Ch 7. Alternating Current Circuits

- Lecture outcomes (what you are supposed to learn):
  - Root Mean Square (tehollisarvot)
  - Phase and Phase shift (vaihekulma ja vaihesiirto)
  - Phasors as vectors and complex numbers (osoitin)
  - Real, Reactive, Complex power (Pätö- , Lois-, Näennäisteho)
  - Power factor (tehokerroin)
  - Electric energy

Introduction

Courtesy of CRC Press/Taylor & Francis Group

- Electricity discovered around 600 BC
- Extensive use started at the end of 19th century

- Batteries produce DC-voltage
- DC generator or Dynamo at beginning of 19th century

- DC power system for lighting 1882
  - Low voltage (100 V)
  - Large losses and voltage drop in wires

- AC motor and generator + transformer
- AC power system with high voltage

Courtesy of CRC Press/Taylor & Francis Group

Why AC won the battle ?

- Voltage level can be adjusted with transformers

- High-voltage allow for long transmission lines
  - Low losses
  - Low wire cross-section

Courtesy of CRC Press/Taylor & Francis Group

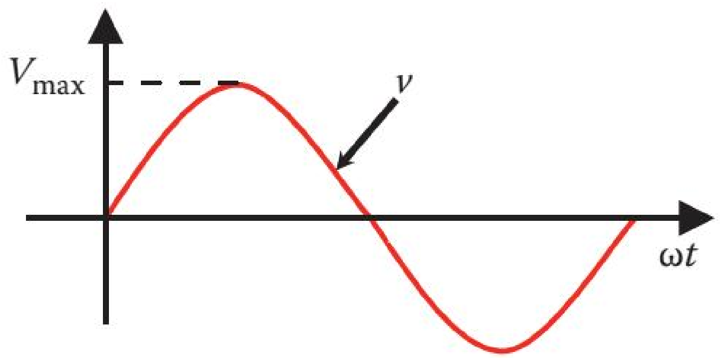
- AC system produce rotating field necessary to spin motors

- Is the battle over? Rectifiers, inverters, HVDC, local DC

Where dc-system won?

Basic quantities

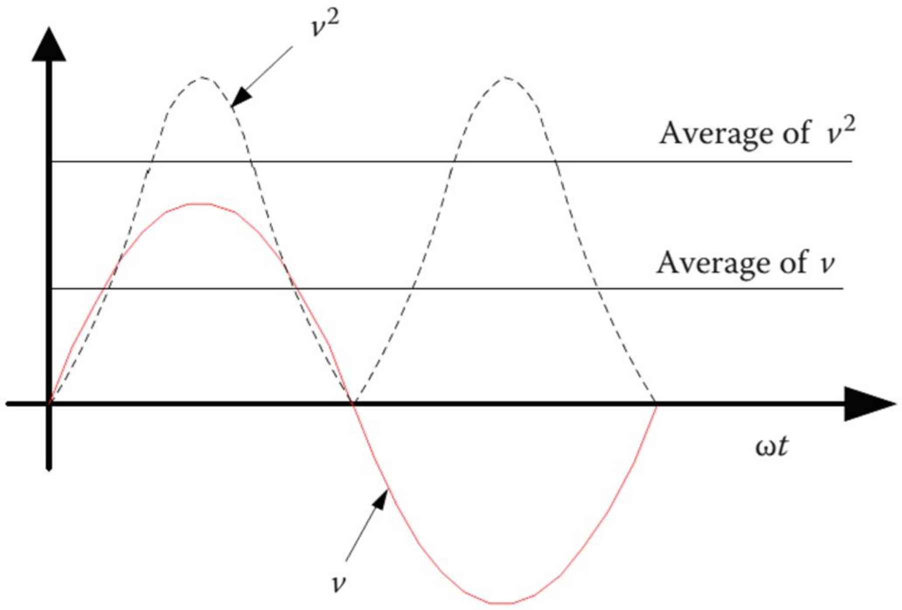
- Waveform $v = V_{\max} \sin \omega t$



- Frequency (taajuus): $f = \dfrac{\omega}{2\pi}$

- Root Mean Square (tehollisarvo)
  - Makes it possible to quantify distorted waves

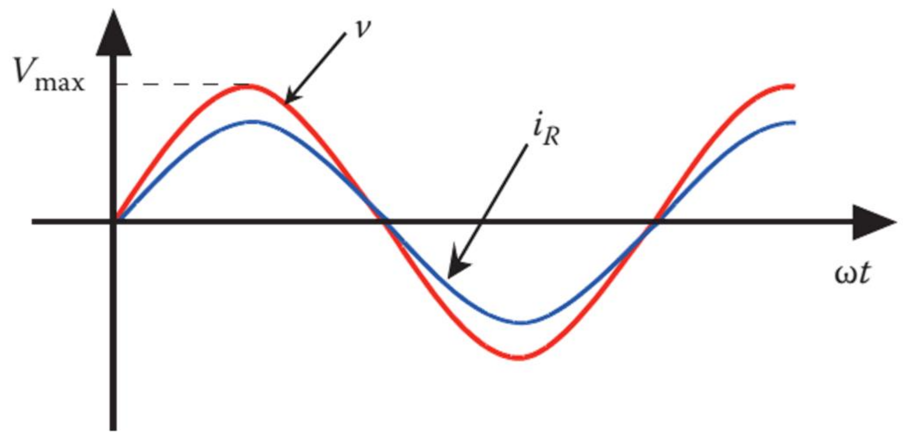
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$



