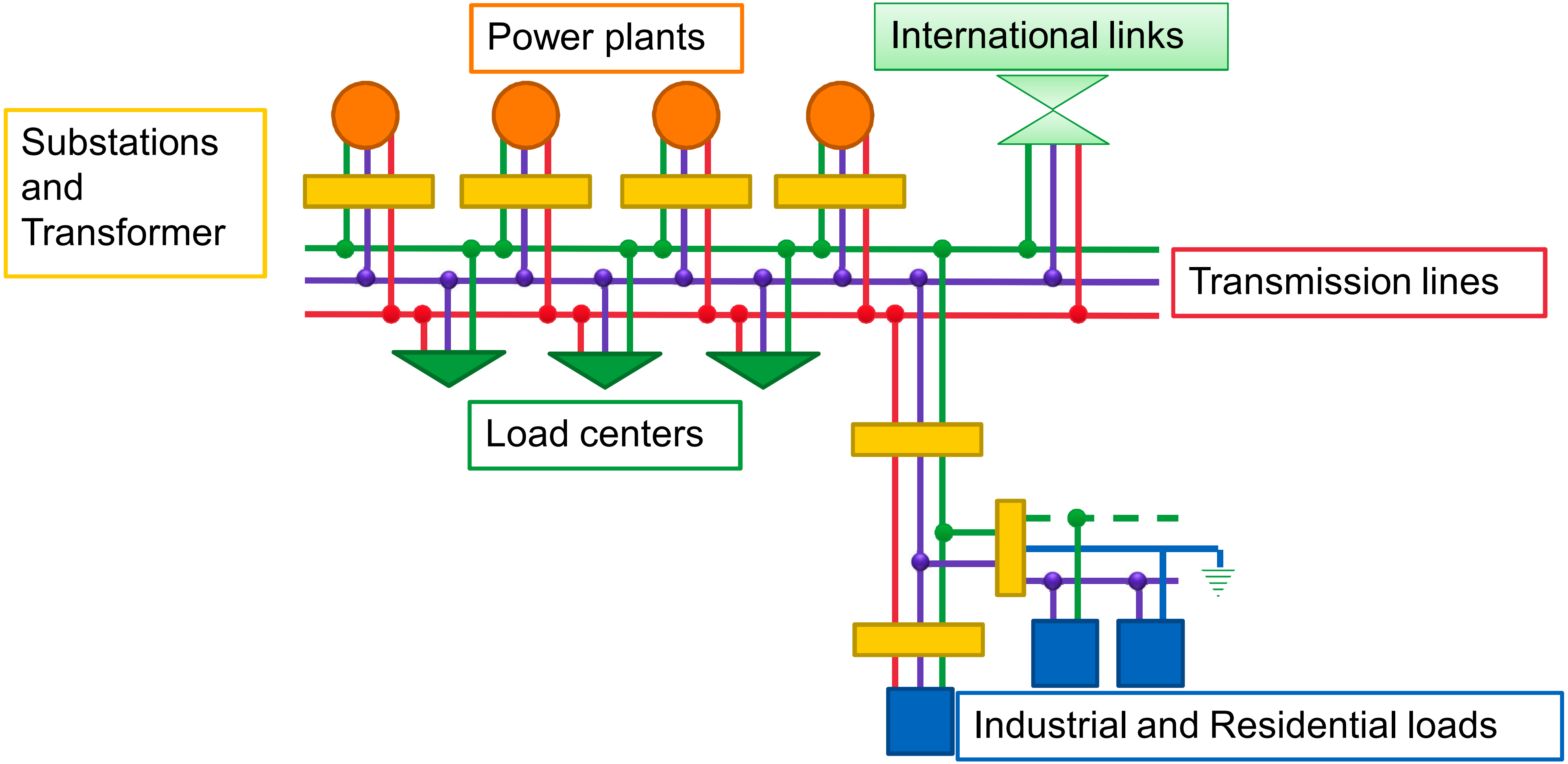
Ch 8. Three-phase systems

- Lecture outcomes (what you are supposed to learn):
  - Generation of three-phase voltages
  - Connection of three-phase circuits
  - Wye-Delta transformation
  - Power of three-phase connected loads

Introduction

- High power equipments are built as three-phase systems.
- Three-phase systems can produce rotating field without special control.
- Three phase generator produce more power than single phase one with the same volume.
- Three-phase systems are more reliable. They can deliver power even if one phase fails.

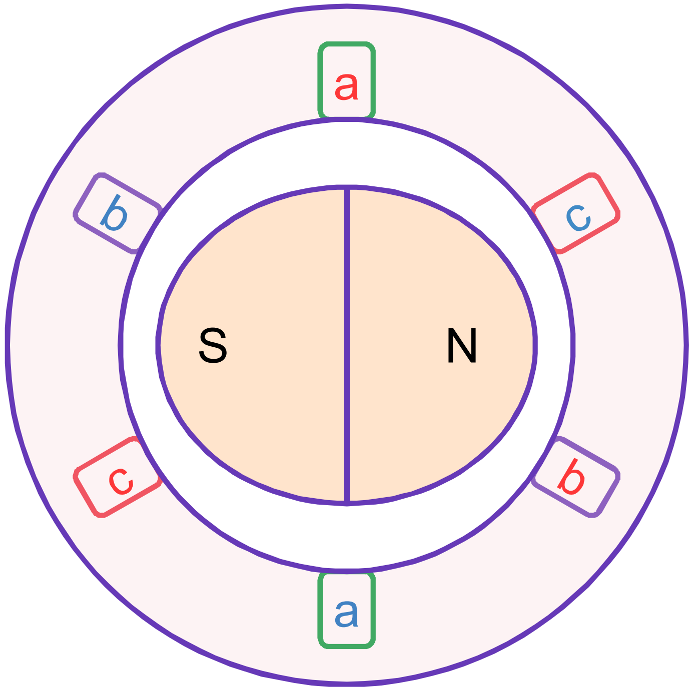
Schematic structure of Power systems

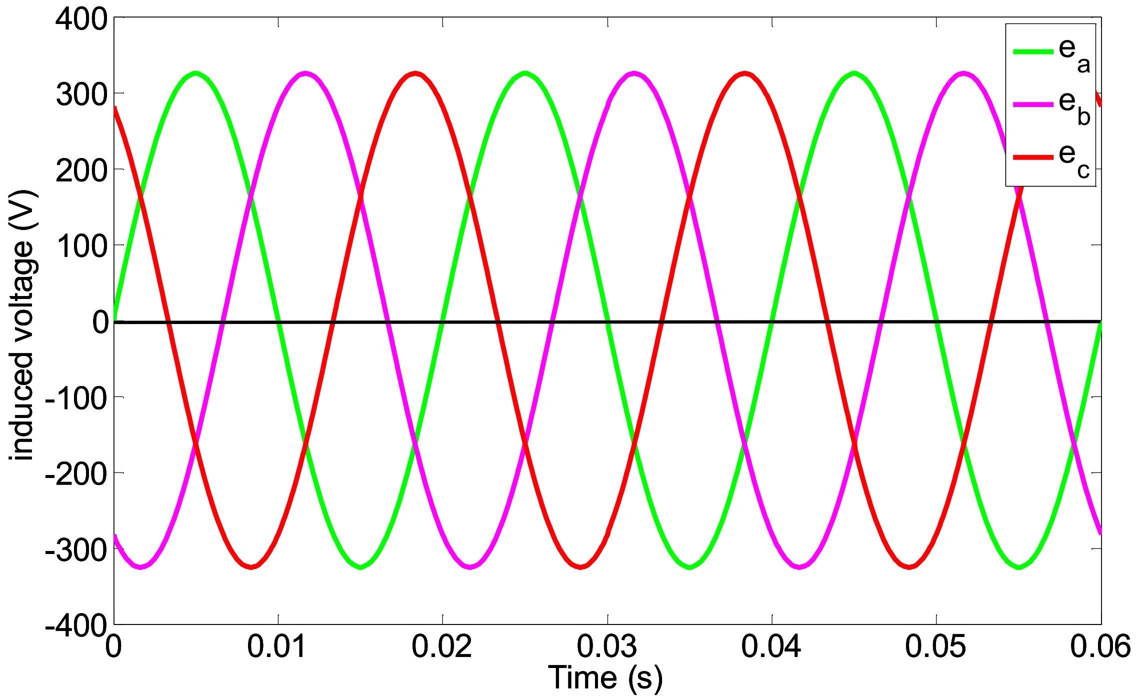


Generation of three-phase voltages (kolmivaihejärjestelmä)

