Outcome of this lecture

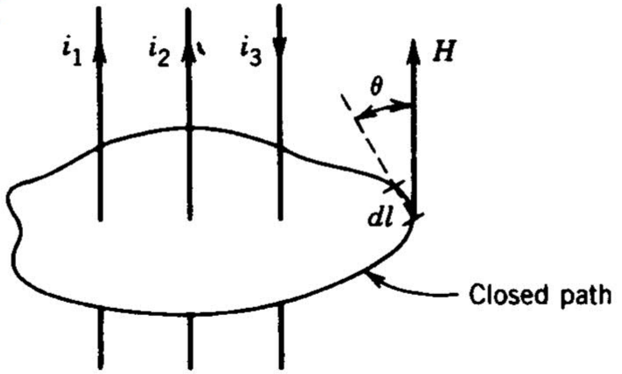
At the end of this lecture you will be able to:

- calculate the magnetomotive force
- model simple magnetic circuits
- calculate magnetic flux densities
- calculate inductances and losses
- apply the above for permanent magnet materials
- you will enhance your understanding of magnetic materials, phenomena and circuits.

Magnetic Circuits

- Ampere's circuit law (i-H relation)

$$\oint \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{A} = \sum_k i_k$$

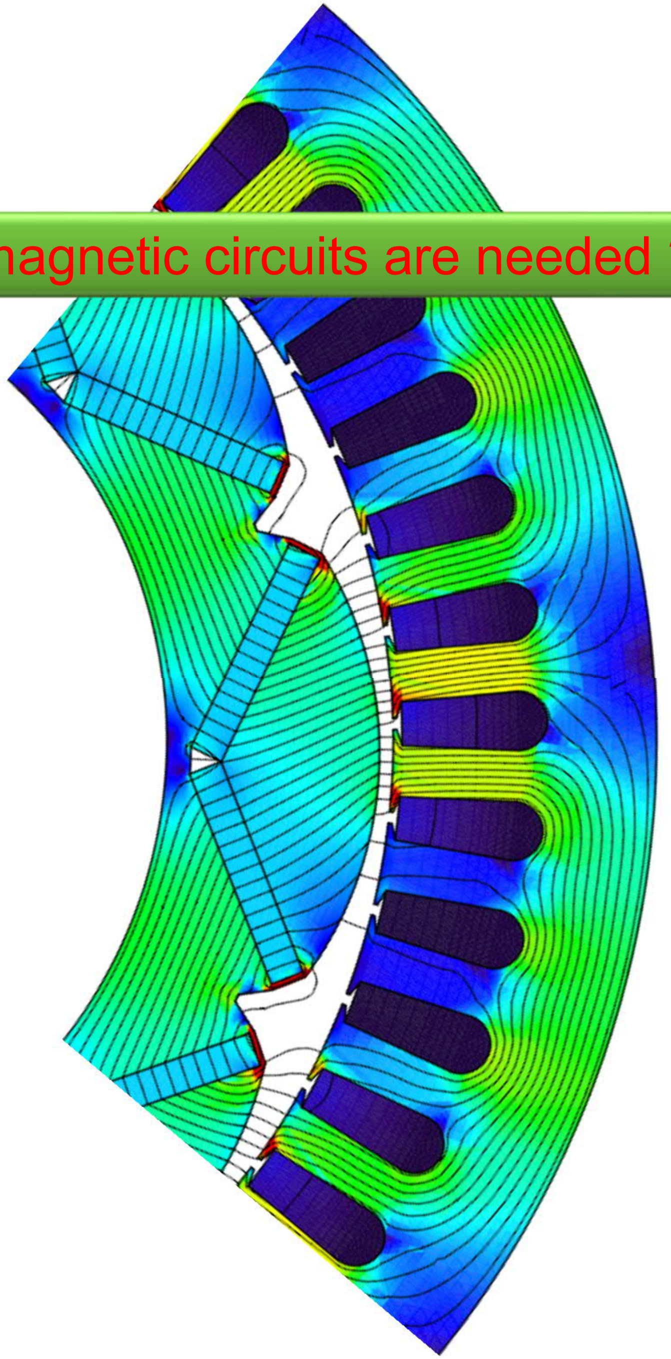


- Permeability (B-H relation)

$$B = \mu H = \mu_r \mu_0 H$$

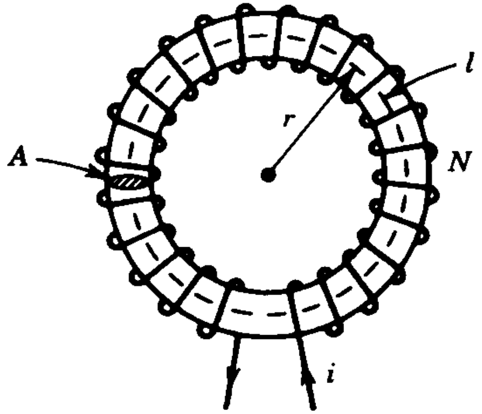
Ferromagnetic materials $\mu_r \approx 2000...6000$

Permeability of free space $\mu_0 = 4\pi 10^{-7}\ henry / meter$



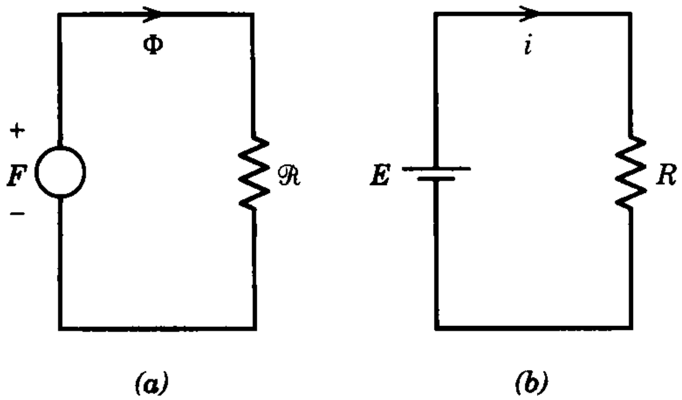
Why magnetic circuits are needed ?