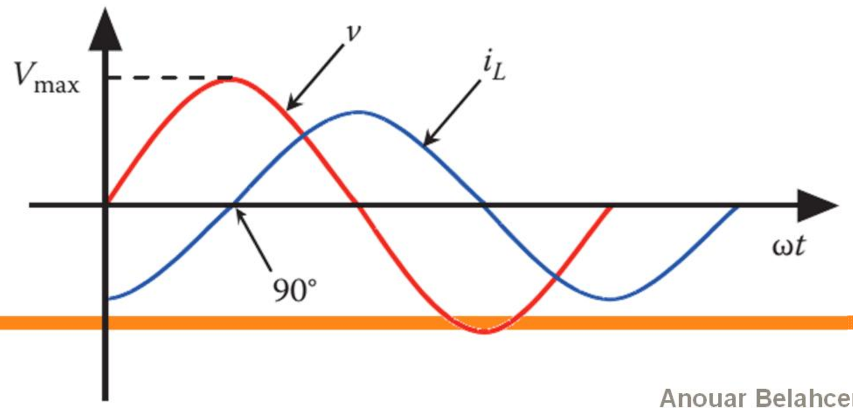
Phase shift

- Waveform $v = V_{\max} \sin \omega t$

- Resistive load: $i = \dfrac{V_{\max}}{R} \sin \omega t$



- Inductive load: $i = \dfrac{1}{L}\int_0^t v dt = -\dfrac{V_{\max}}{\omega L} \cos \omega t$

