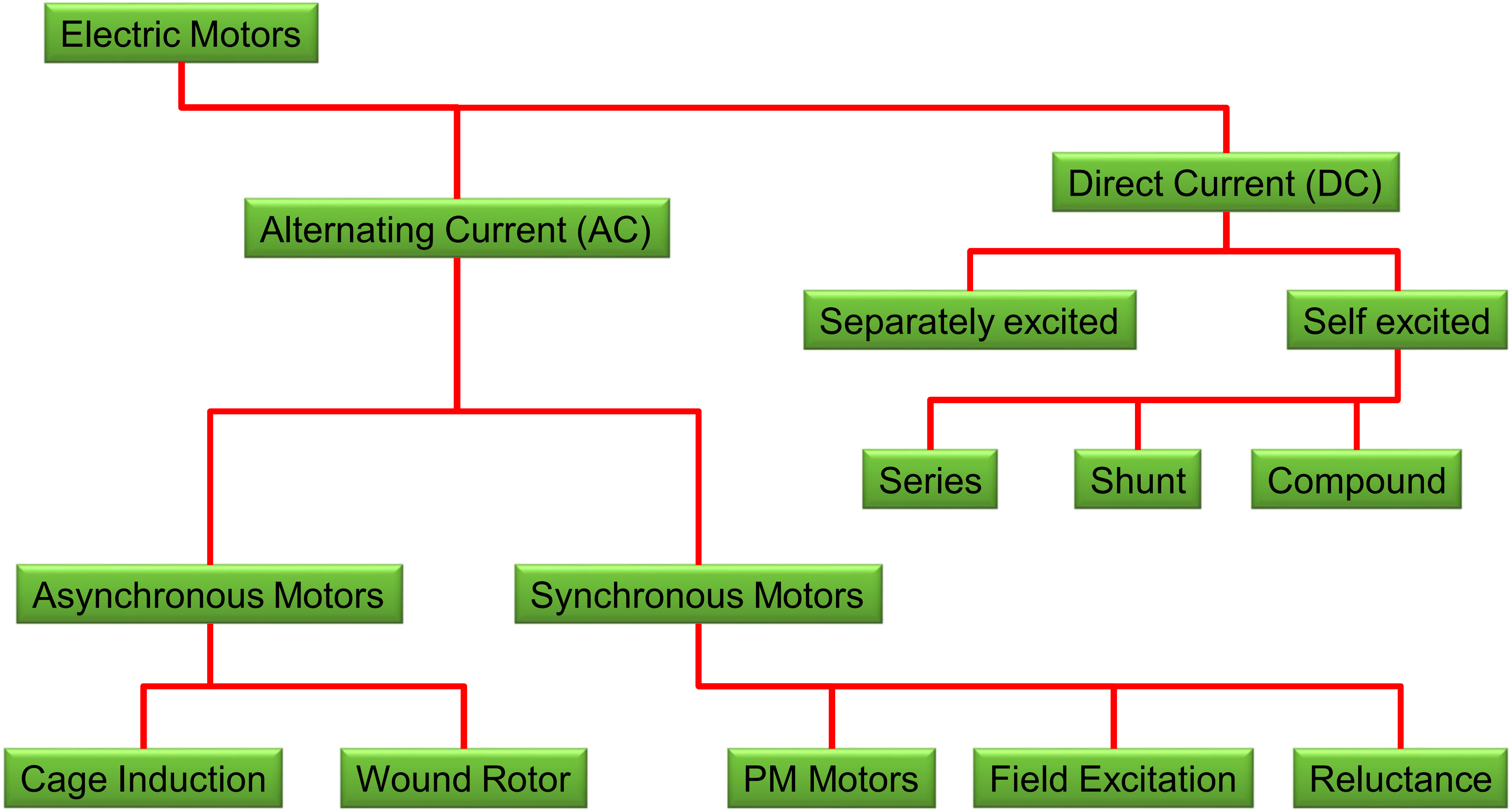
Outcome of this lecture

At the end of this lecture you will be able to:

- Calculate the field energy and co-energy
- Calculate magnetic forces and force densities
- Calculate the torque of an electrical machine
- Understand how the torque is produced in different machines

You will enhance your understanding of the energy conversion process in electrical machines.