Magnetic Equivalent Circuits



Example: toroid with leakage flux neglected

$$\oint \vec{H} \cdot d\vec{l} = Ni$$



- Magnetomotive force (mmf)

$$F = Ni = Hl$$

- Magnetic flux

$$\Phi = \int B \, dA$$

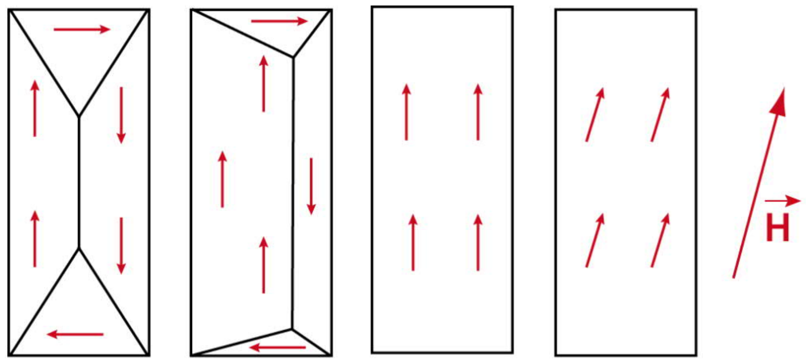
$$\Phi = \frac{F}{\Re}$$

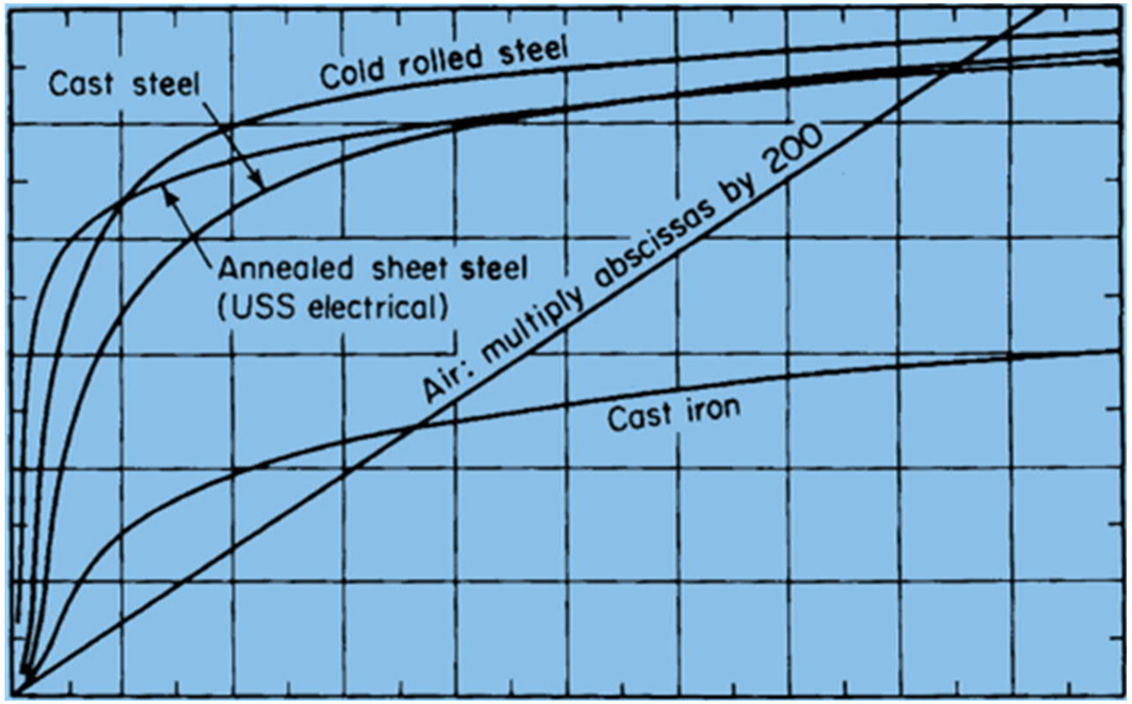
- Reluctance $\Re = \frac{l}{\mu A} = \frac{1}{P}$

- Permeance P

analogy $i = \frac{E}{R}$

Characteristics of electrical steel: Magnetization Curve





Epstein frame for magnetic material characterization

- Near the origin almost linear
- Strong nonlinearity at the knee
- Saturation after the knee

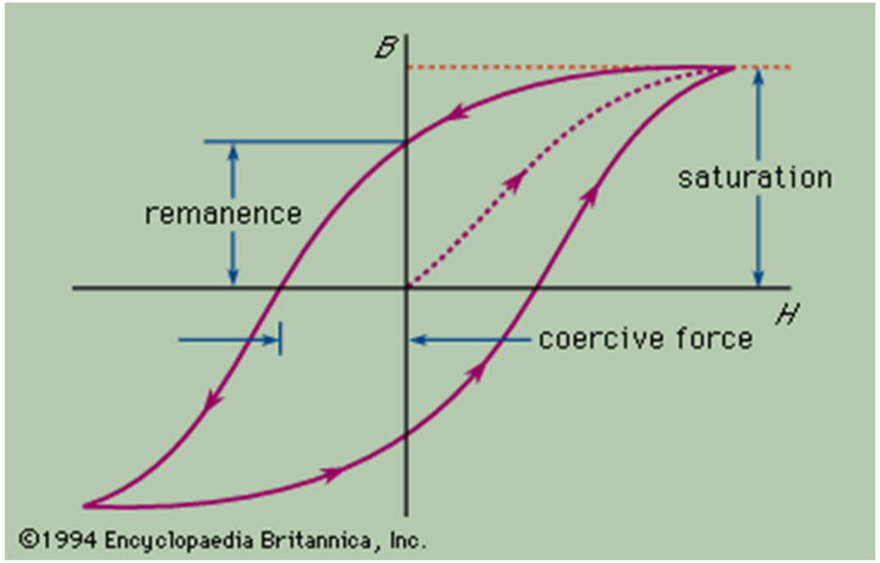
Characteristics of electrical steel: Hysteresis