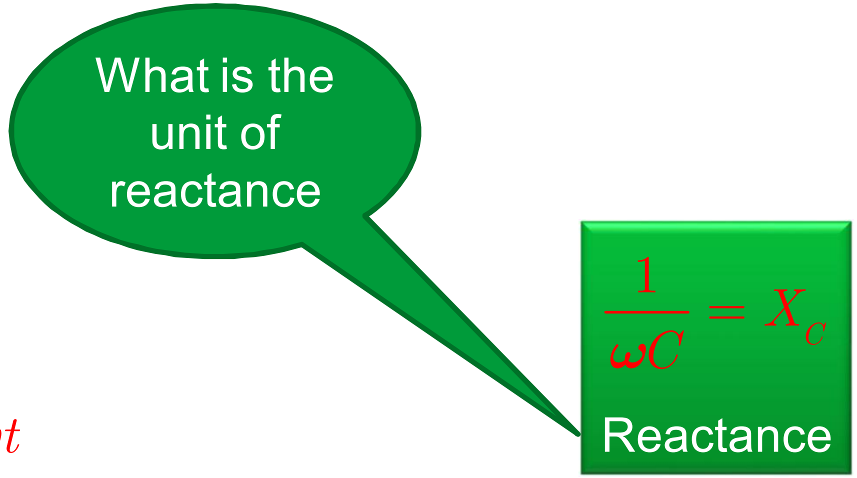


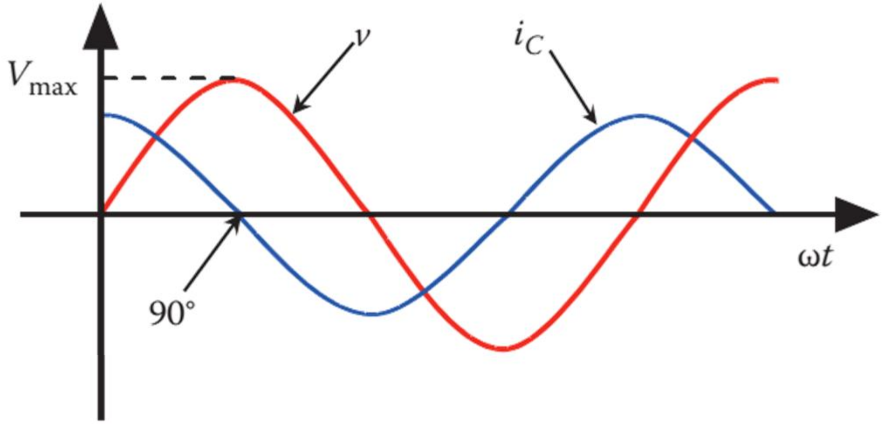
$\omega L = X_L$

Reactance

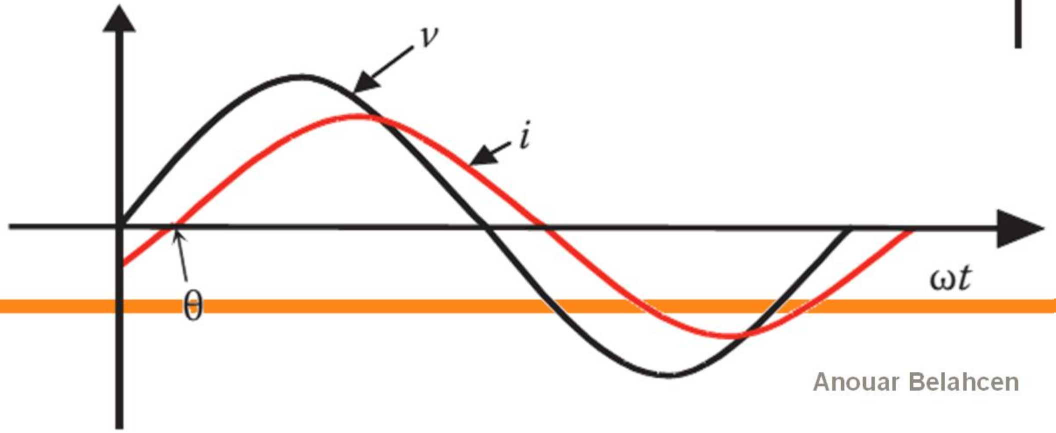
Phase shift

- Waveform $v = V_{\max} \sin \omega t$
- Capacitive load: $i = C\dfrac{dv}{dt} = \omega C V_{\max} \cos \omega t$





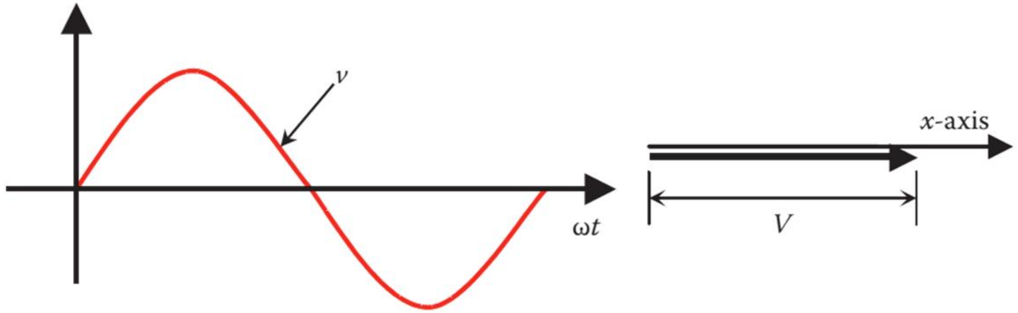
- Complex load: $i = I_{\max} \sin(\omega t - \theta)$



Concept of Phasors

- Graphical representation with mathematical basis

- Give quick information on magnitude and phase shifts

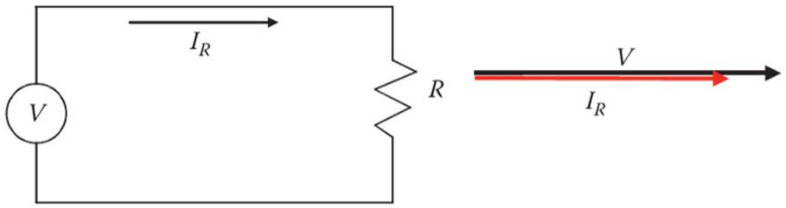


- Vector length proportional to rms value

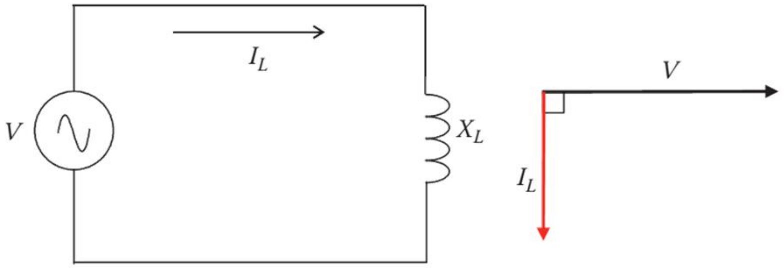
- Angle with respect to x-axis equals to phase shift