Classification of Electric Motors

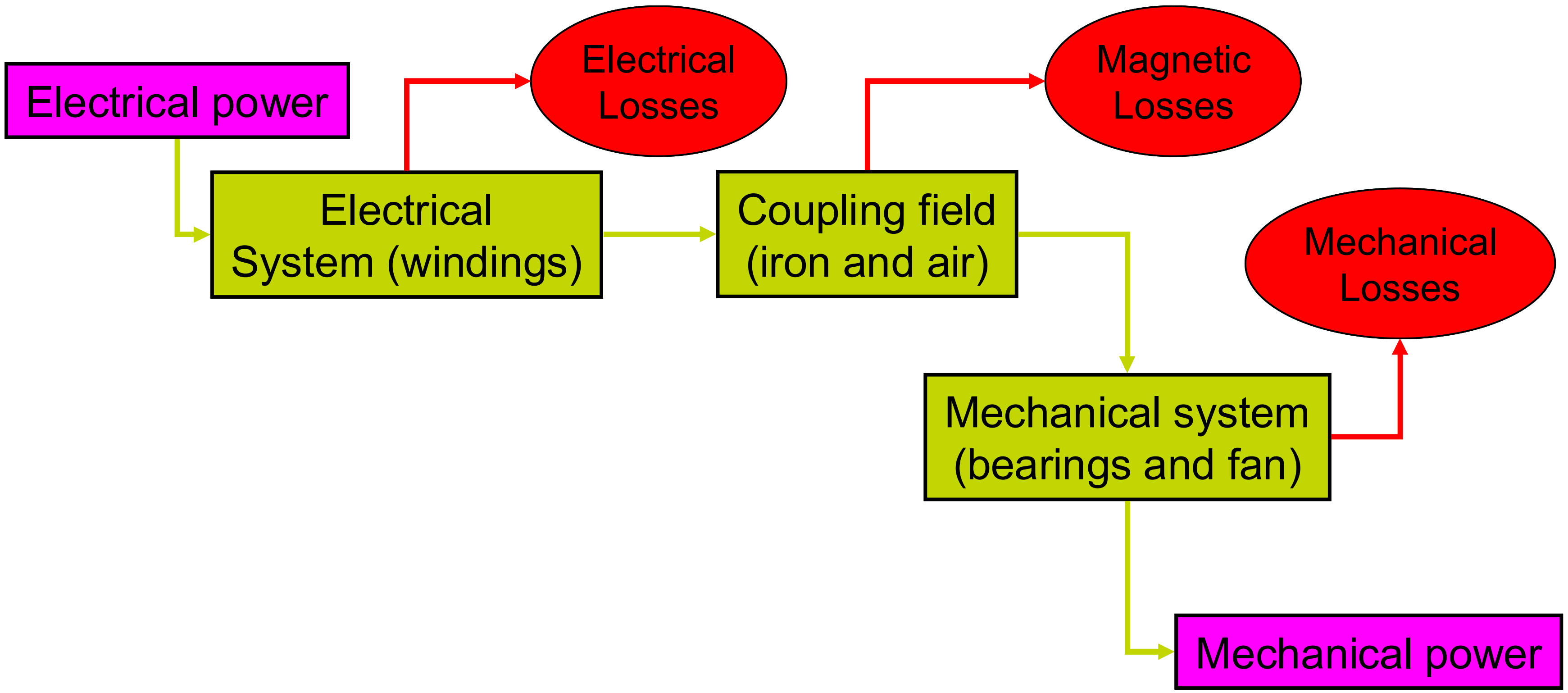


Energy Conversion Process

- Continuous and discrete energy conversion
- Motors
- Generators
- Actuators
- Losses RI^2, P_c