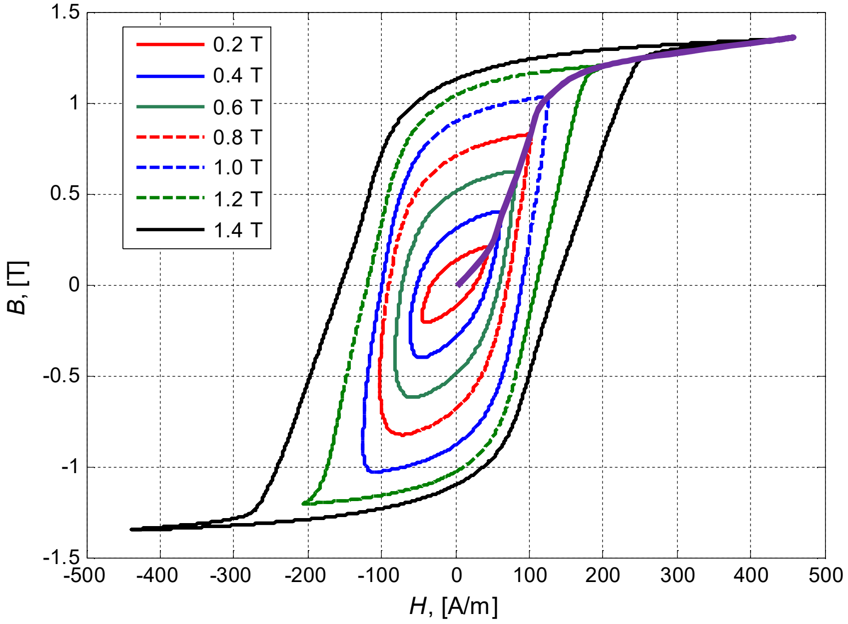
- *B*-*H* relation is nonlinear and multi-valued
- *B* lags behind *H*

- B_r residual flux density (H=0)
- H_c coercitive magnetic field strength (B=0)

- The locus of the tip of the hysteresis loop is

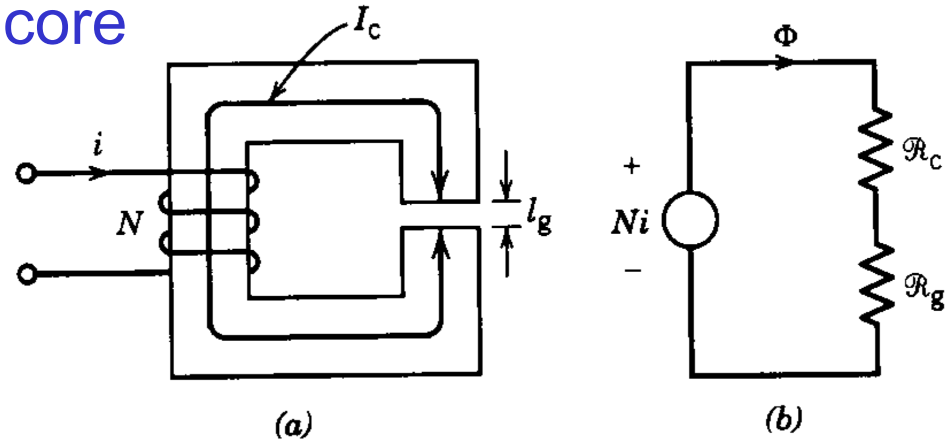
  the magnetization curve





Magnetic Circuit with Air Gap

- Air gap requires more mmf than the core



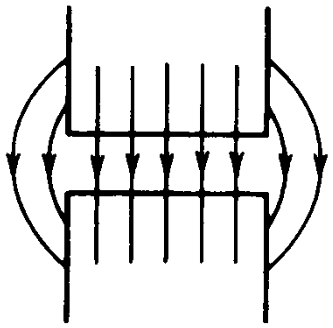
$$Ni = H_c l_c + H_g l_g$$

$$R_c = \frac{l_c}{\mu_c A_c} \qquad R_g = \frac{l_g}{\mu_0 A_g}$$

$$B_c = \frac{\Phi_c}{A_c} \qquad \mu_c = \mu_r \mu_0$$

$$B_g = \frac{\Phi_g}{A_g} \qquad \mu_r \approx 2000...6000$$

- Fringing

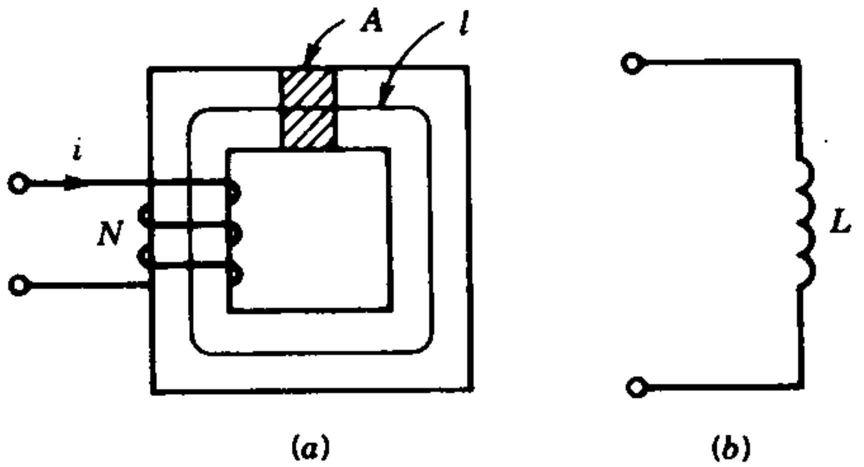


- No fringing

$$A_g = A_c \Rightarrow B_g = B_c$$

Inductance

- A coil is represented by an ideal circuit element



- Flux linkage $\lambda = N\Phi$
- Inductance $L = \dfrac{\lambda}{i}$

What problems the inductance concept presents ?