Concept of Pahsors

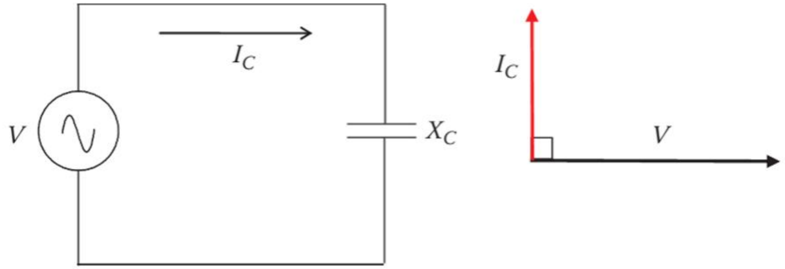
- Resistive load



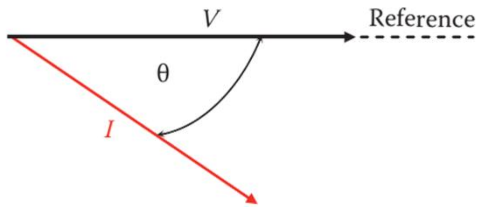
- Inductive load



- Capacitive load

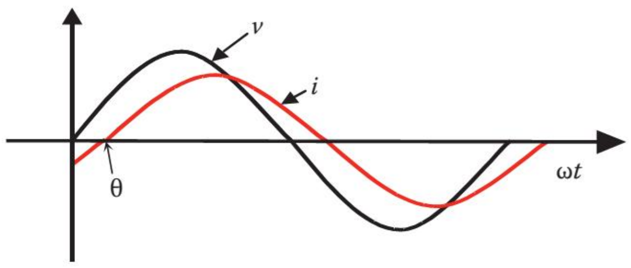


- Complex load

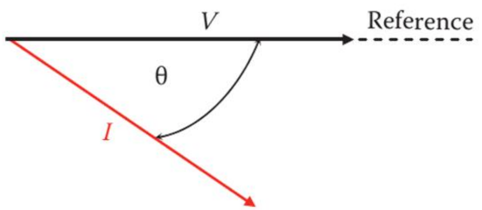


Lagging and leading phase shifts

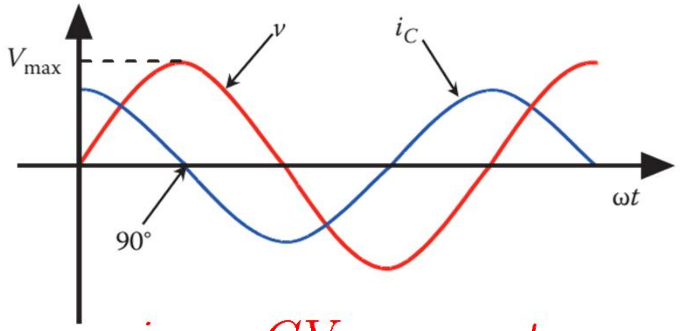
Current lagging voltage



$$i = I_{max} \sin(\omega t - \theta)$$

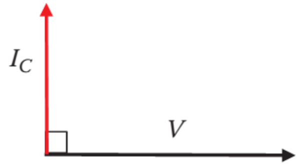


Current leading voltage



$$i = \omega C V_{max} \cos \omega t$$

$$= \omega C V_{max} \sin(\omega t + \frac{\pi}{2})$$

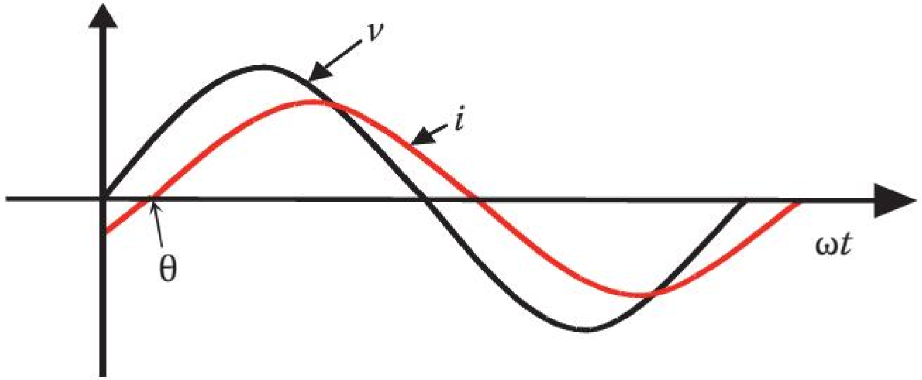


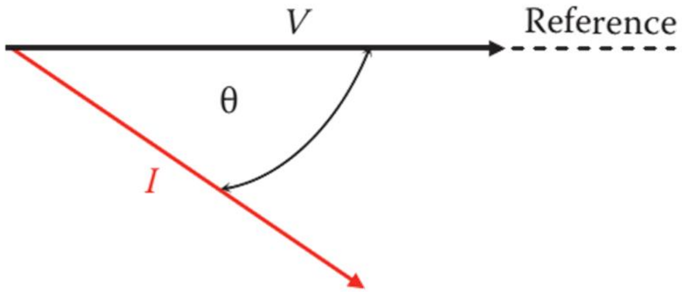
Complex representation of Phasors

- Phasors are good visuals but not convenient to compute
- Vectors can be represented as complex numbers
- Calculus with complex number is handy.

$$\bar{V} = V\angle 0°$$

$$\bar{I} = I\angle -\boldsymbol{\theta}°$$





Complex numbers calculus

$$\bar{A} = A\angle\boldsymbol{\theta}^{\circ} = A\left[\cos\boldsymbol{\theta} + j\sin\boldsymbol{\theta}\right] = X + jY$$