Conservation of energy

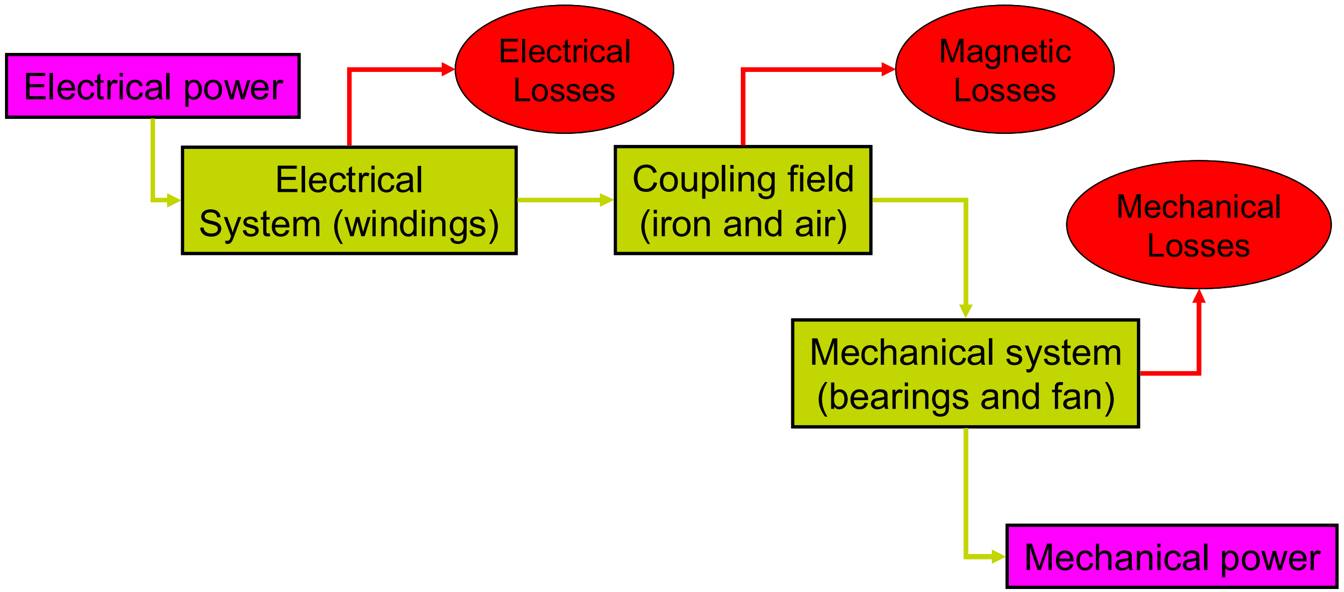
$$dW_e = dW_m + dW_f$$



Energy conversion and losses

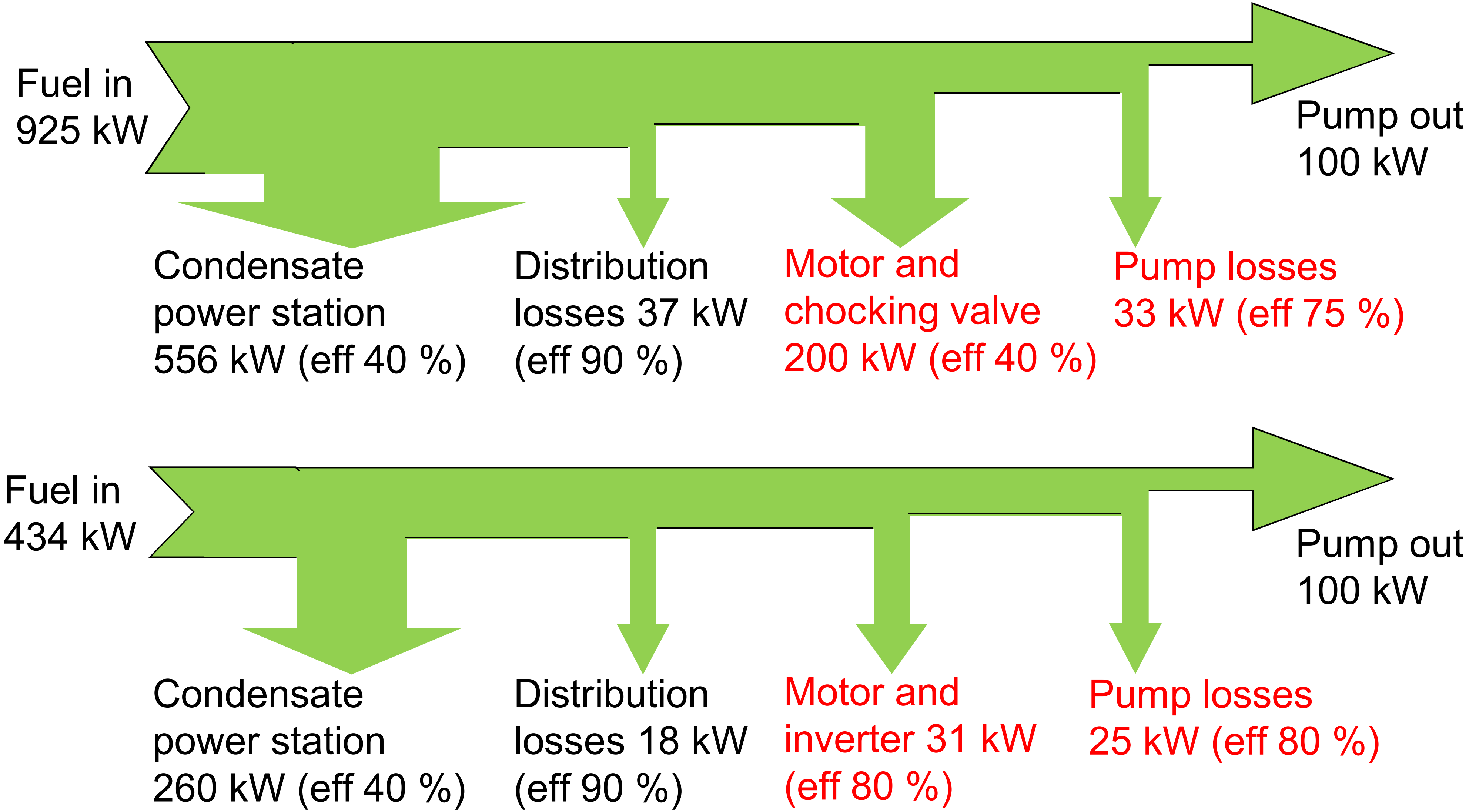
Typical values for induction motors P < 100 kW

Type of loss	Percentage of total loss (100%)
Fixed loss or core	25
Variable loss: stator RI^2	34
Variable loss: rotor RI^2	21
Friction & rewinding loss	15
Stray load loss	5



$$P_{in} = \sqrt{3}\, UI \cos\theta \qquad P_{out} = T_{mech}\,\omega_{mech} \qquad \mathrm{Eff} = \frac{P_{out}}{P_{in}}$$