- Saturation
- Leakage
- Motion

Hysteresis Loss

- Energy transfer: varying magnetic field

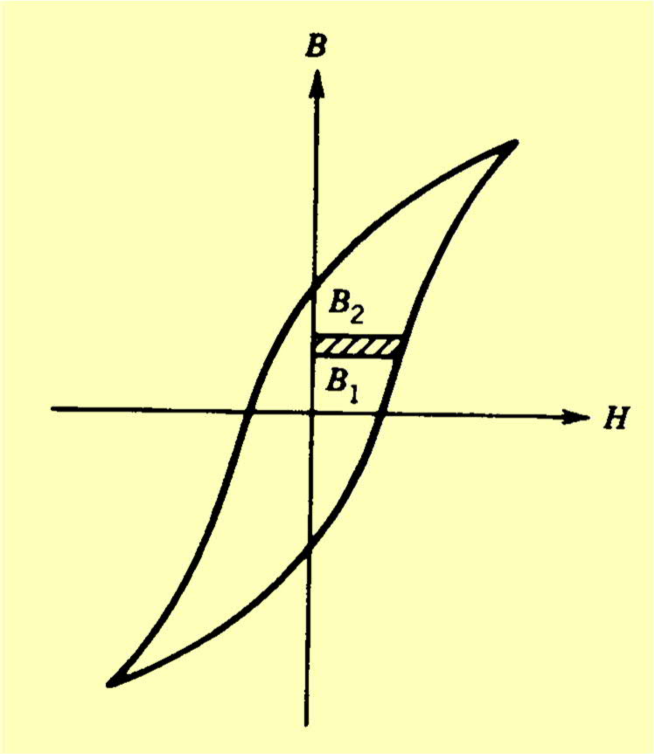
Electric energy

$$W = \int_{t_1}^{t_2} ei\, dt$$

$$i = \frac{Hl}{N} \qquad e = N\frac{d\Phi}{dt} \qquad \Phi = BA$$

Magnetic energy

$$W = V_{core} \int_{B_1}^{B_2} H\, dB$$



Hysteresis Loss

What is the relation between energy and Power ?

- Energy loss during a period

$$W|_{cycle} = V_{core} \oint H\, dB$$

- Loss density in the core $W_h = \oint H\, dB$ Ws/m³ =J/m³

- Power loss $P_h = V_{core} W_h f$ W

- Many ways of computing the power losses in electrical steel

$$P_{FeH} = \int_{V_c} \left(\sum_{n=1}^{N} C_{Hn}(n\omega_s) B_n^2 \right) dV$$

$$P_{FeE} = \int_{V_c} \left(\sum_{n=1}^{N} C_{En}(n\omega_s)^2 B_n^2 \right) dV$$

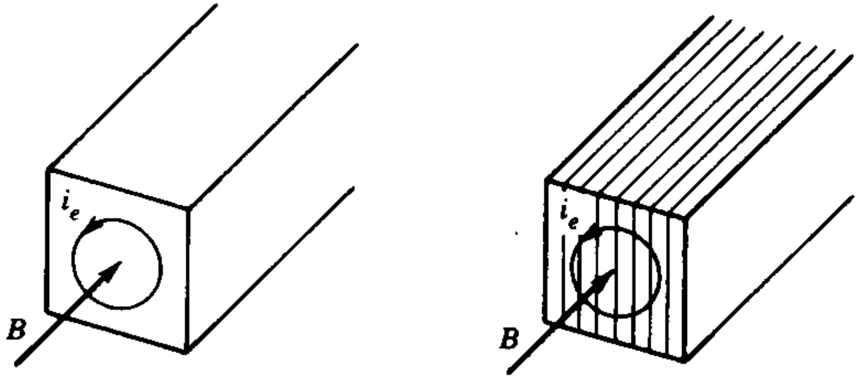
$$P_h = \int_{\Omega} \left[k_h |\boldsymbol{B}| \left| \frac{\partial |\boldsymbol{B}|}{\partial t} \right| + k_r \frac{1 - \frac{|\boldsymbol{B}|}{B_s}}{1 + b(1 - \frac{|\boldsymbol{B}|}{B_s})^2} |\boldsymbol{B}| \left| \frac{\partial \theta}{\partial t} \right| \right] d\Omega$$

$$P_e = \int_{\Omega} k_e \left| \frac{\partial |\boldsymbol{B}|}{\partial t} \right|^{\frac{3}{2}} d\Omega \qquad P_c = \int_{\Omega} k_c \left| \frac{\partial \boldsymbol{B}}{\partial t} \right|^2 d\Omega$$

Eddy Current Loss

- Time varying magnetic field induces eddy currents in conducting material

$$i_e \propto e = \frac{dB}{dt}$$



(a) (b)

- Power loss proportional to Ri^2 will be caused

$$B = B_{\max}\sin(2\pi f t) \Longrightarrow P_e = K_e B_{\max}^2 f^2$$

$$P_c = \int_{\Omega} k_c \left|\frac{\partial \boldsymbol{B}}{\partial t}\right|^2 d\Omega$$