- Simple three-phase generator

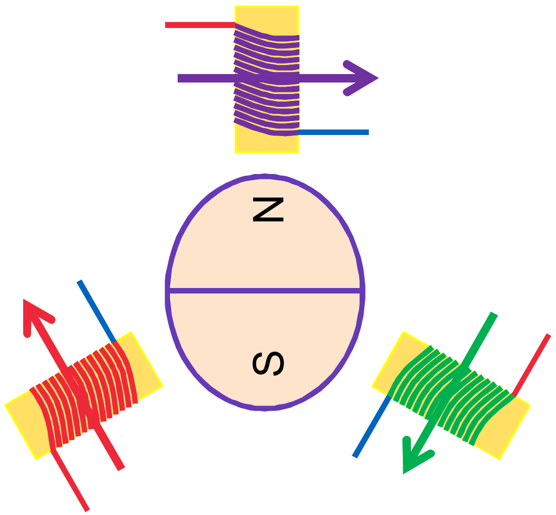
$e = nBl\omega$





Generation of three-phase voltages

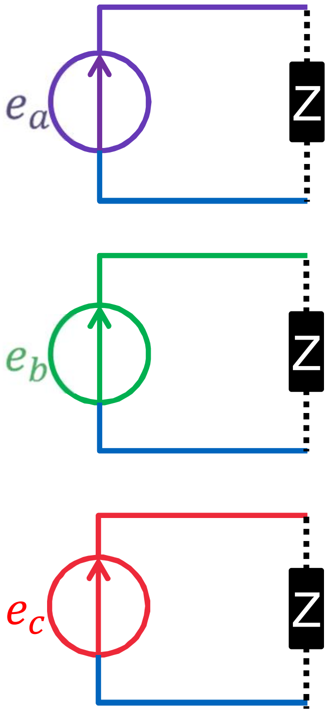
- 3 single-phase circuits at different phase angle!



$$v_{aa'} = e_a = V_{\max} \cos(\omega t)$$

$$v_{bb'} = e_b = V_{\max} \cos(\omega t - 120°)$$

$$v_{cc'} = e_c = V_{\max} \cos(\omega t - 240°)$$
$$= V_{\max} \cos(\omega t + 120°)$$



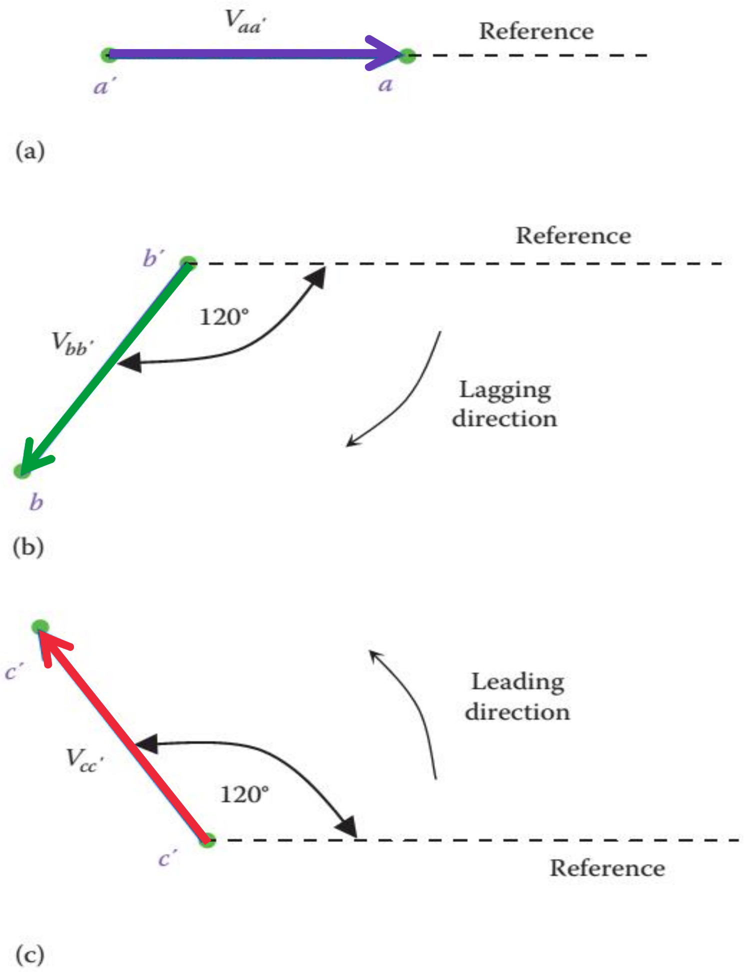
Generation of three-phase voltages

- The voltage of each phase are called phase voltages
- The phase voltages are written in complex form

$$\bar{V}_{aa'} = \frac{V_{max}}{\sqrt{2}} \angle 0° = V\angle 0°$$

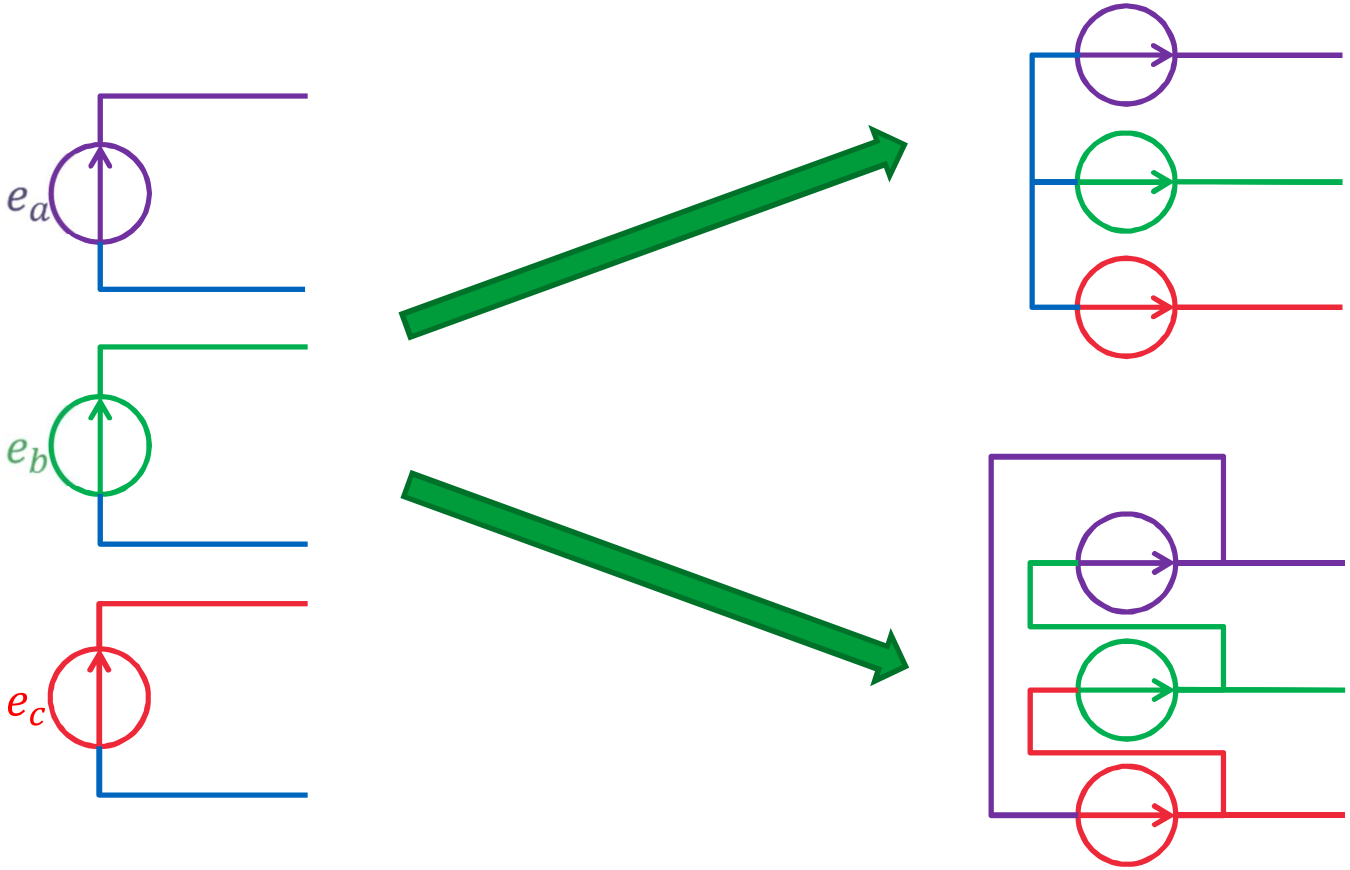
$$\bar{V}_{bb'} = \frac{V_{max}}{\sqrt{2}} \angle -120° = V\angle -120°$$