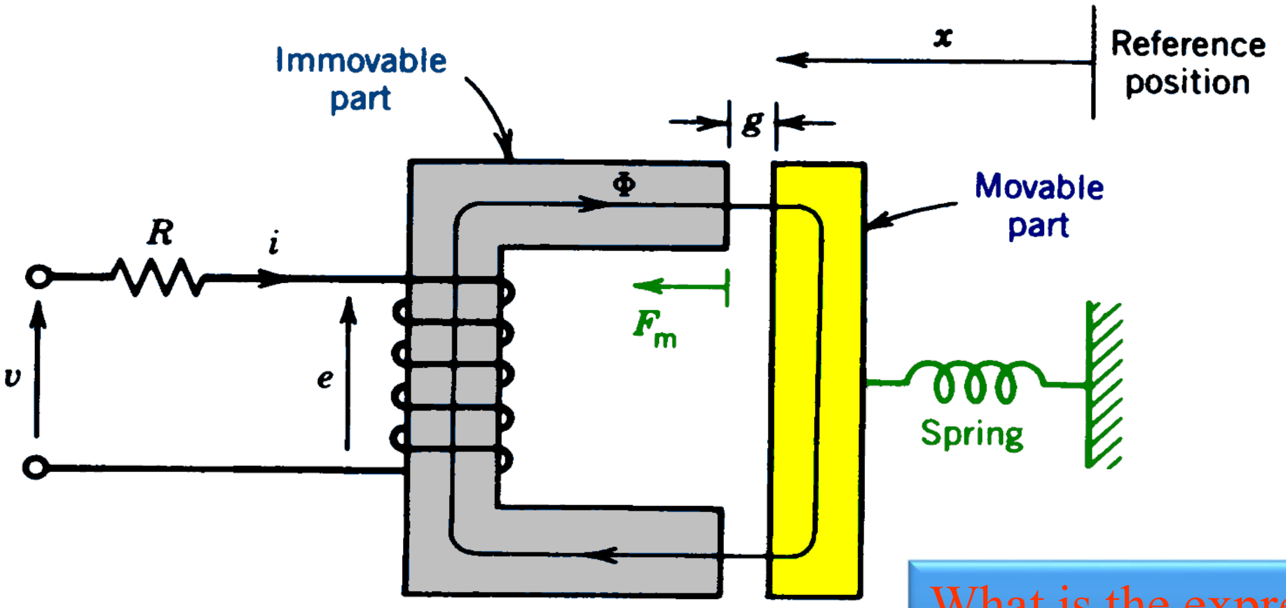
Example: losses in pump drives



Field Energy

No motion

$$\begin{cases} dW_m = 0 \\ dW_e = dW_f \end{cases}$$

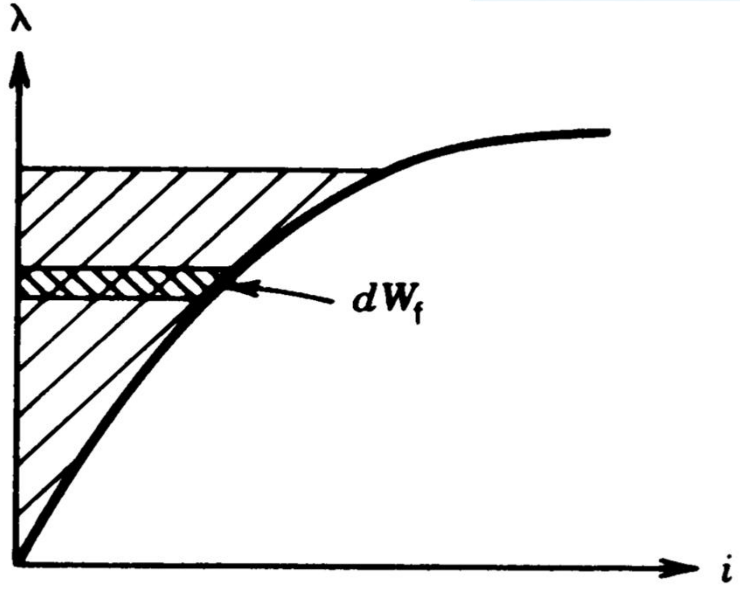


What is the expression for spring potential energy?