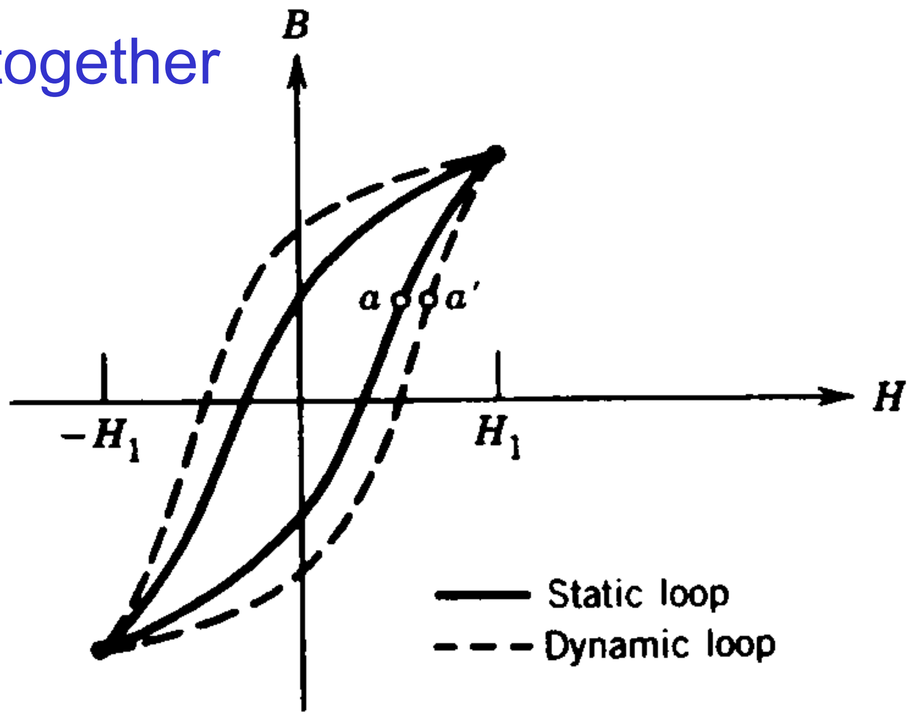
- Eddy current loss can be reduced by
  - increasing the resistivity of the core material
  - using laminated cores

Core Loss

- hysteresis and eddy current loss are lumped together

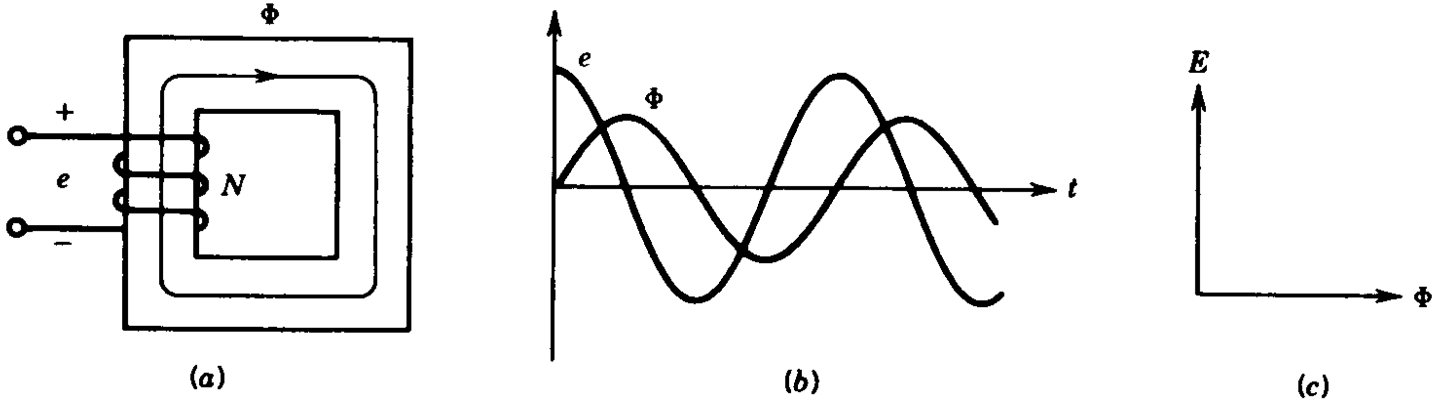
$$P_c = P_h + P_e$$

- Slow variations
  - eddy current loss negligible
  - static loop
- Rapid variations
  - hysteresis loop becomes broader
  - dynamic loop
- The loss appears as heat in the core



$$P_c = V_{core} f \oint_{\text{dynamic loop}} H \, dB$$

Sinusoidal Excitation



- sinusoidal flux $$\Phi(t) = \Phi_{\max} \sin \omega t$$

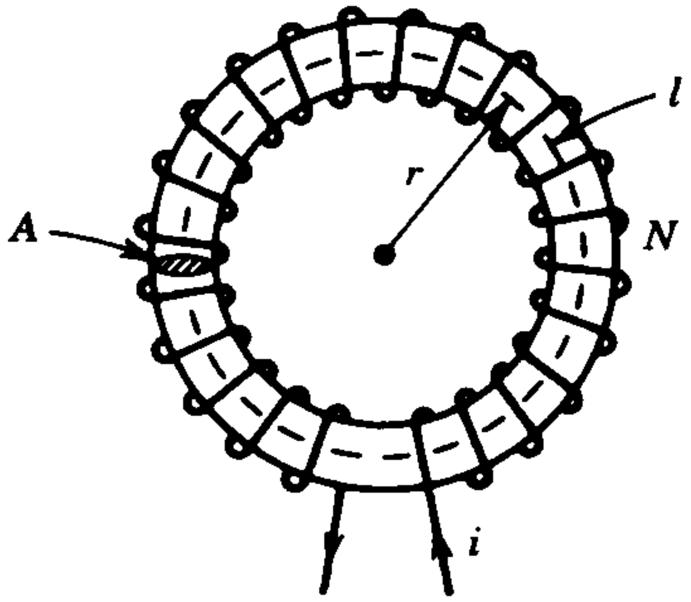
- sinusoidal voltage $$e(t) = N \frac{d\Phi}{dt} = E_{\max} \cos \omega t$$

$$\frac{2\pi}{\sqrt{2}}$$

- root-mean-square value $$E_{rms} = 4{,}44 N f \Phi_{\max}$$

In power applications, coil resistances are small → harmonic flux and voltage

Excitation Current



sinusoidal voltage source

⇓

sinusoidal flux

- Exciting current flows in the coil to establish the flux
- Nonlinear *B-H* characteristic