$$\bar{V}_{cc'} = \frac{V_{max}}{\sqrt{2}} \angle 120° = V\angle 120°$$

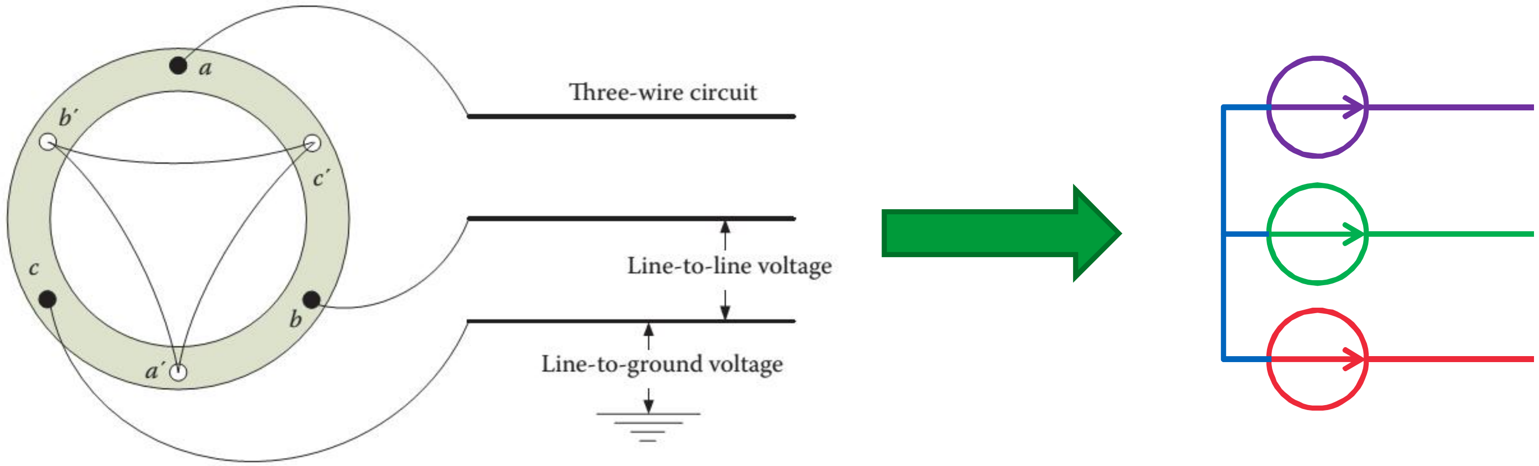


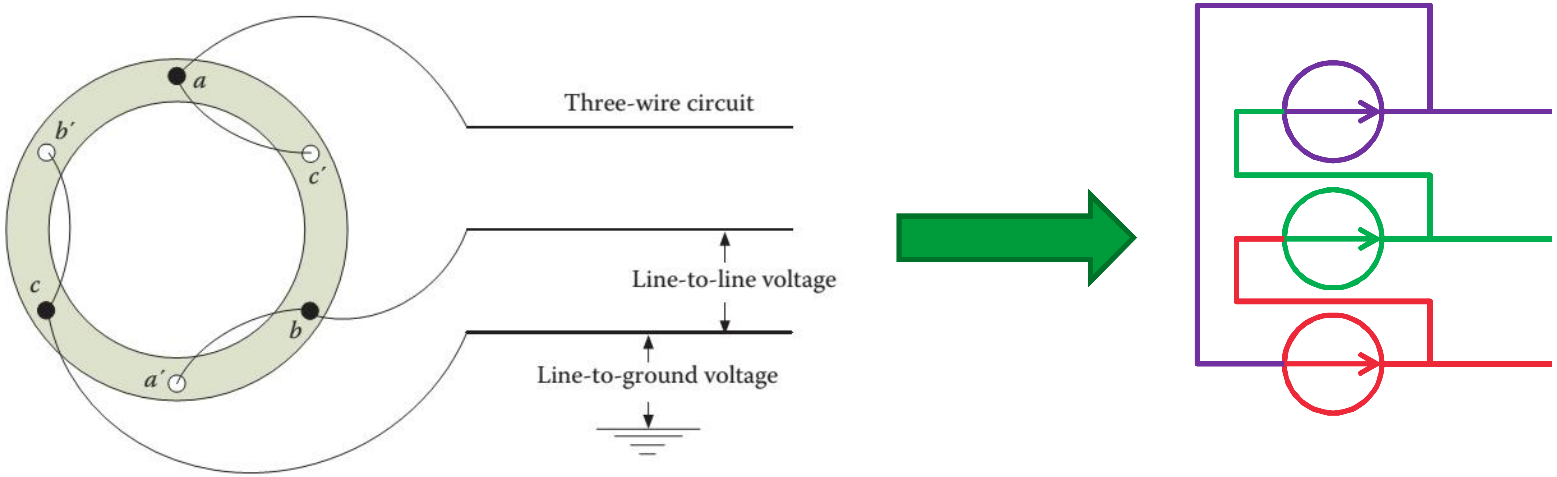
Connecting the 3-phase voltages

- The potential difference is known but not the potentials !



Connecting the 3-phase voltages

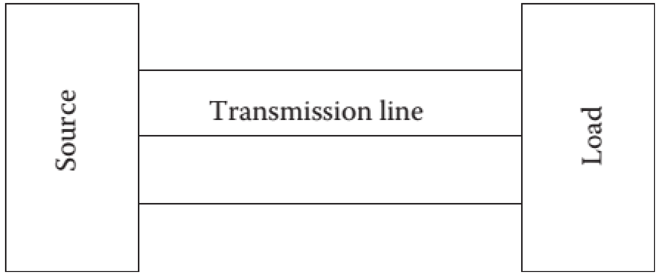




Line-to-line voltage
=
Pääjännitte

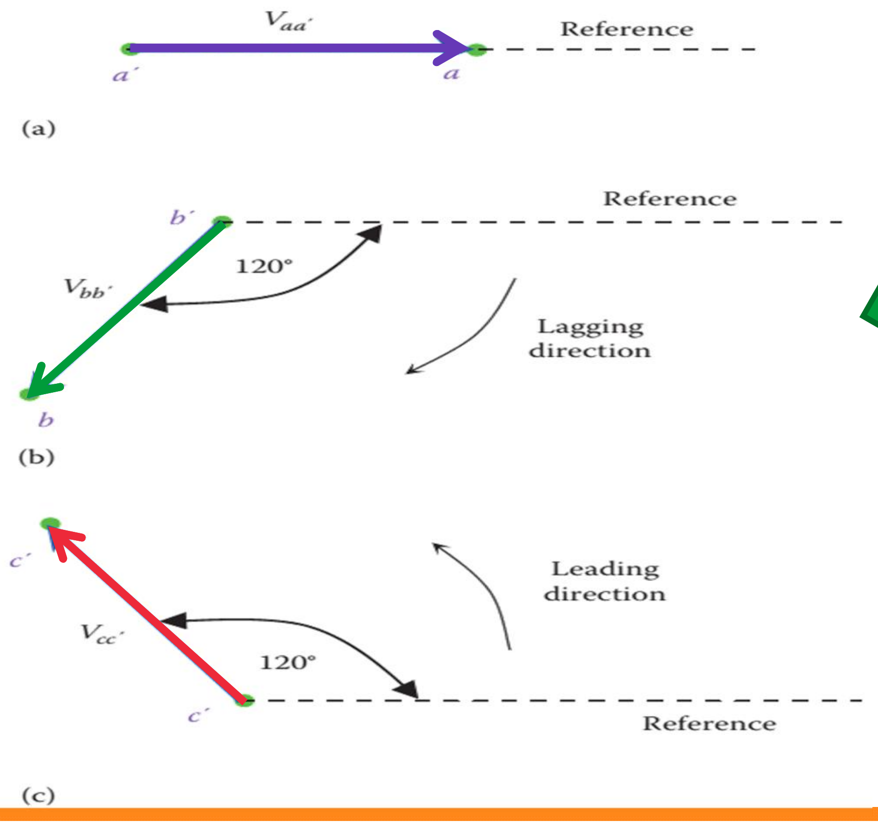
Connecting the source and load

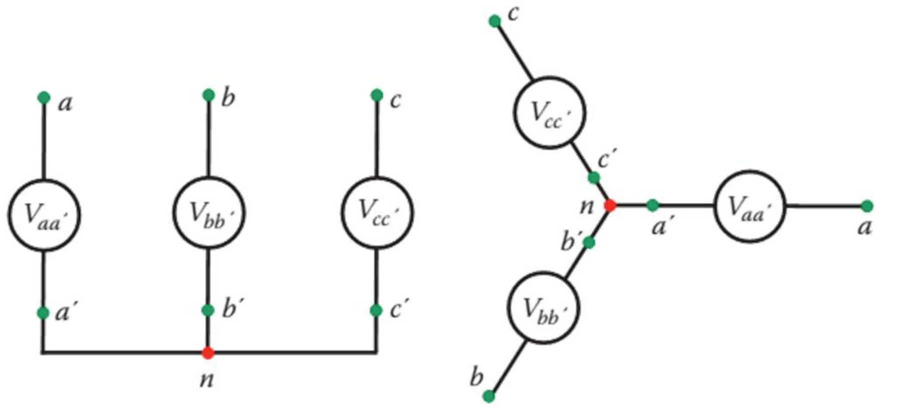
- Only 3 wires are needed to connect the source and load

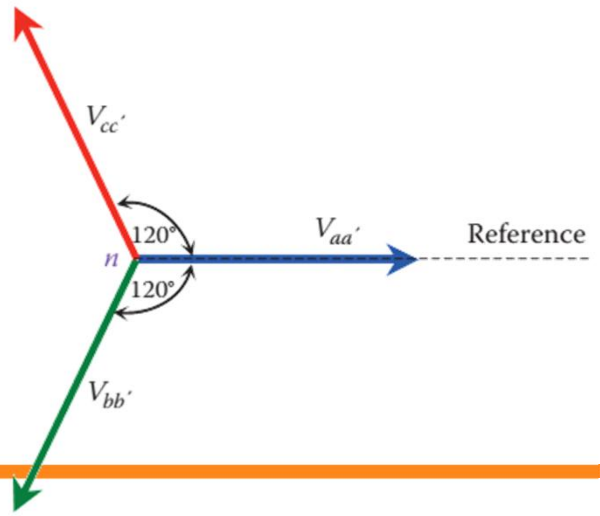


Wye connection (Y- tai tähtikytkentä)

- Three similar terminal of each coil connected to the same point called neutral or N