$$dW_e = ei\,dt \qquad e = \frac{d\lambda}{dt}$$

$$dW_f = i\,d\lambda$$

$$W_f = \int_0^{\lambda} i\,d\lambda$$



Field Energy

Remember

$$\begin{cases} Ni = H_c l_c + H_g l_g \\ \lambda = N\Phi = NAB \end{cases}$$

$$W_f = V_c w_{fc} + V_g w_{fg}$$

Core energy

$$w_{fc} = \int_0^B H_c \, dB$$

Air gap energy

$$w_{fg} = \frac{B^2}{2\mu_0}$$

linear material

$$H_c = \frac{B_c}{\mu_c}$$

$$w_{fc} = \frac{B_c^2}{2\mu_c}$$

Energy – Co energy

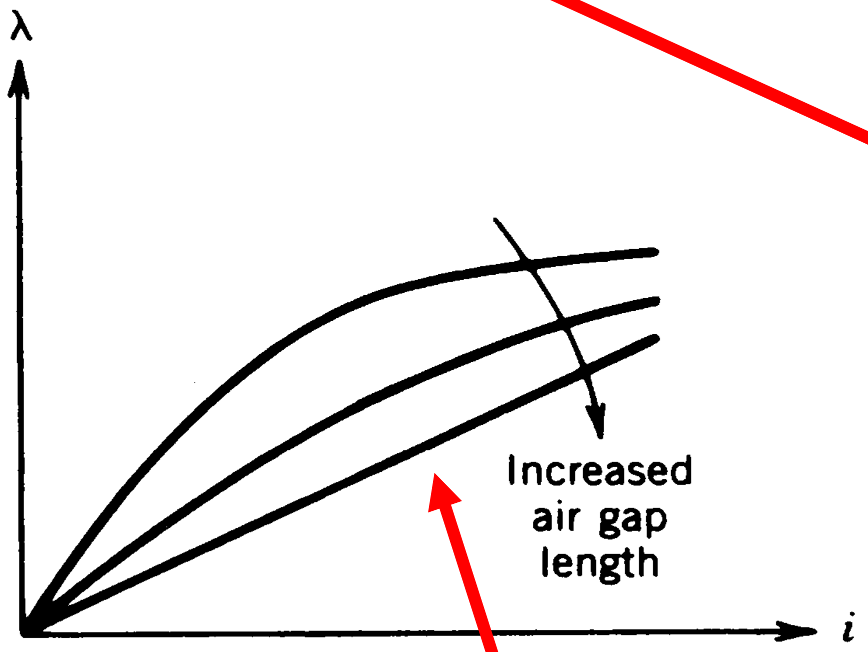
Energy

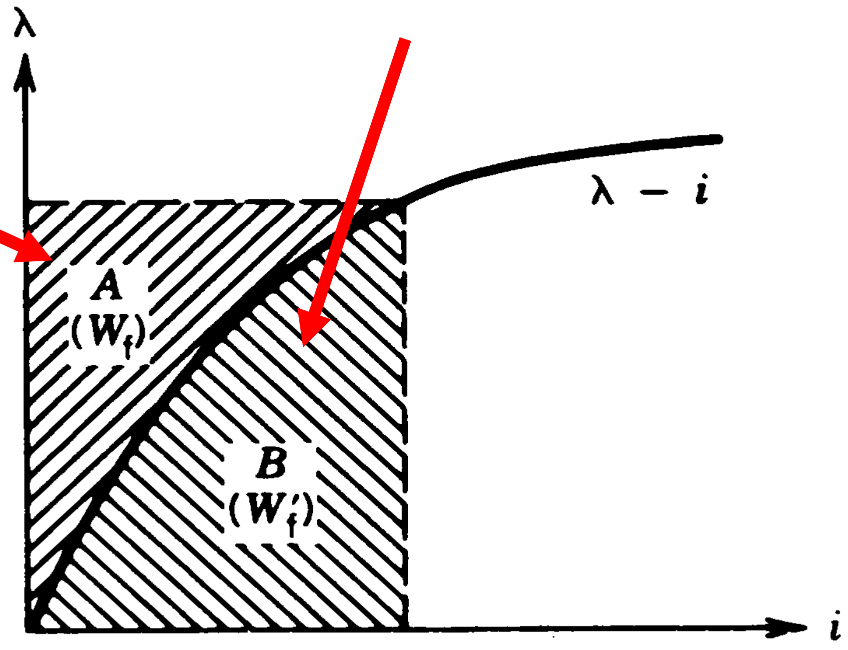
$$W_f = \int_0^\lambda i \, d\lambda$$

$$W'_f + W_f = \lambda i$$

Co energy

$$W'_f = \int^i \lambda \, di$$





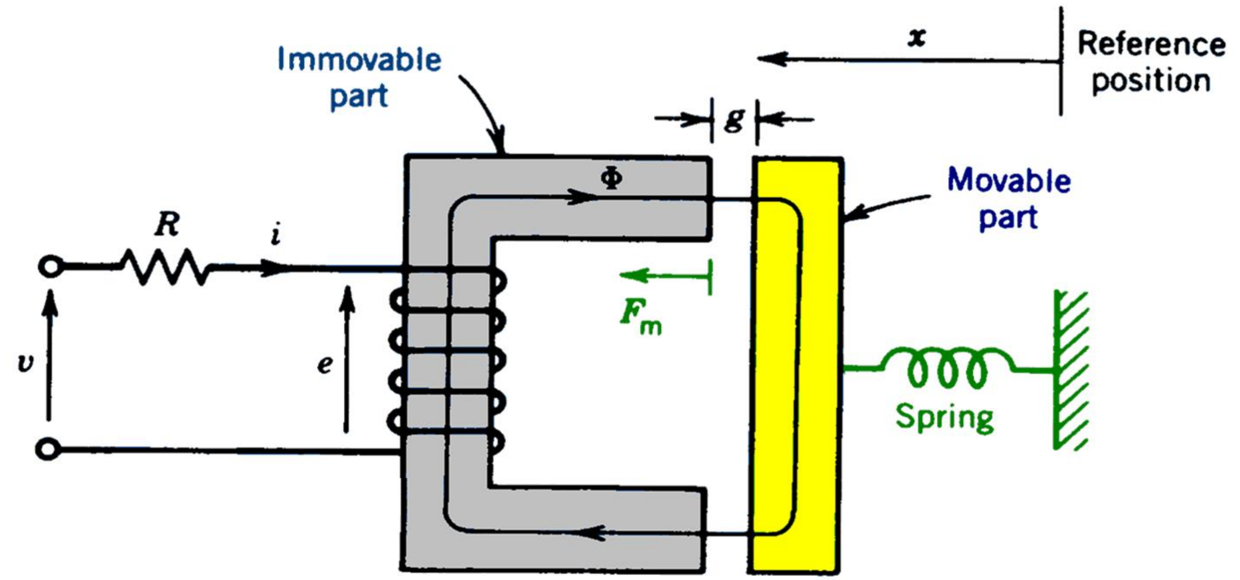
Characteristic depends on the air gap length

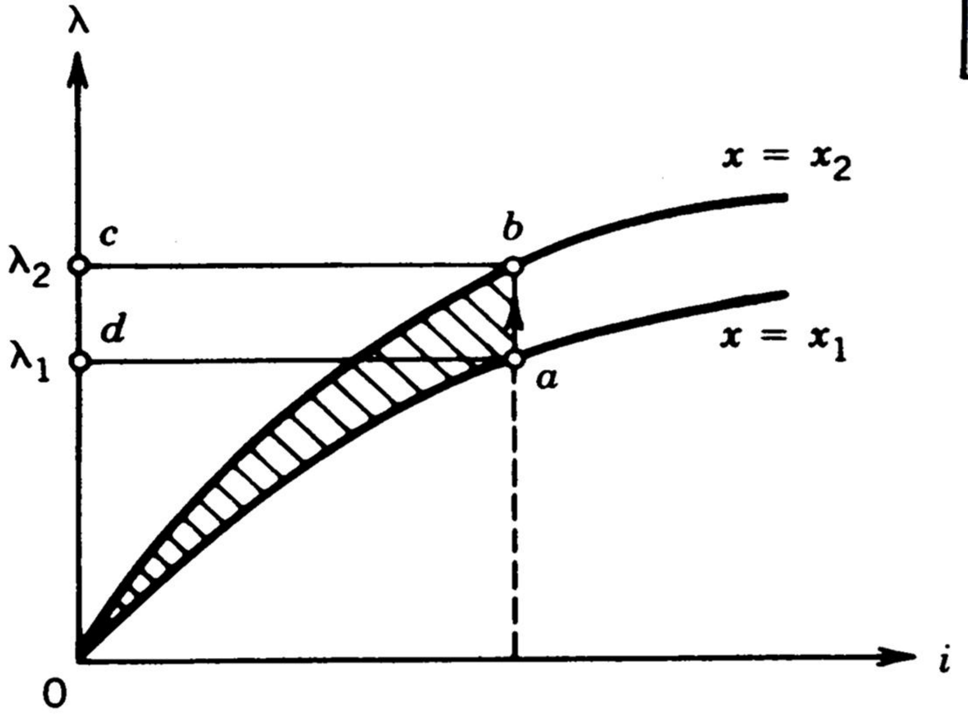
Mechanical Force – Scenario 1

Movable part moves slowly from x_1 to x_2

Constant current

Why ?