⇒ Exciting current will be non sinusoidal

- In a toroid $\Phi = BA \qquad i = \frac{Hl}{N}$
- in general $\Phi \propto B \qquad i \propto H$

The *B-H* curve can be rescaled to *Φ-i* curve

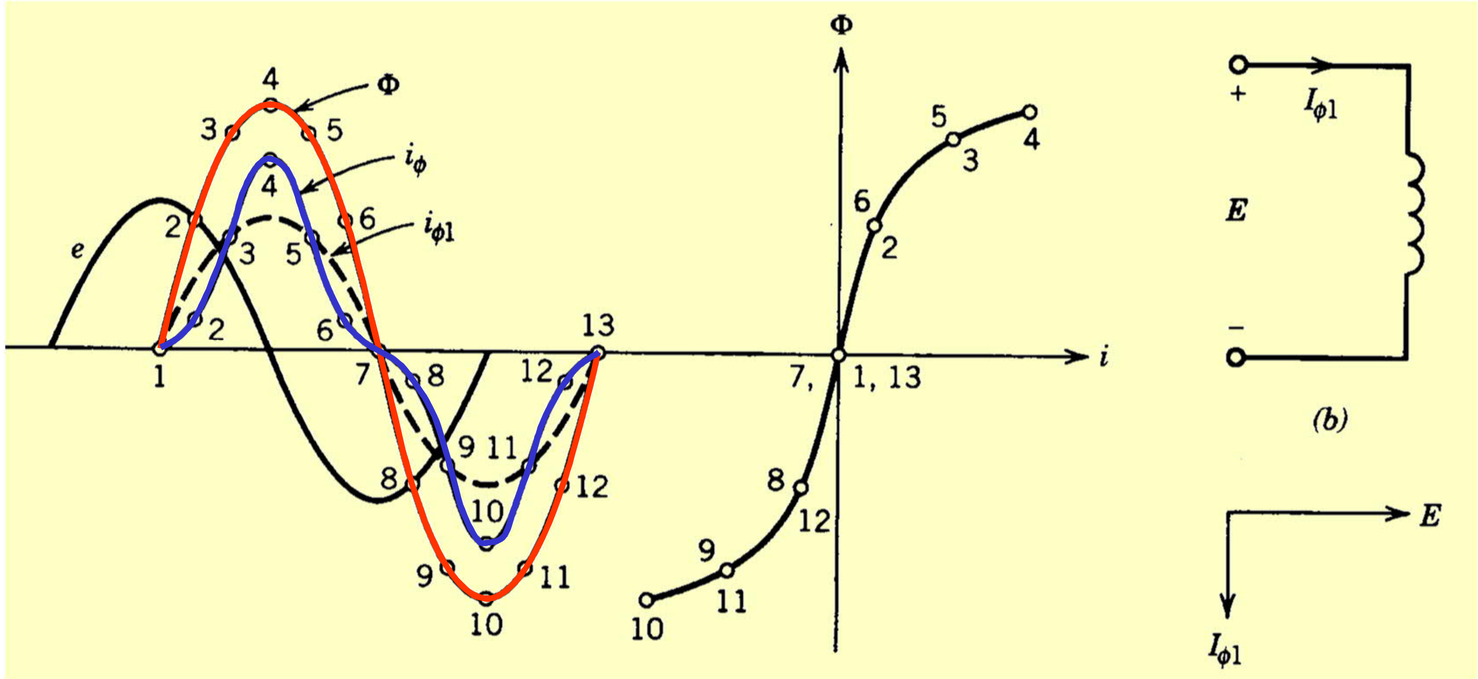
Circuit representation 1:
Nonlinear Material - no Hysteresis

Sinusoidal flux → Non sinusoidal exciting current

- Current wave form obtained from *Φ-i* curve
- Current in phase with flux and symmetric
- Fundamental component lags the voltage by 90°
- No power loss
- Coil can be represented by a pure inductance

What we mean by current in phase

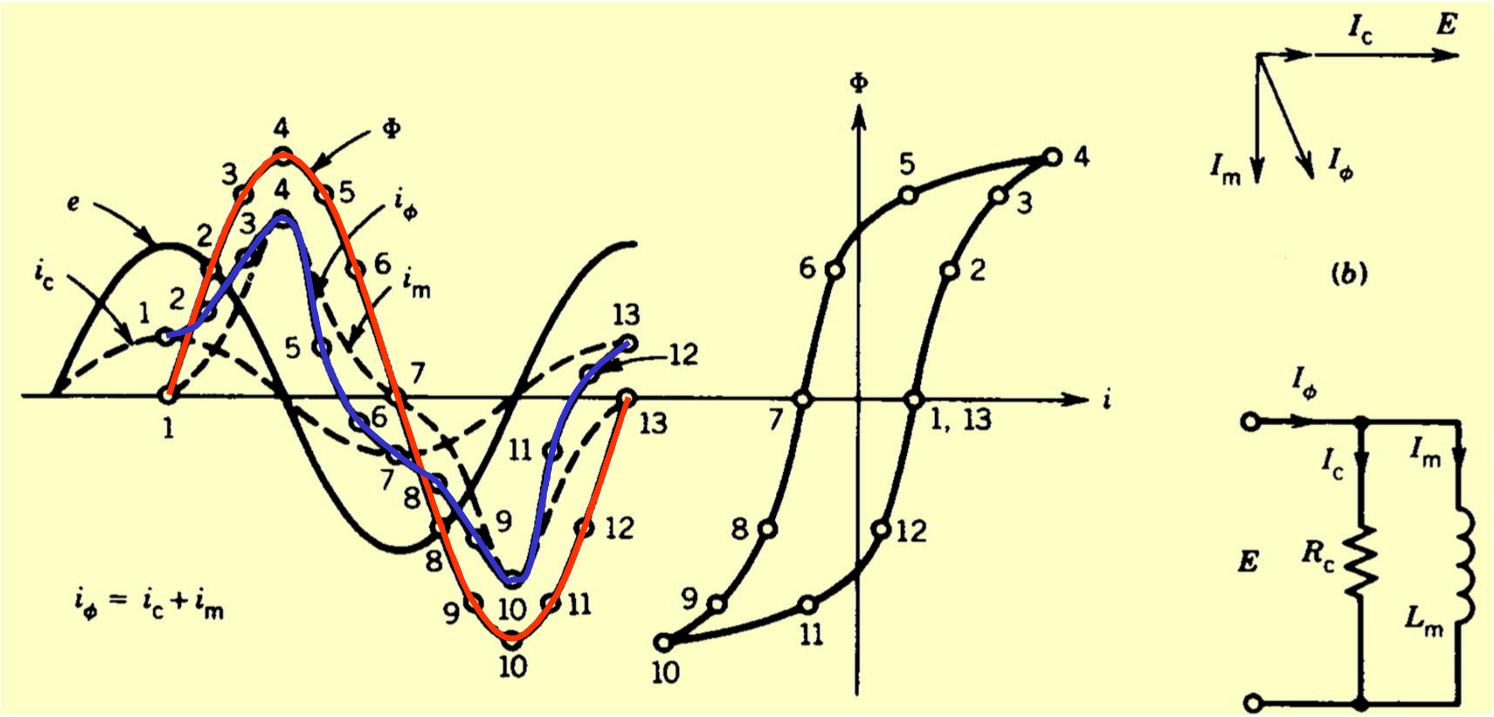
What we mean by the fundamental component of the current?