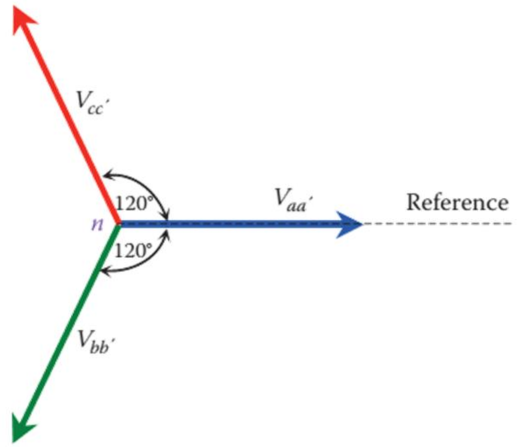




Wye connection (Y- tai tähtikytkentä)

- For balanced symmetrical three phase system:

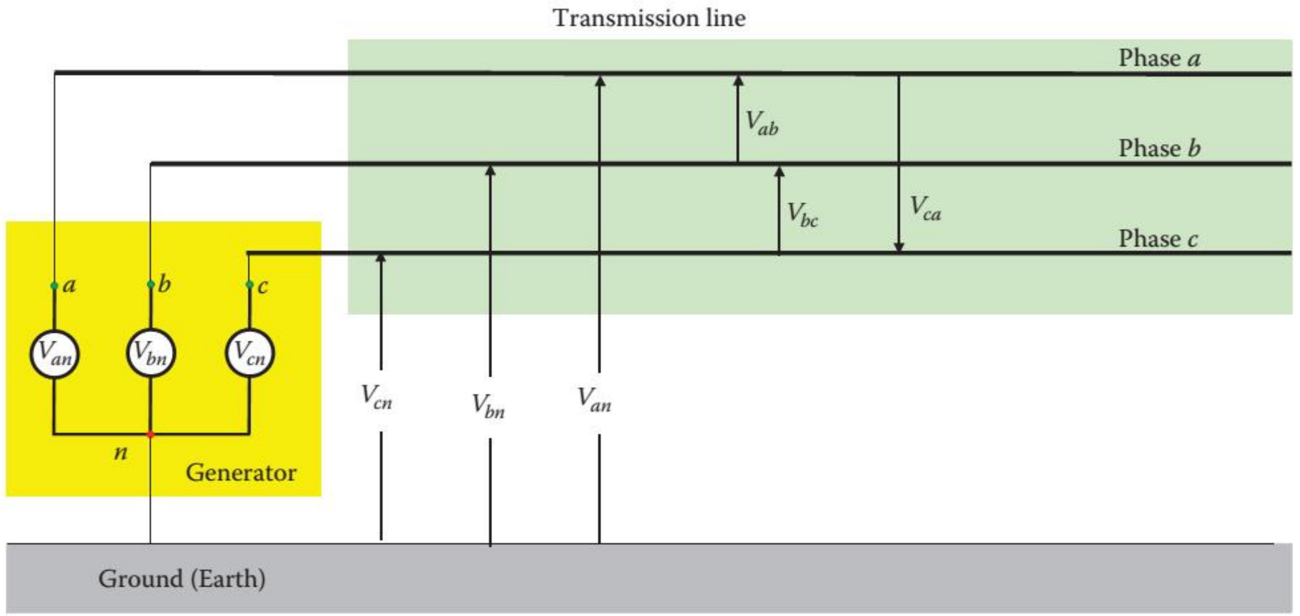
$$V_{an} = V_{bn} = V_{cn} = V_{ph}$$



$$\bar{V}_{an} = V_{ph}\angle 0°$$

$$\bar{V}_{bn} = V_{ph}\angle -120°$$

$$\bar{V}_{cn} = V_{ph}\angle 120°$$



Line-to-line voltages in Wye connection

$$\bar{V}_{an} = V_{ph}\angle 0°$$

$$\bar{V}_{bn} = V_{ph}\angle -120°$$

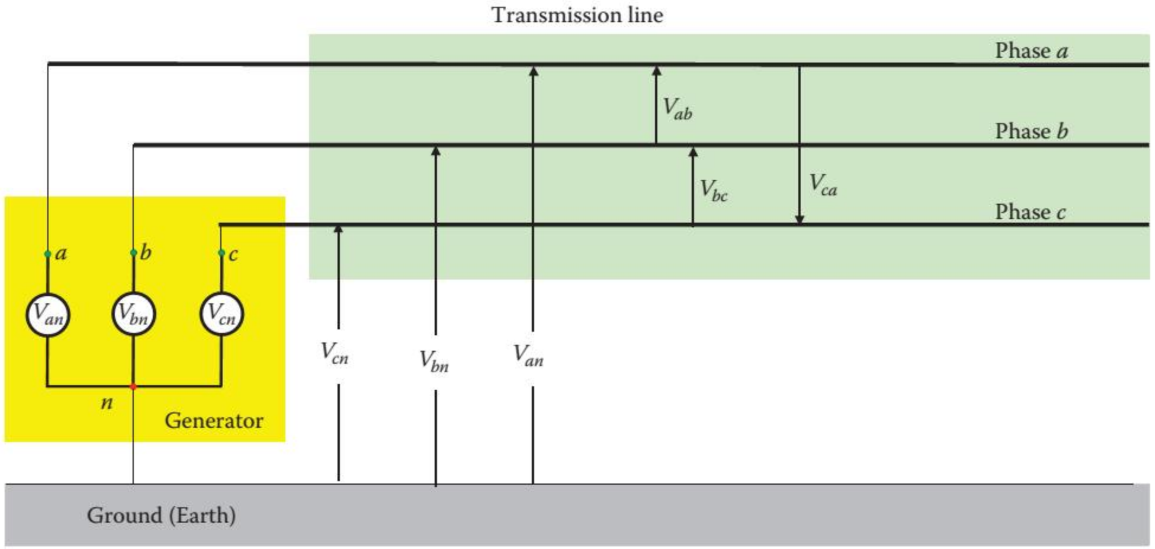
$$\bar{V}_{cn} = V_{ph}\angle 120°$$

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn}$$

$$\bar{V}_{ab} = V_{ph}\angle 0° - V_{ph}\angle -120° = \sqrt{3}V_{ph}\angle 30°$$

- Magnitude of line-to-line voltage $\bar{V}_{ab}$ is larger than the magnitude of phase voltages $\bar{V}_{an}$ by a factor of $\sqrt{3}$

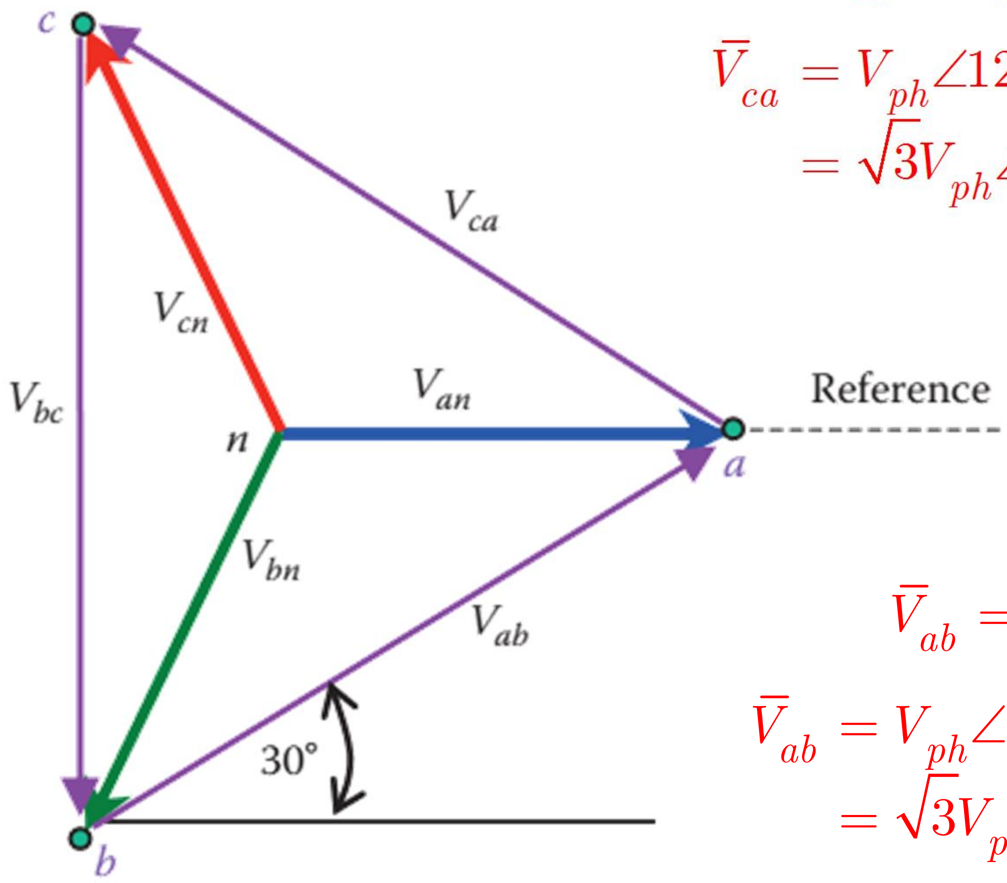
- Line-to-line voltage $\bar{V}_{ab}$ leads $\bar{V}_{an}$ by $30°$



Line-to-line voltages in Wye connection

$$\bar{V}_{ca} = \bar{V}_{cn} - \bar{V}_{an}$$

$$\bar{V}_{ca} = V_{ph}\angle 120° - V_{ph}\angle 0° = \sqrt{3}V_{ph}\angle 150°$$



$$\bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn}$$

$$\bar{V}_{bc} = V_{ph}\angle -120° - V_{ph}\angle 120° = \sqrt{3}V_{ph}\angle -90°$$