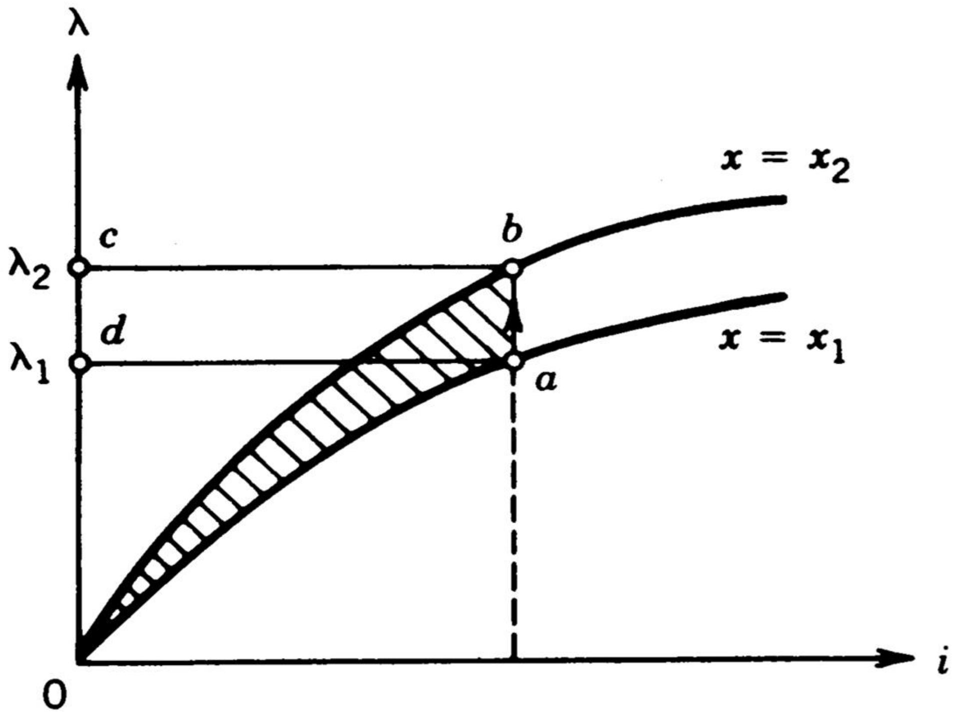


Mechanical Force – Scenario 1

$$\Delta W_{\mathrm{e}} = \int ei\,dt$$

$$\Delta W_{\mathrm{f}} = \int_{\lambda_1}^{\lambda_2} i\,d\lambda$$

$$\Delta W_{\mathrm{m}} = \Delta W_{\mathrm{e}} - \Delta W_{\mathrm{f}}$$



Differential displacement dx

$$f_{\mathrm{m}}\,dx = dW_{\mathrm{m}} = dW_{\mathrm{f}}'$$

$$f_{\mathrm{m}} = \left.\frac{\partial W_{\mathrm{f}}'(i,x)}{\partial x}\right|_{i=\mathrm{constant}}$$

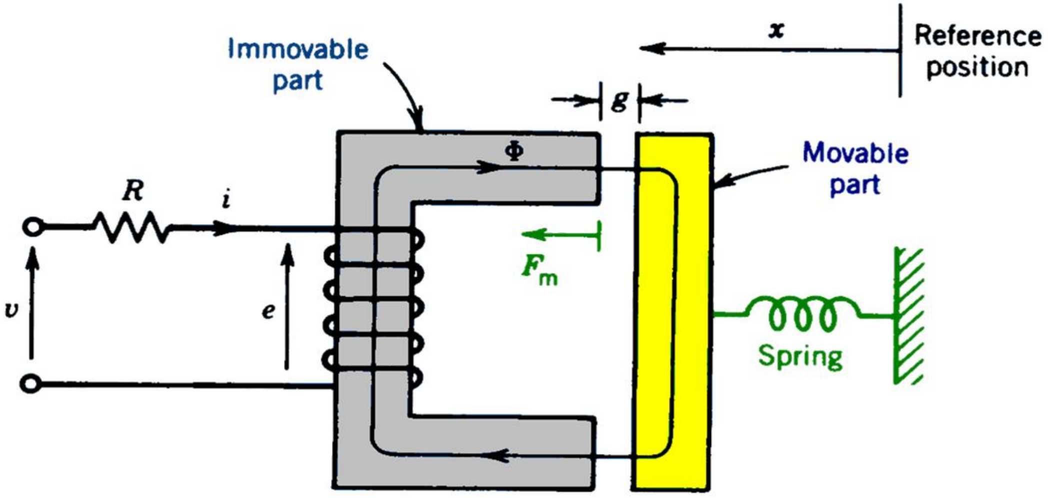
Mechanical Force – Scenario 2

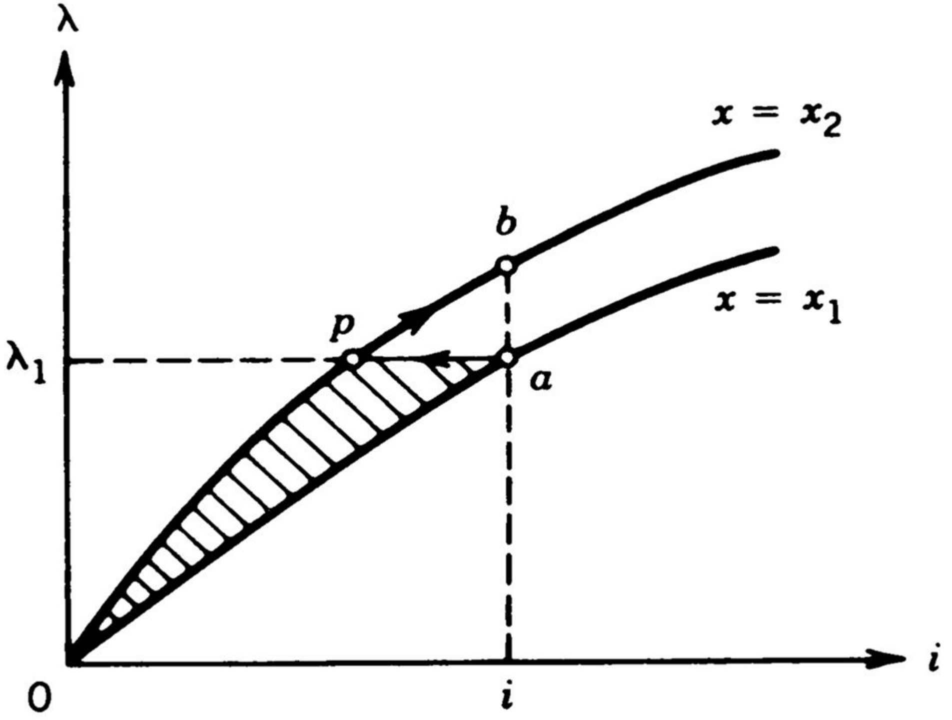
Movable part moves quickly from x_1 to x_2

$$dW_e = 0$$

$$f_m \, dx = dW_m = -dW_f$$

$$f_m = -\left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda=\text{constant}}$$





Force in a Linear System

Force from field energy

$$\lambda = L(x)i \quad \Rightarrow \quad W_{\mathrm{f}} = \frac{1}{2}L(x)i^2$$

$$f_{\mathrm{m}} = -\frac{\partial}{\partial x}\left(\frac{\lambda^2}{2L(x)}\right)\Bigg|_{\lambda=\text{constant}} \quad \Rightarrow \quad f_m = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