- Real component

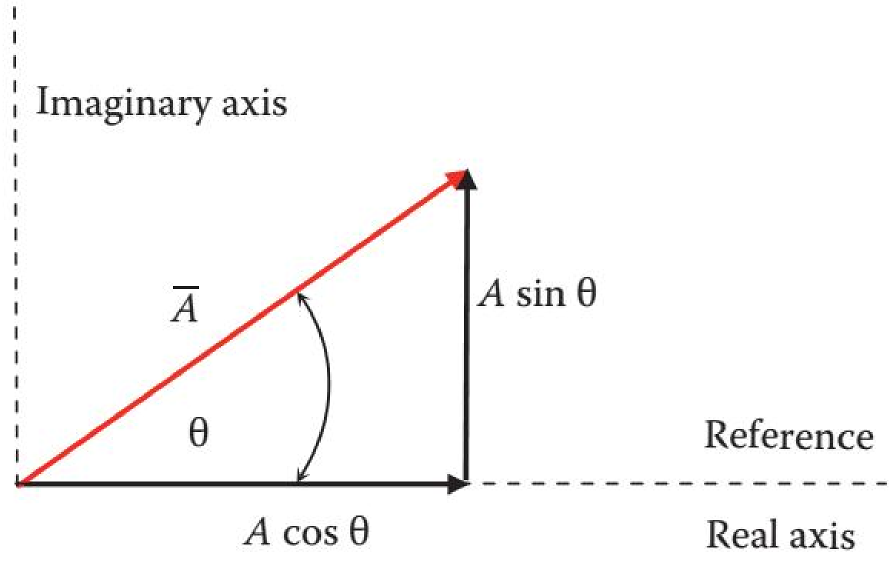
$$A\cos\boldsymbol{\theta} = X$$

- Imaginary component

$$A\sin\boldsymbol{\theta} = Y$$

- Conjugate

$$\bar{A}^{*} = X - jY = A\angle-\boldsymbol{\theta}^{\circ}$$



Impedances

- Basic impedances
  - Resistance

$$R = \frac{\bar{V}}{\bar{I}_R} = \frac{V\angle 0}{I_R\angle 0} = R\angle 0$$

  - Inductance

$$\bar{X}_L = \frac{\bar{V}}{\bar{I}_L} = \frac{V\angle 0}{I_L\angle -90^\circ} = X_L\angle 90^\circ$$

  - Capacitance

$$\bar{X}_C = \frac{\bar{V}}{\bar{I}_C} = \frac{V\angle 0}{I_C\angle 90^\circ} = X_C\angle -90^\circ$$

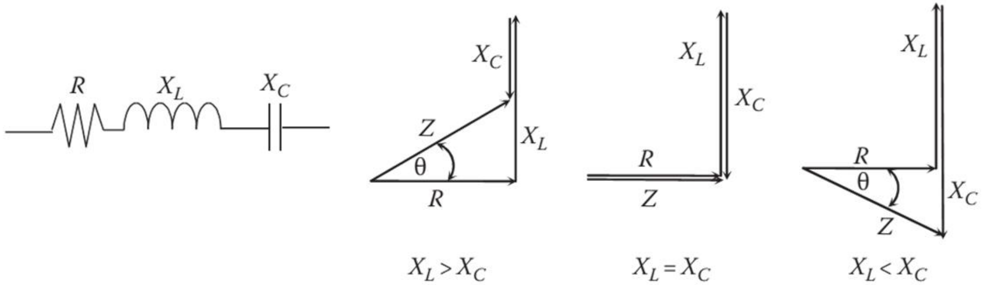
- Complex
  - Series and parallel connections of basic impedances

Admittance

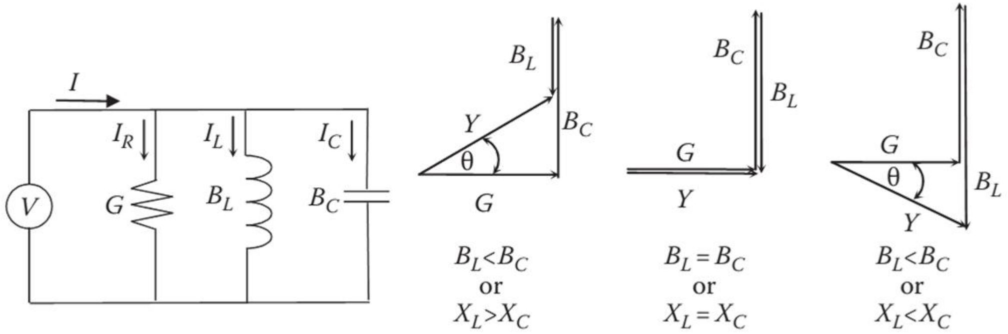
$$\bar{Y} = \frac{1}{\bar{Z}}$$

What is the unit of admittance

- Series connected impedances are additive



- Parallel connected impedances are not additive
- Parallel connected admittances are additive



$$\bar{G} = \frac{1}{\bar{R}} = \frac{1}{R}\angle 0^\circ$$

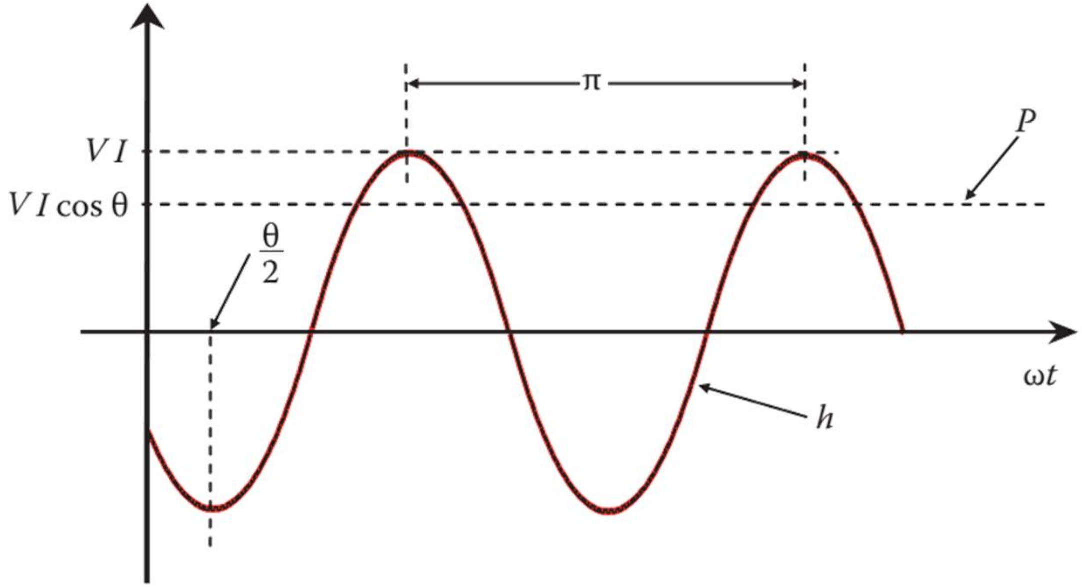
$$\bar{B}_L = \frac{1}{\bar{X}_L} = \frac{1}{X_L}\angle -90^\circ$$