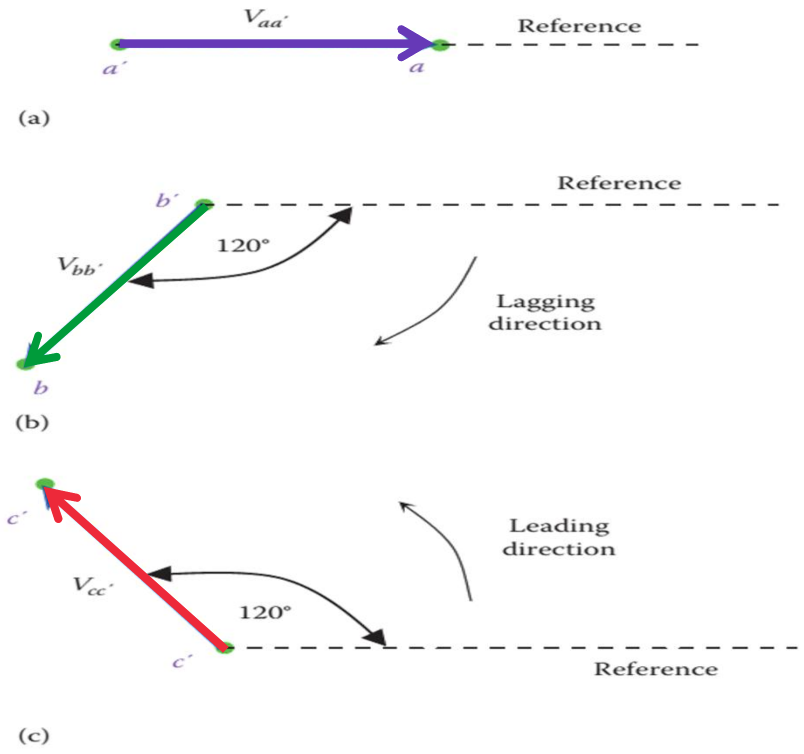
$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn}$$

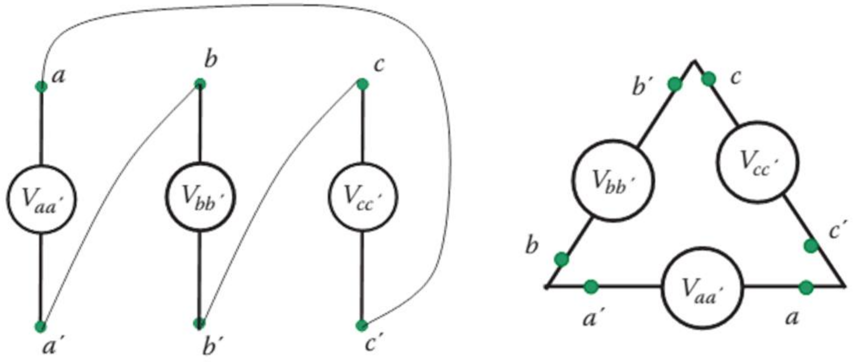
$$\bar{V}_{ab} = V_{ph}\angle 0° - V_{ph}\angle -120° = \sqrt{3}V_{ph}\angle 30°$$

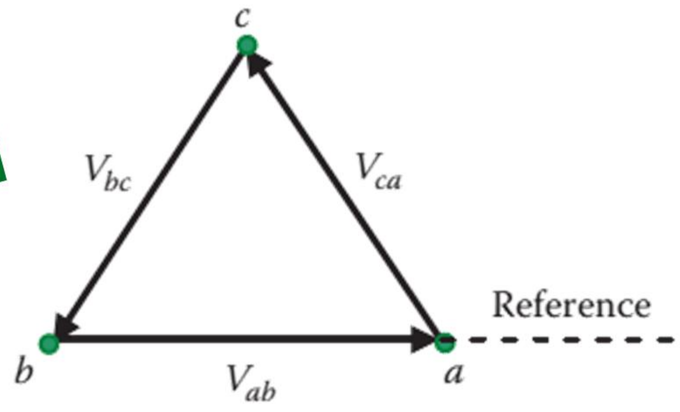
$$V_{ll} = \sqrt{3}V_{ph}$$

Delta connection (kolmiokytkentä)

- The entrance terminal of one coil is connected to the end terminal of the next coil







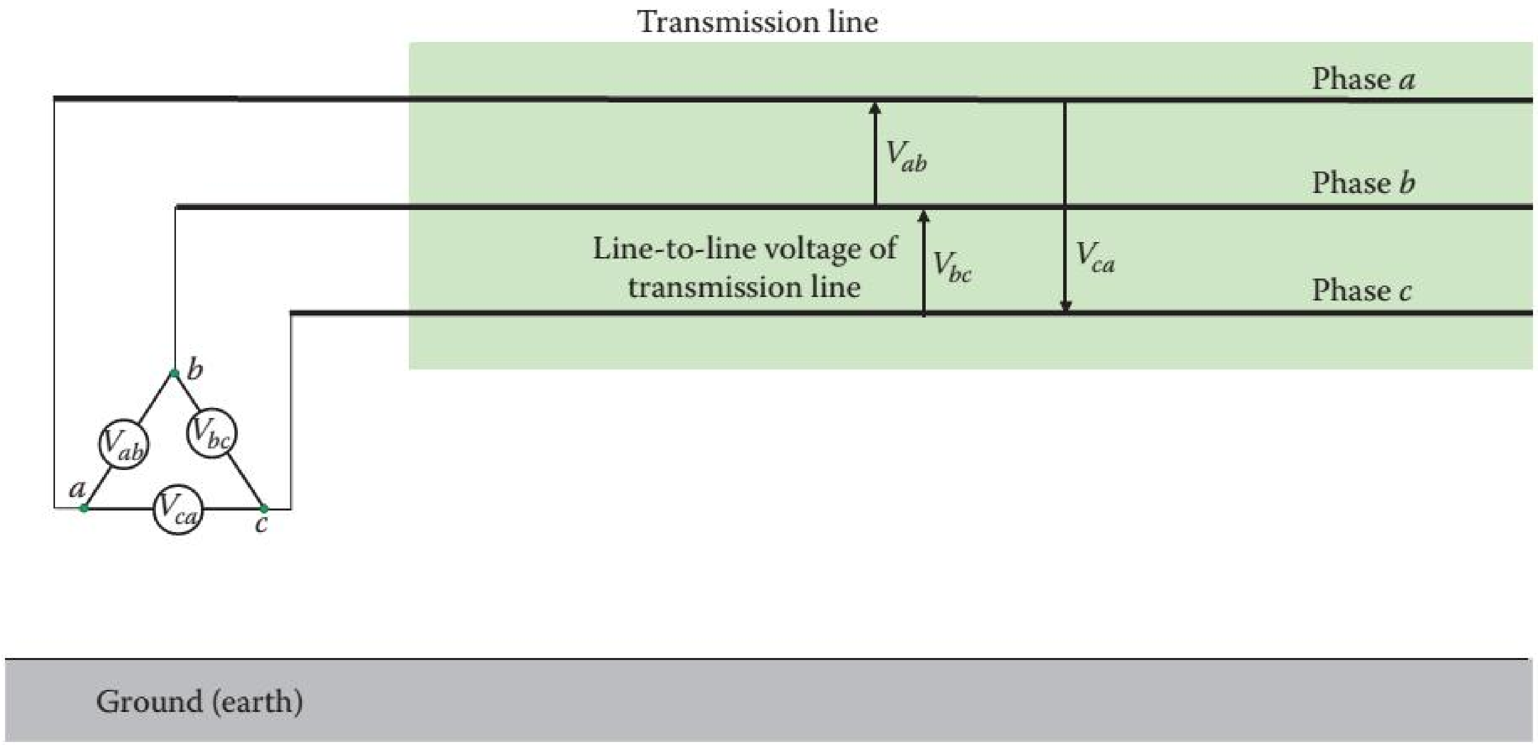
Delta connection

- Absence of a neural point i.e. floating potentials
- Phase voltages are identical with line-to-line voltages

$$\bar{V}_{aa'} = \bar{V}_{ab}$$

$$\bar{V}_{bb'} = \bar{V}_{bc}$$

$$\bar{V}_{cc'} = \bar{V}_{ca}$$



Single and three phases loads

- Residential loads are usually single phase loads (230 V)
- Industrial and commercial loads are mostly three phases loads (400 V)