Force from co energy

$$W_{\mathrm{f}} = W_{\mathrm{f}}' = \frac{1}{2}L(x)i^2$$

$$f_{\mathrm{m}} = \frac{\partial}{\partial x}\left(\frac{1}{2}L(x)i^2\right)\Bigg|_{i=\text{constant}} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

Linear System (Rc << Rg)

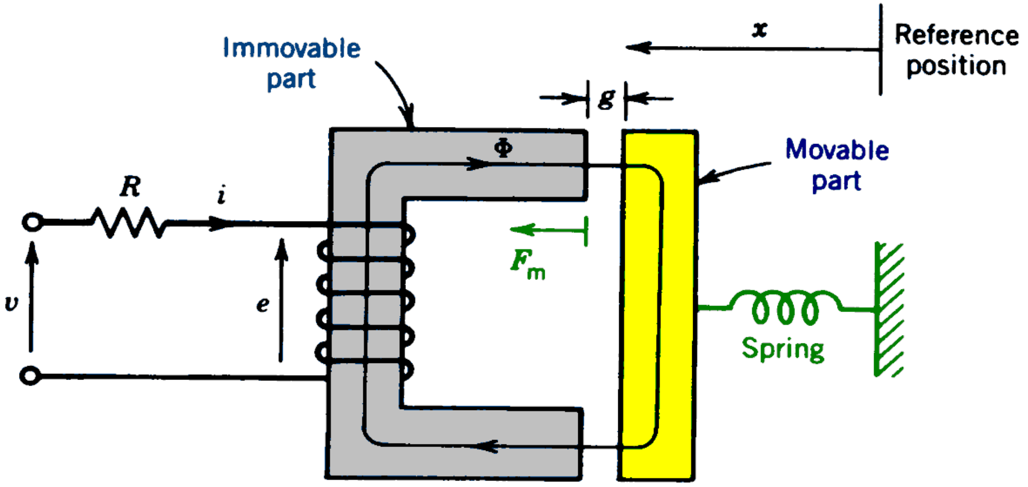
$$Ni = H_g 2g = \frac{B_g}{\mu_0} 2g$$

$$\mu_c = \infty$$
$$H_c = 0$$

$$W_f = \frac{B_g^2}{2\mu_0} V_g = \frac{B_g^2}{\mu_0} A_g g$$

Energy confined in the air gap

$$f_m = \frac{\partial}{\partial g}\left(\frac{B_g^2}{\mu_0} A_g g\right) = \frac{B_g^2}{\mu_0} A_g$$



Force pressure

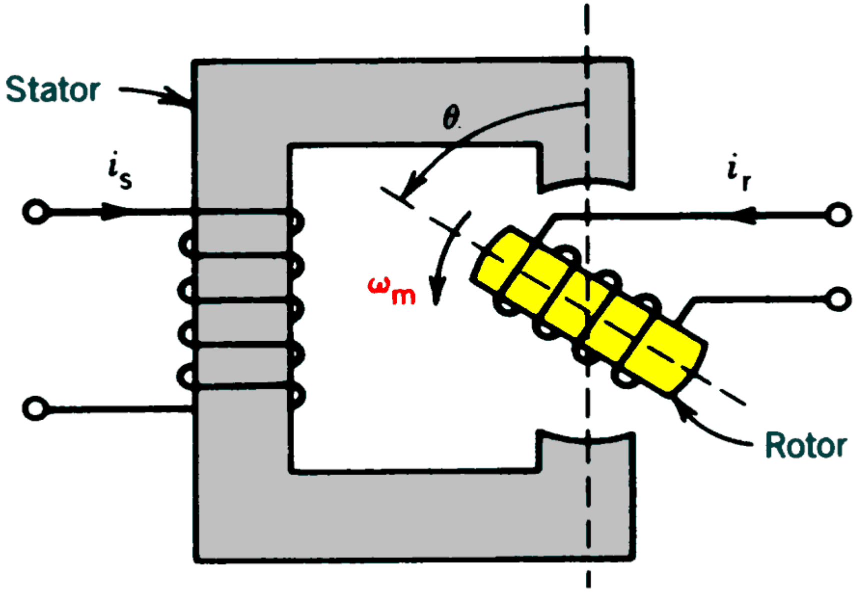
$$F_m = \frac{B_g^2}{2\mu_0}$$

Rotating Machines

Stored field energy

$$dW_f = e_s i_s \, dt + e_r i_r \, dt$$
$$= i_s \, d\lambda_s + i_r \, d\lambda_r$$

$$\lambda_s = L_{ss} i_s + L_{sr} i_r$$
$$\lambda_r = L_{rs} i_s + L_{rr} i_r$$



$$dW_f = L_{ss} i_s \, di_s + L_{rr} i_r \, di_r + L_{sr} \, d(i_s i_r)$$

$$\Rightarrow \quad W_f = \frac{1}{2} L_{ss} i_s^2 + \frac{1}{2} L_{rr} i_r^2 + L_{sr} i_s i_r$$

Torque

$$T = \left.\frac{\partial W_f'(i,\theta)}{\partial \theta}\right|_{i=\text{constant}}$$

$$W_f = \frac{1}{2}L_{ss}i_s^2 + \frac{1}{2}L_{rr}i_r^2 + L_{sr}i_s i_r$$

$$T = \frac{1}{2}i_s^2\frac{dL_{ss}}{d\theta} + \frac{1}{2}i_r^2\frac{dL_{rr}}{d\theta} + i_s i_r\frac{dL_{sr}}{d\theta}$$

For linear systems the energy is equal to the co-energy

First two terms represent reluctance torque; variation of self-inductance

Third term represents the torque produced due to the variation of mutual inductance