$$\bar{B}_C = \frac{1}{\bar{X}_C} = \frac{1}{X_C}\angle 90^\circ$$

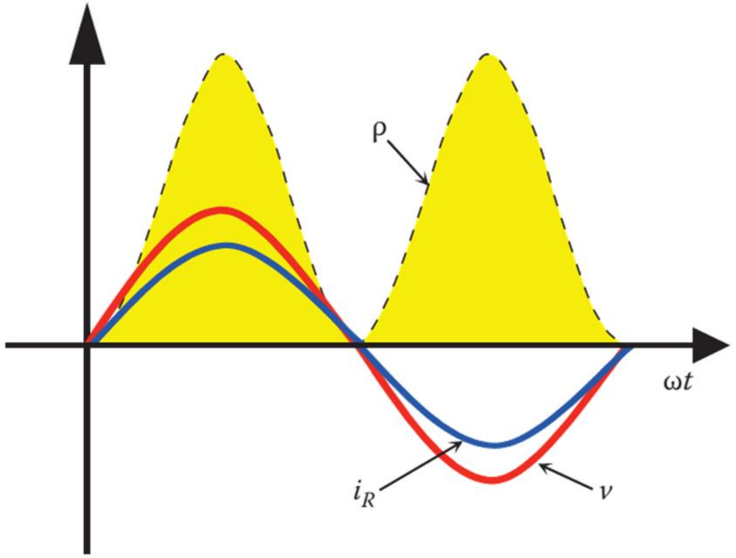
Electric power

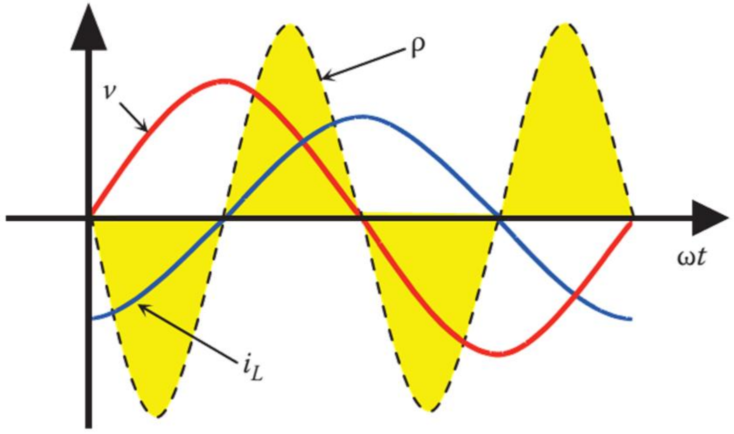
- Instantaneous power $p = vi$

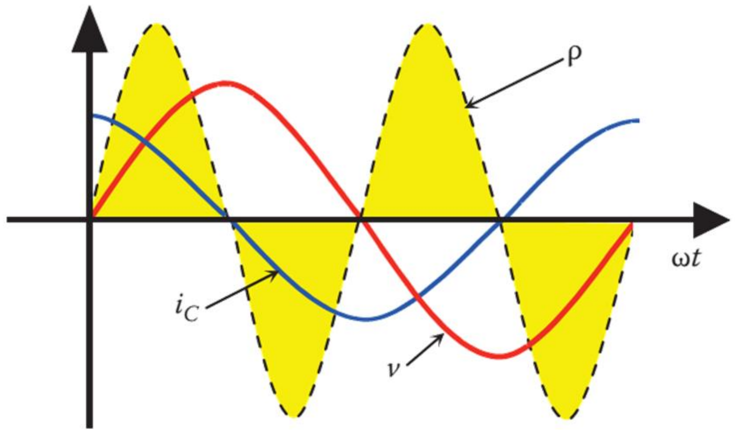
- For a complex load
$$p = \left[V_{\max}\sin(\omega t)\right]\left[I_{\max}\sin(\omega t - \theta)\right]$$
$$= VI\left[\cos(\theta) - \cos(2\omega t - \theta)\right]$$