- Clustered residential areas are powered by three phases

- Single houses might be powered by single phase

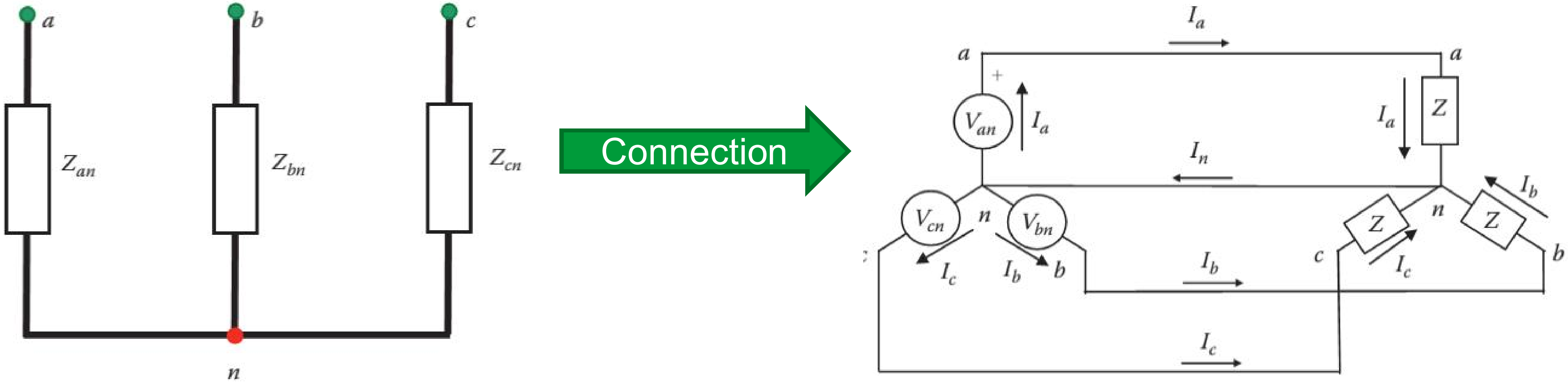
- Neutral point is grounded to ensure that all loads are powered regardless of fluctuations in current

Wye connected load

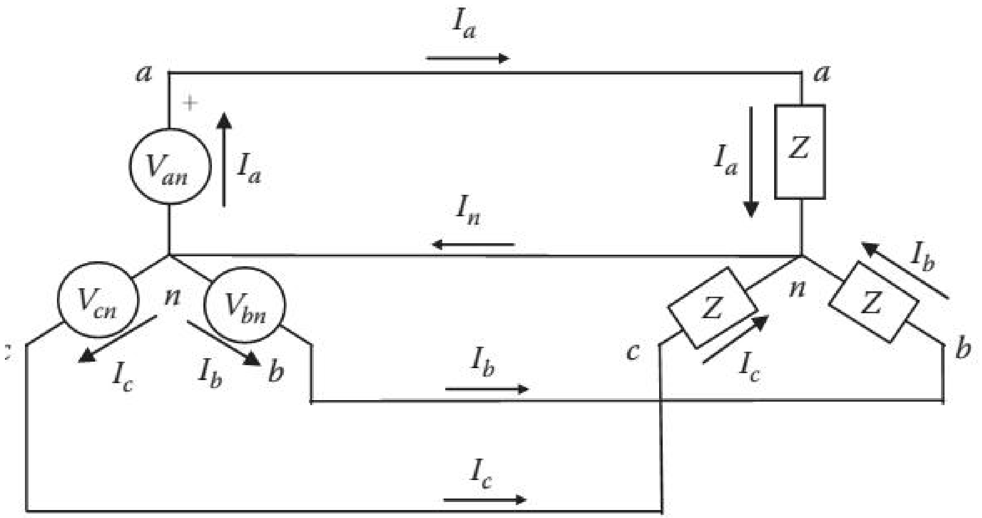
- Load impedances connected to a common neutral point from one terminal

$$\bar{Z}_{an} = \bar{Z}_{bn} = \bar{Z}_{cn} = \bar{Z}$$

- The load is powered by a three phases source



Wye connected load



- Phase currents

$$\bar{I}_a = \frac{\bar{V}_{an}}{\bar{Z}} = \frac{V_{ph}\angle\boldsymbol{\theta}}{Z\angle\varphi} = \frac{V_{ph}}{Z}\angle(\boldsymbol{\theta} - \varphi)$$

$$\bar{I}_b = \frac{\bar{V}_{bn}}{\bar{Z}} = \frac{V_{ph}\angle(\boldsymbol{\theta} - 120)}{Z\angle\varphi} = \frac{V_{ph}}{Z}\angle(\boldsymbol{\theta} - \varphi - 120)$$

$$\bar{I}_c = \frac{\bar{V}_{cn}}{\bar{Z}} = \frac{V_{ph}\angle(\boldsymbol{\theta} + 120)}{Z\angle\varphi} = \frac{V_{ph}}{Z}\angle(\boldsymbol{\theta} - \varphi + 120)$$

- Equal magnitudes, 120 phase shifts

Wye connected load

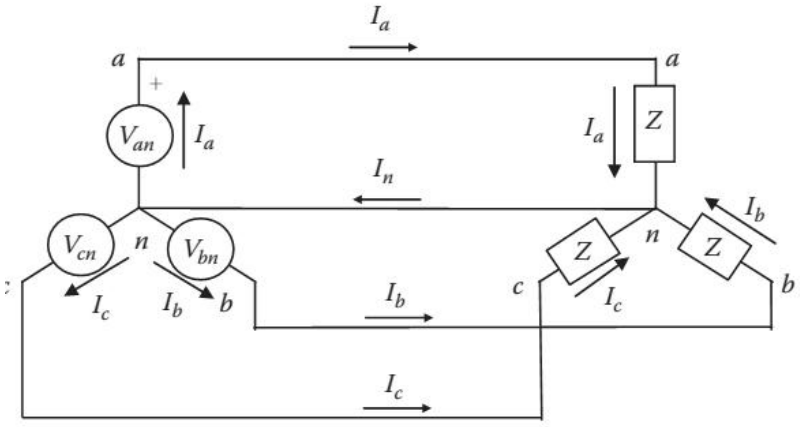
- Neutral current

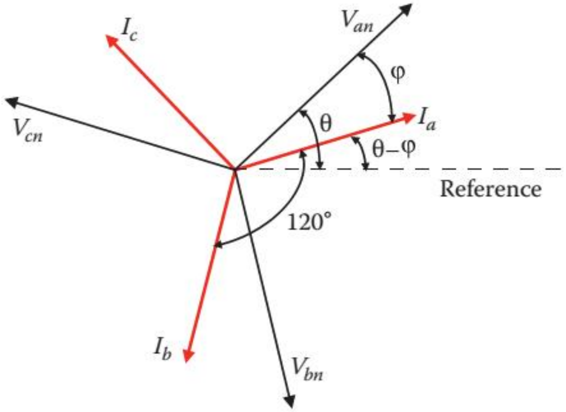
$$\overline{I}_n = \overline{I}_a + \overline{I}_b + \overline{I}_c$$

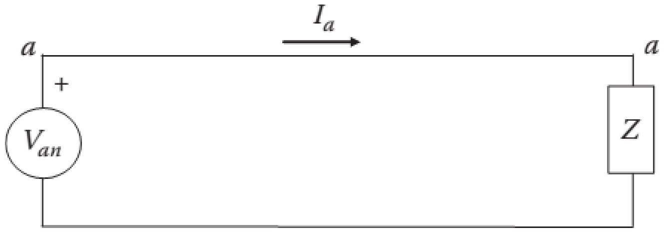
$$\overline{I}_n = \overline{I}_a + \overline{I}_a\angle -120 + \overline{I}_a\angle 120 = 0$$

- If source and loads are balanced the neutral current is zero

- In transmission this means no need for a neutral line







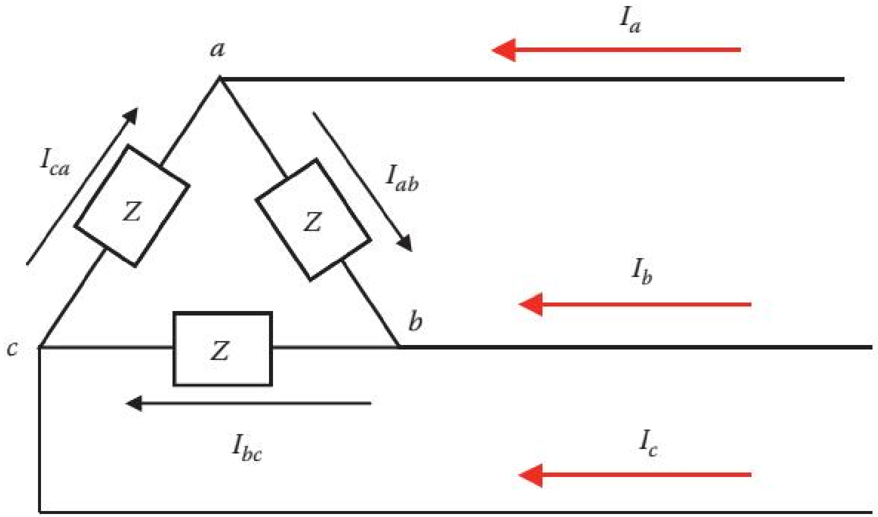
Delta-connected load

- Loads connected between two transmission lines
- Voltage across the single load is the line-to-line voltage

