdot MPa$

Strain of the strand just before the transfer $\qquad \varepsilon_{pi} := \frac{\sigma_{pi}}{E_p} \qquad \varepsilon_{pi} = 6.41\,‰$

Prestressing force just before the transfer $\qquad P_i := \sigma_{pi} \cdot A_p \qquad P_i = 1250\,kN$

Concrete C35/45 (characteristic cylinder /characteristic cubic compressive strength)

Characteristic strength $\qquad f_{ck} := 35MPa$

Mean strength $\qquad f_{cm} := f_{ck} + 8 \cdot MPa \qquad f_{cm} = 43\,MPa$

Elastic modulus of concrete $\qquad E_{cm} := 22000 \cdot MPa \cdot \left(\frac{f_{cm}}{10 \cdot MPa}\right)^{0.3} \qquad E_{cm} = 34077\,MPa$

Transfer strength usually about 70 % of the final strength

Development of concrete strengh according to EC2

Cement type parameter $\qquad s := 0.2$ (rapid-cement)

Development strength of strength $\qquad \beta_{cc}(t) := e^{s\left(1-\sqrt{\frac{28}{t}}\right)} \qquad$ EC2, equation(3.4)

Mean strength, when age of concrete is t $f_{cmt}(t) := \beta_{cc}(t) \cdot f_{cm}$

At transfer: $f_{cmi} := 0.7 \cdot f_{cm}$ $f_{cmi} = 30.1\,MPa$

=> $\beta_{cci} := 0.7$

Development of elastic modulus as a function of time $E_{cmt}(t) := \left(\frac{f_{cm}(t)}{f_{cm}}\right)^{0.3} \cdot E_{cm}$

$\frac{f_{cmt}(t)}{f_{cm}} := \beta_{cc}(t)$ $\frac{f_{cmi}}{f_{cm}} = 0.7$ EC2 equation (3.5)

Elastic modulus of concrete at transfer $E_{cmi} := \left(\frac{f_{cmi}}{f_{cm}}\right)^{0.3} \cdot E_{cm}$ $E_{cmi} = 30619\,MPa$

Ratio of elastic modulies $n_e := \frac{E_p}{E_{cmi}}$ $n_e = 6.369$

Area of the **transformed cross-section** $A_m := A + (n_e - 1) \cdot A_p$ $A_m = 0.255\,m^2$

Netto area of concrete $A_c := A - A_p$ $A_c = 0.249\,m^2$

Axial rigidity of the beam cross-section $EA := E_{cmi} \cdot A_c + E_p \cdot A_p$

$EA := E_{cmi} \cdot (A - A_p) + E_p \cdot A_p$ $EA = 7819\,MN$

$EA := E_{cmi} \cdot A + (E_p - E_{cmi}) \cdot A_p$

$EA := E_{cmi} \cdot \left[A + \left(\frac{E_p}{E_{cmi}} - 1\right) \cdot A_p\right]$

$EA := E_{cmi} \cdot A_m$

Concrete strain $\varepsilon_c := \frac{-P_i}{EA}$ $\varepsilon_c := \frac{-P_i}{E_{cmi} \cdot A_m}$ $\varepsilon_c = -0.16\,‰$

Concrete stress $\sigma_c := \varepsilon_c \cdot E_{cmi}$ $\sigma_c = -4.895\,MPa$

Netto area of concrete $A_c := A - A_p$ $A_c = 0.249\,m^2$

Concrete stress resultant $N_c := A_c \cdot \sigma_c$ $N_c = -1218.827\,kN$

Change of strain of the strands $\Delta\varepsilon_p := \varepsilon_c$ $\Delta\varepsilon_p = -0.16\,‰$

Change of stress of the strands due to the elastic shortening $\Delta\sigma_{pe} := \Delta\varepsilon_p \cdot E_p$

$\Delta\sigma_{pe} = -31.173\,MPa$

Stress of the strands just after the transfer $\sigma_p := \sigma_{pi} + \Delta\sigma_{pe}$ $\sigma_p = 1218.8\,MPa$

Force of the strands just after the transfer $P := \sigma_p \cdot A_p$ $P = 1218.827\,kN$

Equilibrium $P + N_c = 0\,N$