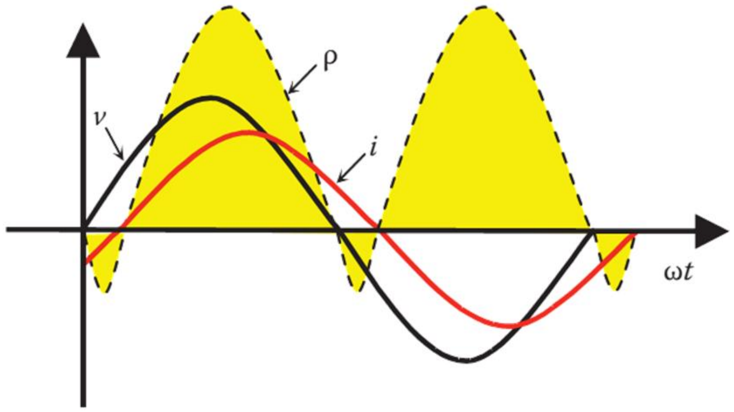


Instantaneous power



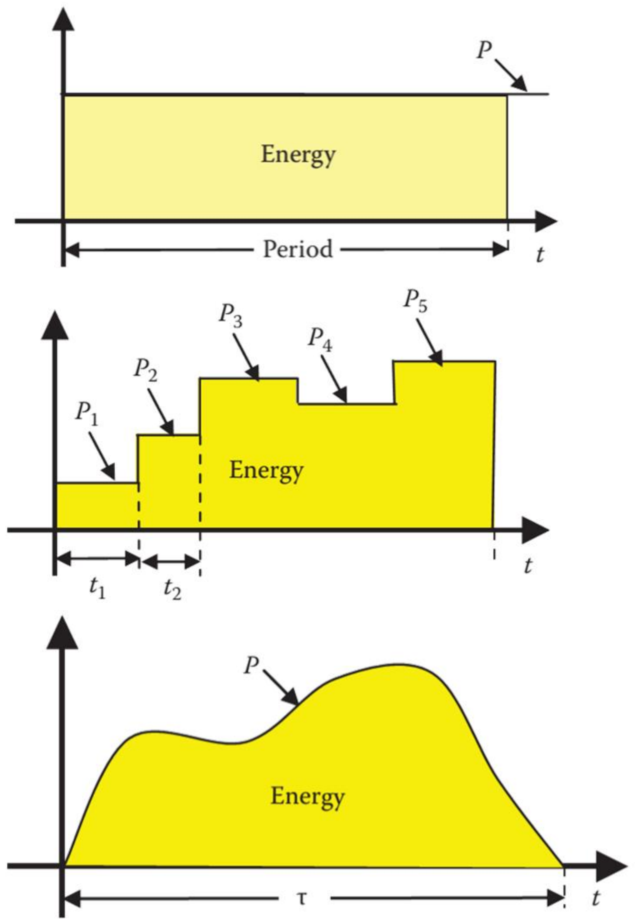






Electric energy

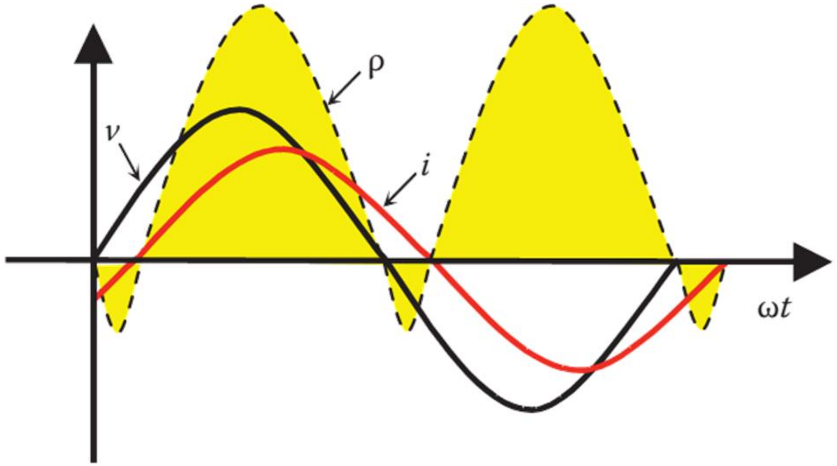
- Energy is the time integral of the power
  - Constant power $E = pT$
  - Discrete power $E = \sum_i p_i T_i$
  - Variable power $E = \int_0^T p\,dt$



Active, Reactive and Complex Power

- The power that produces energy is the average of $p = vi$

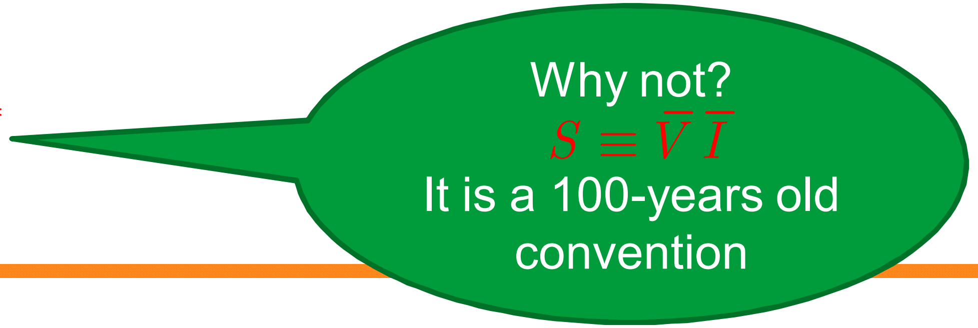
$$P = \frac{1}{2\pi}\int_0^{2\pi} p d(\omega t) = VI\cos(\theta)$$