$$\overline{I}_{ab} = \overline{I}_a + \overline{I}_{ca}$$

$$\overline{I}_{bc} = \overline{I}_b + \overline{I}_{ab}$$

$$\overline{I}_{ca} = \overline{I}_c + \overline{I}_{bc}$$



$$\overline{I}_{ab} = \frac{\overline{V}_{ab}}{\overline{Z}}$$

$$\overline{I}_{bc} = \frac{\overline{V}_{bc}}{\overline{Z}}$$

$$\overline{I}_{ca} = \frac{\overline{V}_{ca}}{\overline{Z}}$$

- Balanced load and source ⟹ balanced currents

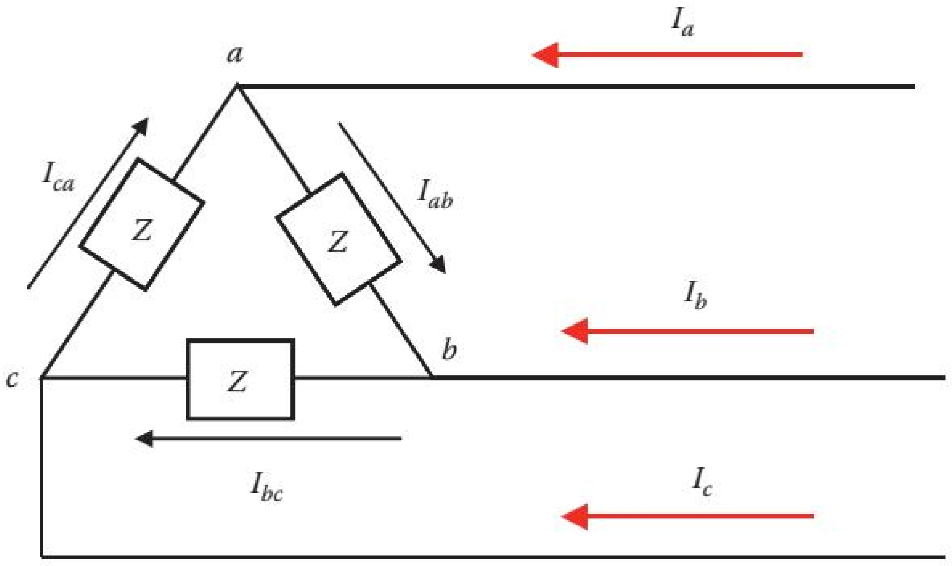
Delta-connected load

- If we choose $\overline{I}_{ab}$ as reference

$$\overline{I}_{ab} = I\angle 0°$$

$$\overline{I}_{bc} = I\angle -120°$$

$$\overline{I}_{ca} = I\angle 120°$$



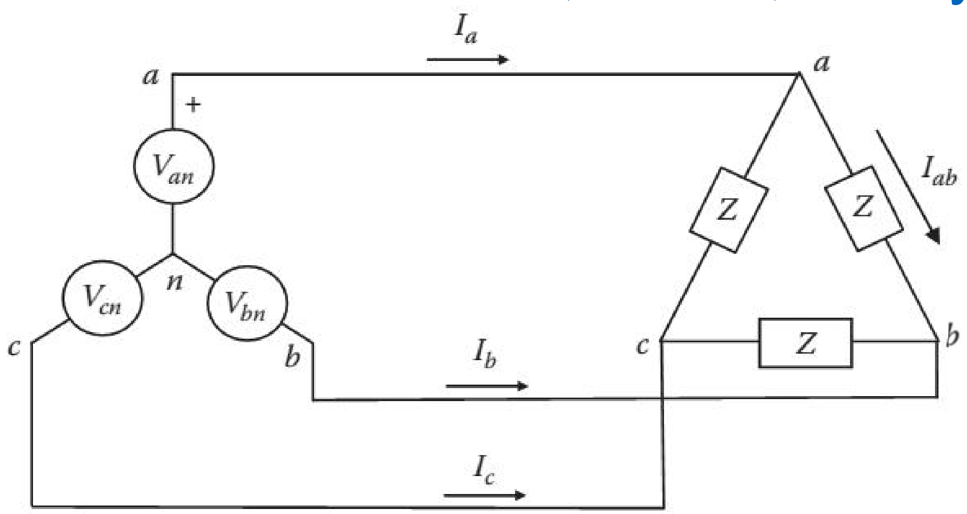
$$\overline{I}_a = \sqrt{3}I\angle -30° = \sqrt{3}\overline{I}_{ab}\angle -30°$$

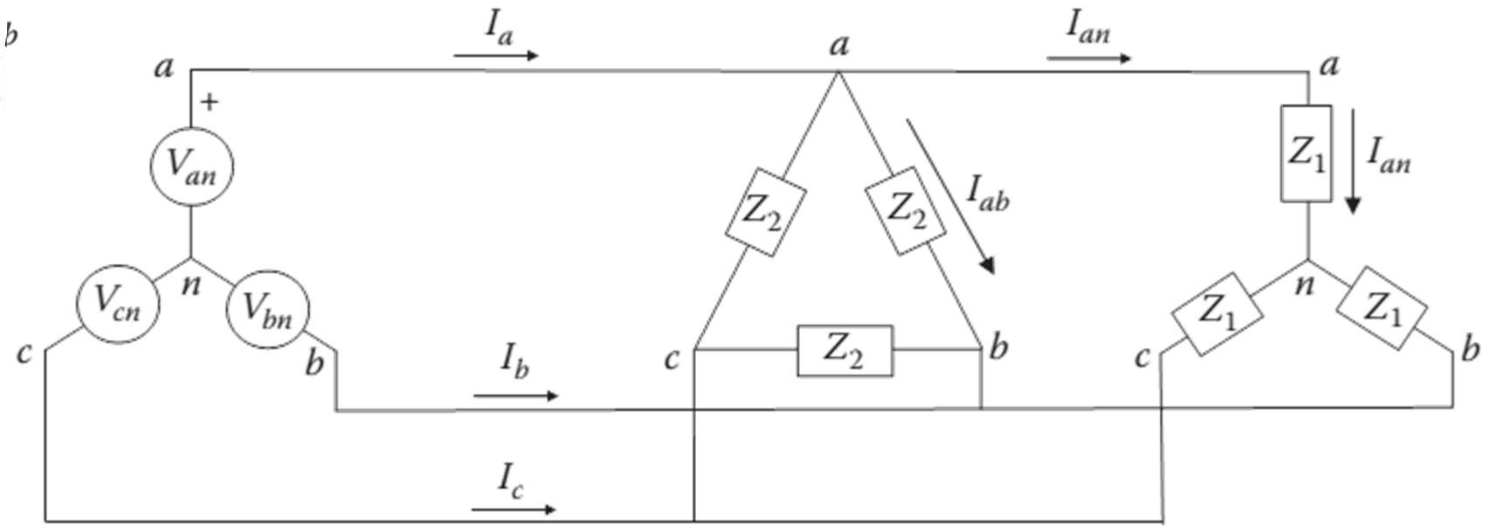
$$\overline{I}_b = \sqrt{3}I\angle -150° = \sqrt{3}\overline{I}_{bc}\angle -30°$$

$$\overline{I}_c = I\angle 90° = \sqrt{3}\overline{I}_{ca}\angle -30°$$

Circuits with mixed connections

- In general circuits can be so that the source or the load or both are connected in Y-, Delta-, or any combination