Cylindrical Machines

- No reluctance torque

$$T = i_s i_r \frac{dL_{sr}}{d\theta}$$

- Mutual inductance

$$L_{sr} = M \cos\theta$$

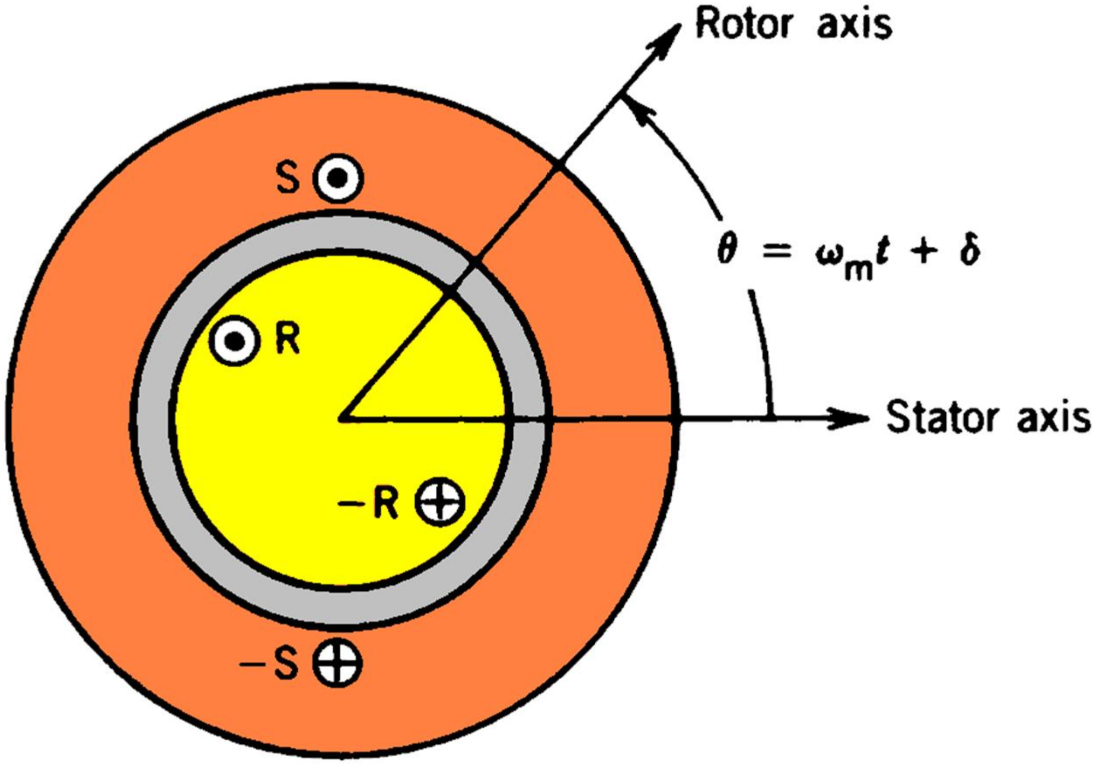
- Currents

$$i_s = I_{sm} \cos\omega_s t$$

$$i_r = I_{rm} \cos(\omega_r t + \alpha)$$

- Rotor position

$$\theta = \omega_m t + \delta$$



Basis for Synchronous and Asynchronous Machines

$$T = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4} \begin{bmatrix} \sin\left\{\left(\omega_{\mathrm{m}} + \left(\omega_{\mathrm{s}} + \omega_{\mathrm{r}}\right)\right)t + \alpha + \delta\right\} + \\ \sin\left\{\left(\omega_{\mathrm{m}} - \left(\omega_{\mathrm{s}} + \omega_{\mathrm{r}}\right)\right)t - \alpha + \delta\right\} + \\ \sin\left\{\left(\omega_{\mathrm{m}} + \left(\omega_{\mathrm{s}} - \omega_{\mathrm{r}}\right)\right)t - \alpha + \delta\right\} + \\ \sin\left\{\left(\omega_{\mathrm{m}} - \left(\omega_{\mathrm{s}} - \omega_{\mathrm{r}}\right)\right)t + \alpha + \delta\right\} \end{bmatrix}$$

- Torque in general varies sinusoidally with time
- Average value of each term is zero unless the coefficient of t is zero
- Nonzero average torque exists only if $\omega_{\mathrm{m}} = \pm\left(\omega_{\mathrm{s}} \pm \omega_{\mathrm{r}}\right)$

$$\left|\omega_{\mathrm{m}}\right| = \left|\omega_{\mathrm{s}} \pm \omega_{\mathrm{r}}\right|$$

Synchronous Machines

synchronous machine | $\omega_r = 0$ | $\omega_m = \omega_s$ | $\alpha = 0$

$$T = -\frac{I_{sm} I_R M}{2} \left\{ \sin\left(2\omega_s t + \delta\right) + \sin\delta \right\}$$

$$T_{avg} = -\frac{I_{sm} I_R M}{2} \sin\delta$$

- Single-phase machines, 1 winding at the stator
    - Pulsating Torque : NOT OK for larger machines!
    - Poly-phase machines to minimize pulsating torque

- $\omega_m = 0$ ➔ $T_{avg} = 0$ → Not self starting

Asynchronous Machines

asynchronous machine

$\omega_{\mathrm{m}} = \omega_{\mathrm{s}} - \omega_{\mathrm{r}}$ | $\omega_{\mathrm{m}} \neq \omega_{\mathrm{r}}$ | $\omega_{\mathrm{m}} \neq \omega_{\mathrm{s}}$

$$T = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4}\begin{bmatrix} \sin(2\omega_{\mathrm{s}} t + \alpha + \delta) + \sin(-2\omega_{\mathrm{r}} t - \alpha + \delta) + \\ \sin(2\omega_{\mathrm{s}} t - 2\omega_{\mathrm{r}} t - \alpha + \delta) + \sin(\alpha + \delta) \end{bmatrix}$$

$$T_{\mathrm{avg}} = -\frac{I_{\mathrm{sm}} I_{\mathrm{rm}} M}{4}\sin(\alpha + \delta)$$

- Single-phase machines
  - Pulsating Instantaneous Torque
  - Not self-starting
- Poly-phase machines minimize pulsating torque and self starting