- This is called Active or Real power

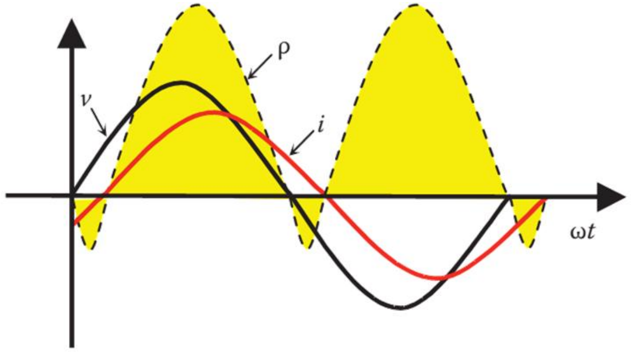
- The Complex or Apparent power is defined as

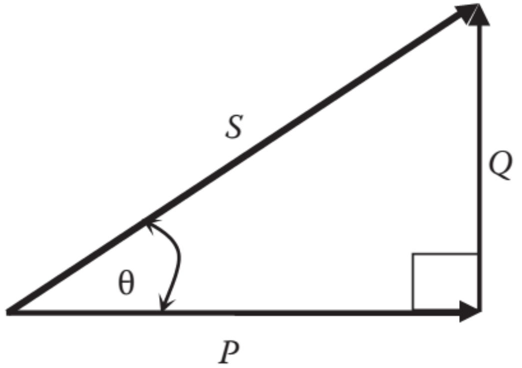
$$S \equiv \bar{V}\,\bar{I}^*$$



Active, Reactive and Complex Power

- Part of the instantaneous power does not produce energy, it is called Reactive or Imaginary power $Q \equiv VI\sin(\boldsymbol{\theta})$





$$S = \bar{V}\,\bar{I}^* = VI\cos(\boldsymbol{\theta}) + jVI\sin(\boldsymbol{\theta})$$

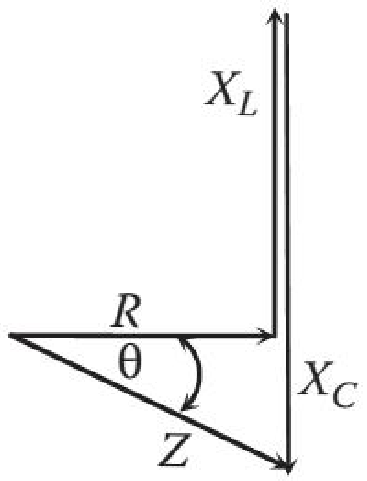
- In case of sinusoidal quantities the reactive power is well defined but not obvious in case of distorted quantities !

Power factor

- The power factor is defined as the ratio of real to apparent power

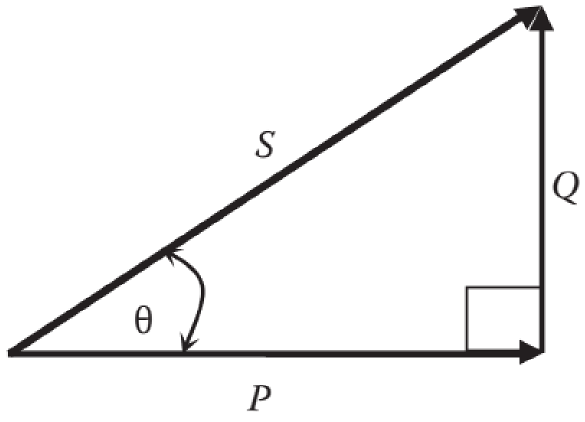
$$pf = \frac{P}{S} = \cos(\boldsymbol{\theta})$$

- In an electric circuit it can be calculated as



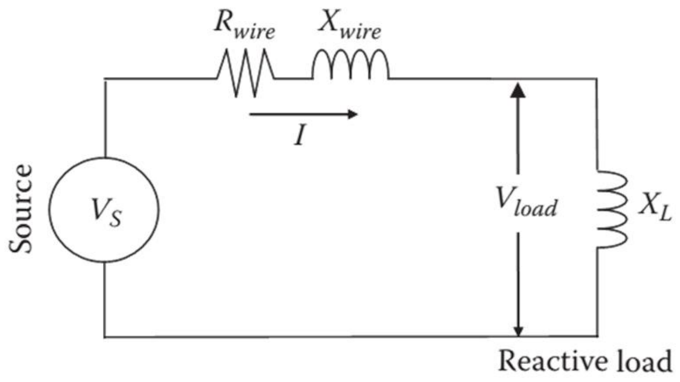
$$pf = \frac{R}{Z} = \frac{R}{\sqrt{X^2 + R^2}}$$

$$pf = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}$$



Problems with power factor

- Increases losses in the transmission line
- Reduces the sparse capacity of transmission line
- Reduces the voltage across the load
- Need for power factor correction



$$\bar{I} = \frac{\bar{V}_s}{R_{wire} + j(X_{wire} + X_{load})}$$

$$P_{loss} = I^2 R_{wire}$$

$$V_{load} = \frac{V_s}{\sqrt{\left(\frac{R_{wire}}{X_L}\right)^2 + \left(1 + \frac{X_{wire}}{X_L}\right)^2}}$$

Summary of the lecture

- Sinusoidal voltage and current represented by phasors
- Phasor has amplitude and phase ⟹ complex number
- The nature of load (resistive, reactive) produces phase shift between voltage and current ⟹ power factor
- Real power produces work, reactive power is a parasitic effect that increases the line losses
- Real mean square useful measure of AC-quantities even under distortion.