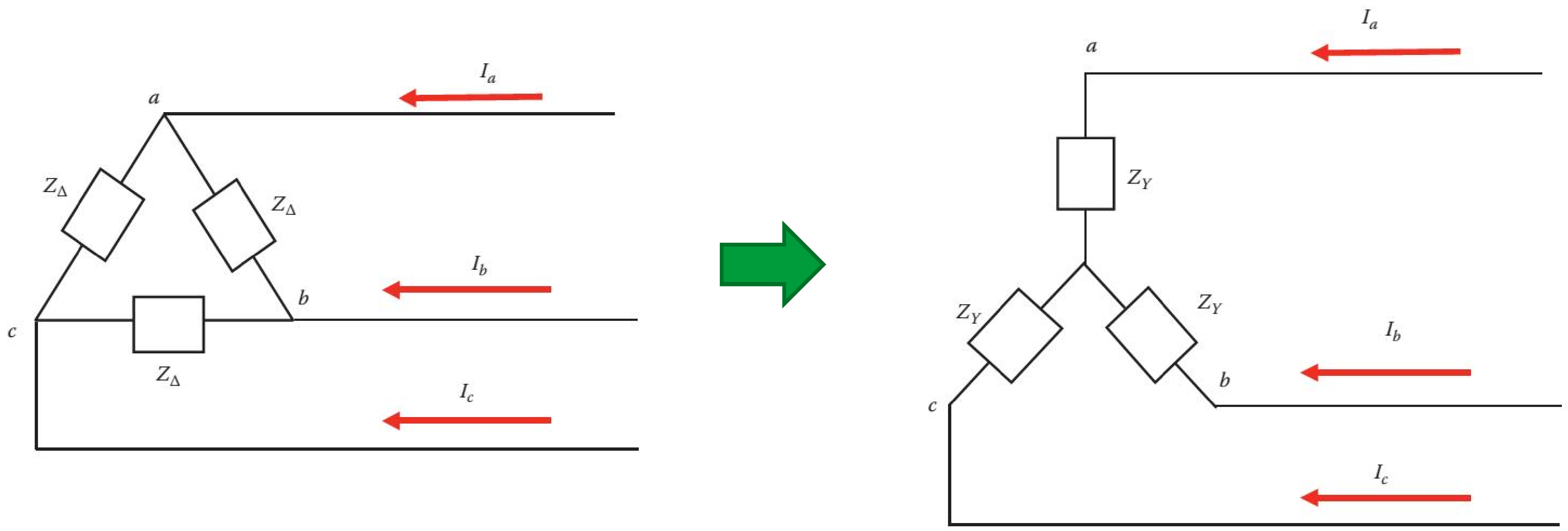




Circuits with mixed connections

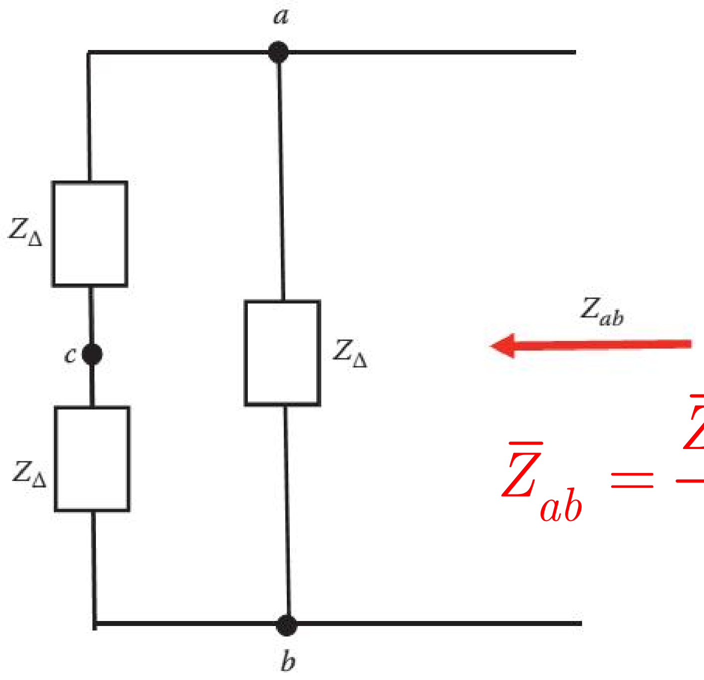
- Such circuits requires the load and source to be in the same connection

→ Y-Delta transformation

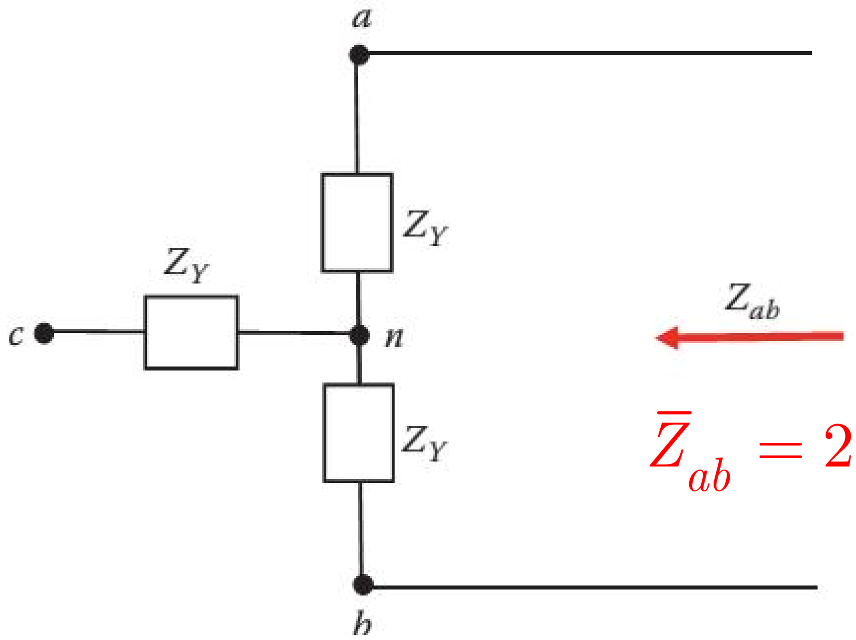


Y-Delta transformation

- Consider the impedance seen by the source terminals



$$\bar{Z}_{ab} = \frac{\bar{Z}_\Delta(2\bar{Z}_\Delta)}{3\bar{Z}_\Delta} = \frac{2}{3}\bar{Z}_\Delta$$



$$\bar{Z}_{ab} = 2\bar{Z}_Y$$

- Transformation valid in both directions

$$\bar{Z}_Y = \frac{\bar{Z}_\Delta}{3}$$

Power of three phases system

- The power of a three phase load is the sum of the powers of each load (each phase)
- Phases related quantities are called phase quantities (phase current, phase voltage, phase power)

$$P_{ph} = V_{ph} I_{ph} \cos(\boldsymbol{\theta})$$

$$Q_{ph} = V_{ph} I_{ph} \sin(\boldsymbol{\theta})$$

$$P = 3P_{ph} = 3V_{ph} I_{ph} \cos(\boldsymbol{\theta})$$

$$Q = 3Q_{ph} = 3V_{ph} I_{ph} \sin(\boldsymbol{\theta})$$

Power of Y-connected three phases load

- Phase current equal to line current
- Phase voltage different

$$I_{ph} = I_l$$

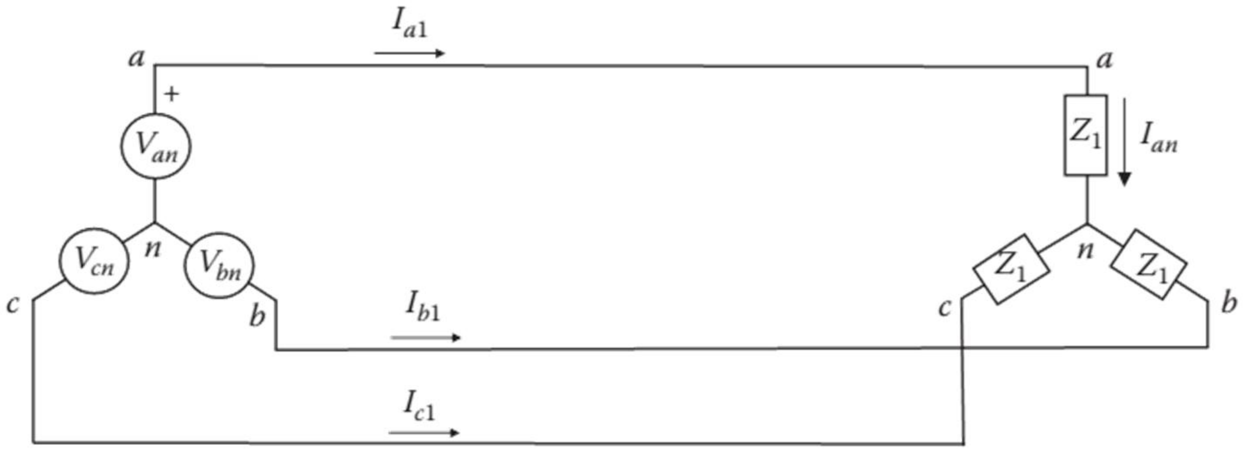
$$V_{ph} = \frac{V_{ll}}{\sqrt{3}}$$

$$P = 3P_{ph} = 3V_{ph}I_{ph}\cos(\boldsymbol{\theta})$$

$$= \sqrt{3}V_{ll}I_l\cos(\boldsymbol{\theta})$$

$$Q = 3Q_{ph} = 3V_{ph}I_{ph}\sin(\boldsymbol{\theta})$$

$$= \sqrt{3}V_{ll}I_l\sin(\boldsymbol{\theta})$$



Power of Delta-connected load

- Phase voltage equal to line-to-line voltage
- Phase current different

$$V_{ph} = V_{ll}$$

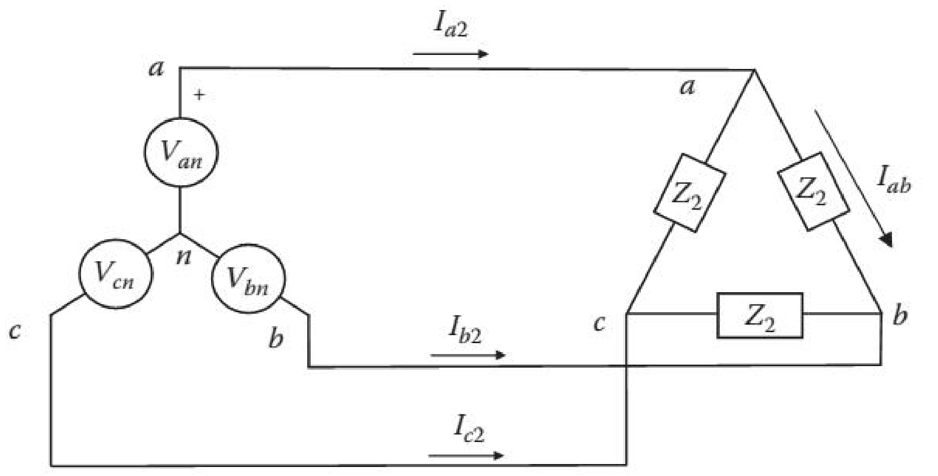
$$I_{ph} = \frac{I_l}{\sqrt{3}}$$

$$P = 3P_{ph} = 3V_{ph}I_{ph}\cos(\boldsymbol{\theta})$$

$$= \sqrt{3}V_{ll}I_l\cos(\boldsymbol{\theta})$$

$$Q = 3Q_{ph} = 3V_{ph}I_{ph}\sin(\boldsymbol{\theta})$$

$$= \sqrt{3}V_{ll}I_l\sin(\boldsymbol{\theta})$$



Summary of the lecture

- Three different coils shifted by 120 deg in space generate balanced three voltages
- The coils can be connected in Y or Delta
- Loads also can be connected in Y or Delta
- Basic equations for single phase system holds also for three phases system (remember the phase shifts)
- In y-connection, line current equals phase current
- In Delta-connection line-to-line voltage equals phase voltage
- Remember to divide by $\sqrt{3}$ for the other quantities.