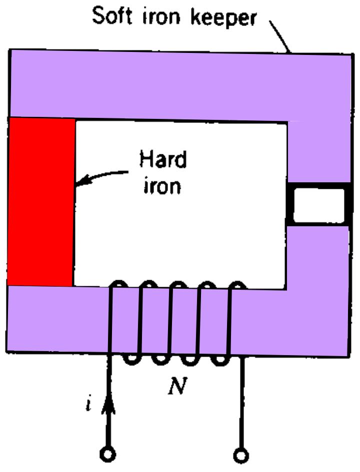
Circuit representation 2: Nonlinear Material and Hysteresis

- Sinusoidal flux → Non sinusoidal – non symmetrical Current
  - Current can be split into two components:
    - I_c in phase with the voltage e
    - I_m in phase with flux Φ
  - Coil can be represented by a resistance and an inductance in parallel

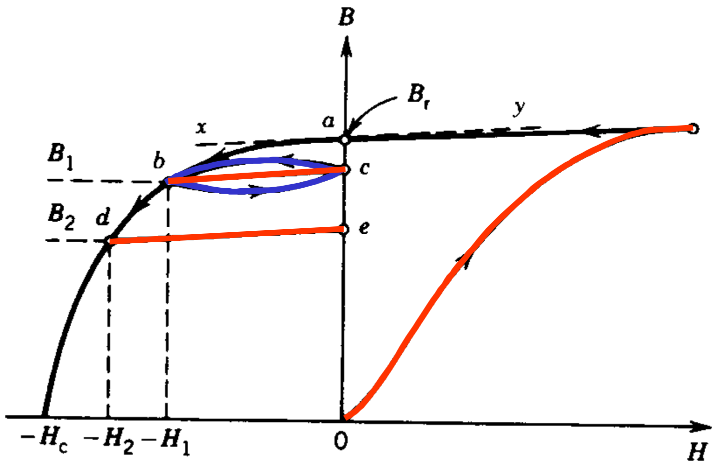
How the core losses are seen in the circuit ?



Permanent Magnets



(a)



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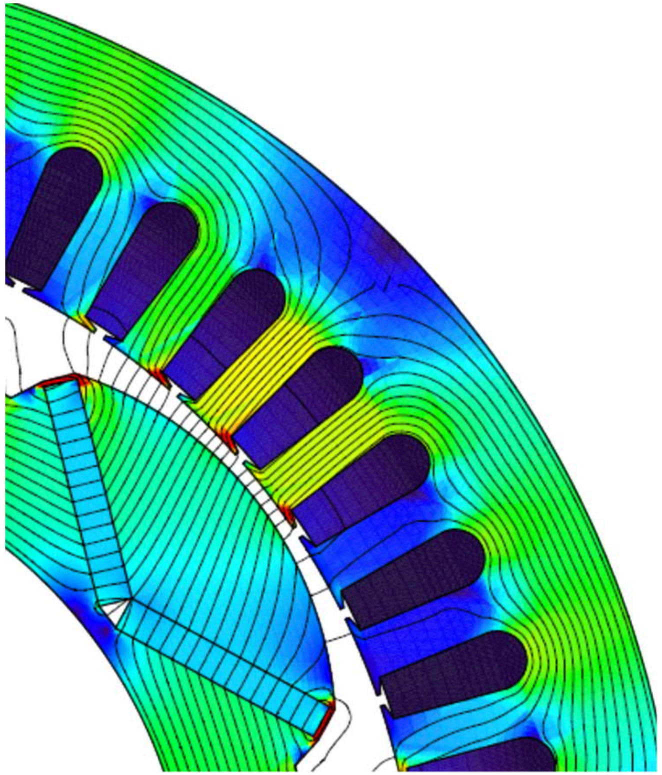


- Magnetization: Application of large mmf

  On its removal flux density will remain at residual value

- Normal operation:

  Reversed magnetic field intensity → Operating point b

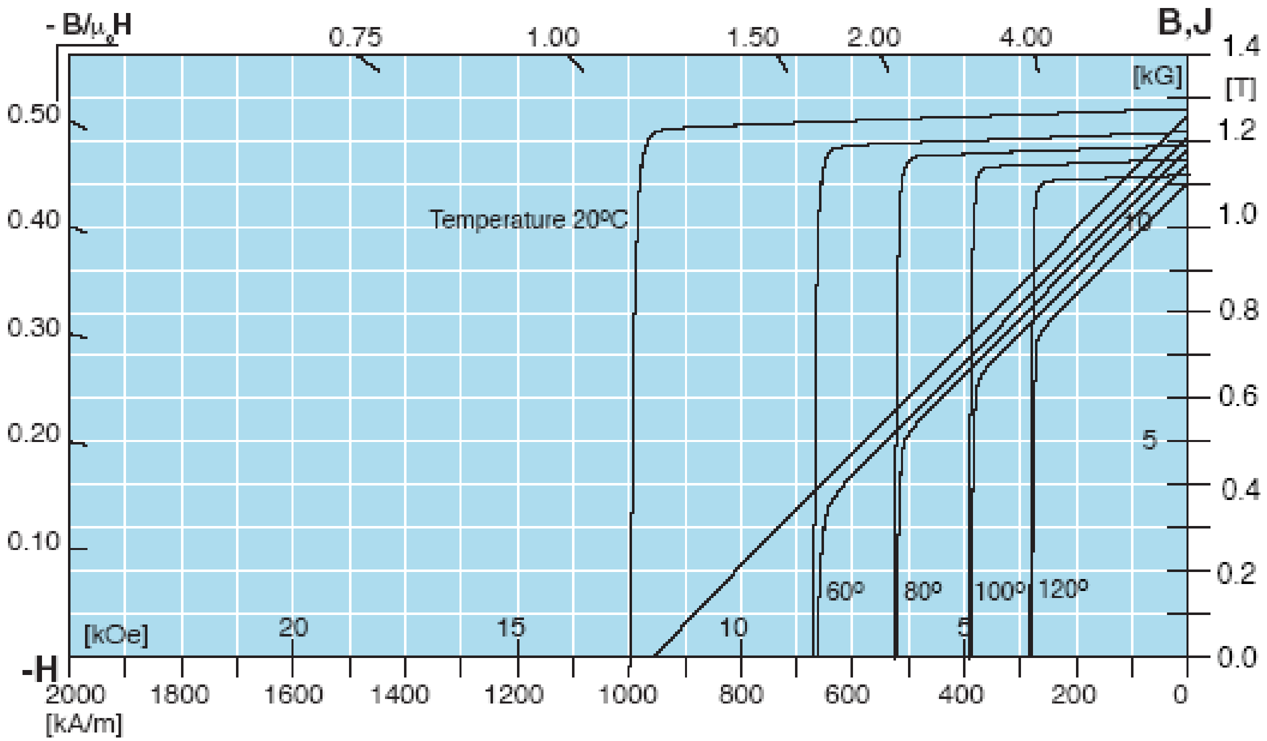
  Field removed and reapplied → Minor loop → recoil line b-c

- Demagnetization

Large reversed field → New operating point d → recoil line d-e

Example of Permanent Magnet characteristic

NEOREM 400a



NEOREM 400a

B_r	1.28 T	12.8 kG
Coercivity		
$_BH_c$	970 kA/m	12.2 kOe
$_JH_c$	1000 kA/m	12.6 kOe
$(BH)_{max}$	310 kJ/m^3	39 MGOe

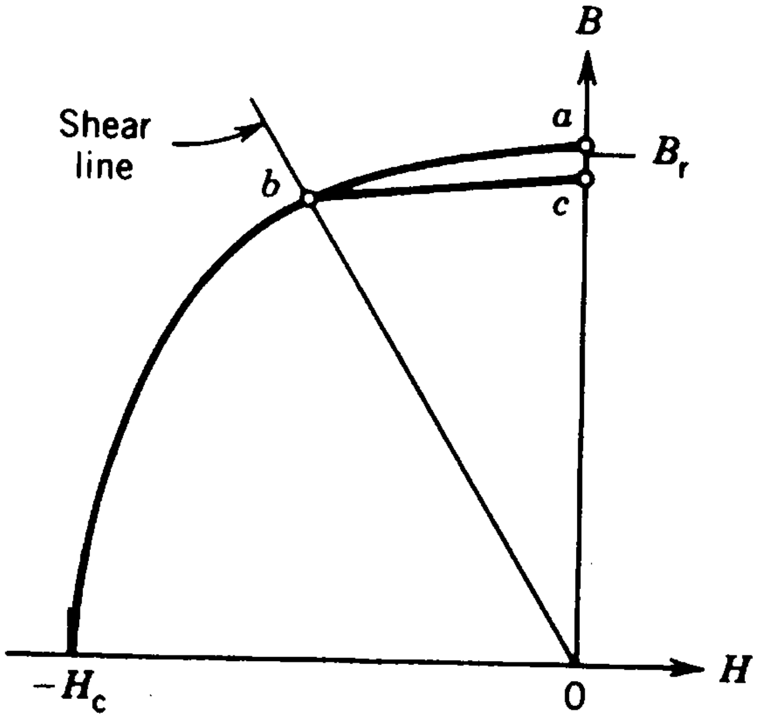
Nominal Values at 20ºC

Approximate design of Permanent Magnets

- no leakage or fringing flux
- no mmf required for soft iron

$$H_m l_m + H_g l_g = 0$$

$$\Phi = B_m A_m = B_g A_g$$



- shear line $$B_m = \mu_0 \frac{A_g}{A_m} \frac{l_m}{l_g} H_m$$

- Find the minimum volume of PM that gives $\max(B_m H_m)$

$$V_m = \frac{B_g^2 V_g}{\mu_0 B_m H_m}$$

Permanent Magnet Materials

- AlNiCo-alloys
  - high residual flux density
  - rather low coersive force
- ferrite-alloys
  - lower residual flux density
  - very high coersitivity force
- rare-earth alloys
  - high residual flux density
  - very high coersitivity force