Transformers

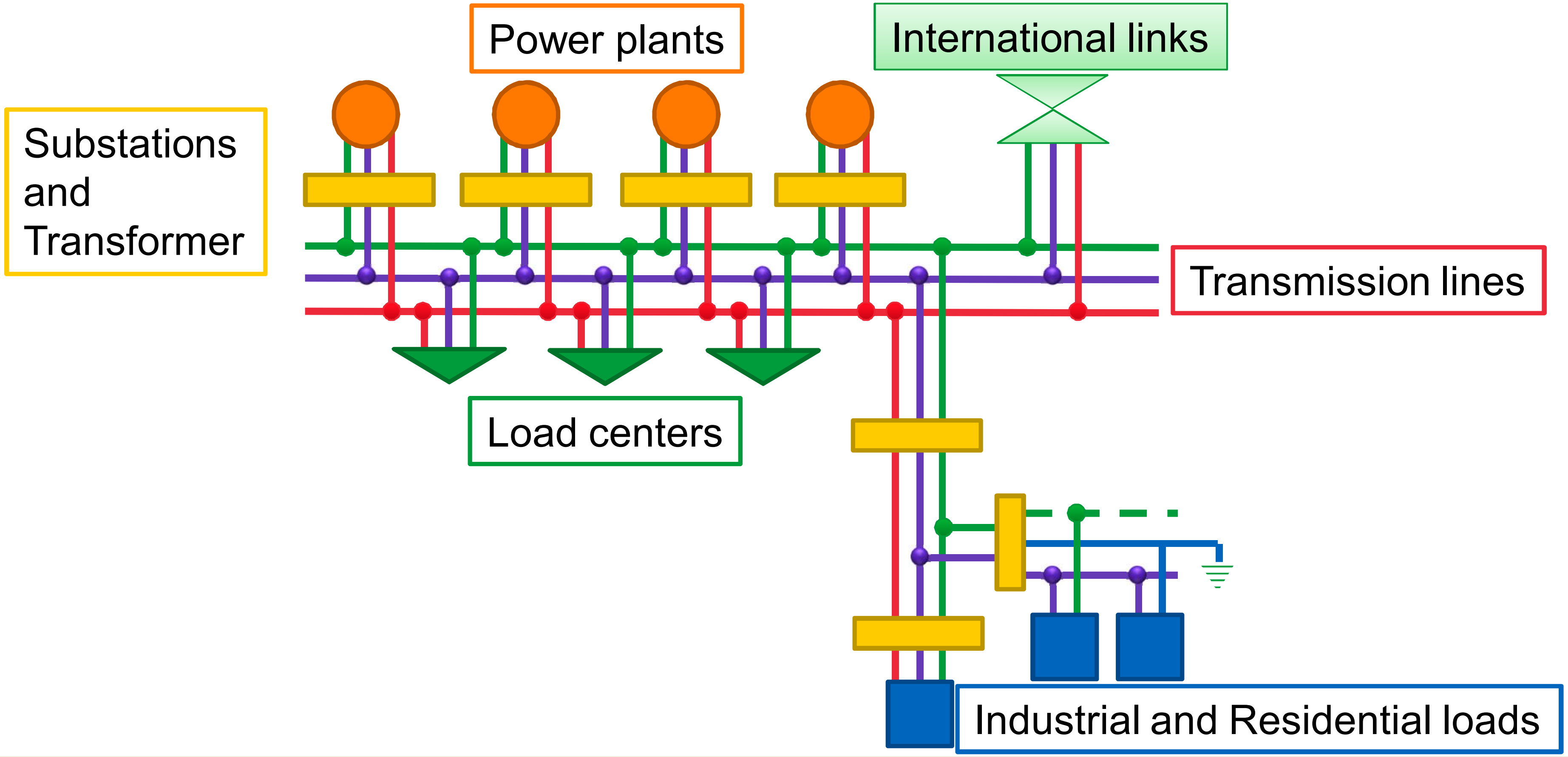
- Lecture outcomes (what you are supposed to learn):
  - Describe the construction of transformers
  - Explain the operation principle of the transformer
  - How to connect the windings of three phase transformer
  - Derive the equivalent circuit of the transformer
  - Analyze the transformer operation with the equivalent circuit

Introduction

- Transformers are used to step up or step down the voltage in a circuit
- Large transformers are used in electrical energy transmission and distribution (three phase)
- Smaller transformers are used in electronic circuits and they are coupled to rectifiers for DC-voltage production (single phase)
- Transformer is a static electromagnetic energy conversion device

Introduction

- Different kind of transformers are needed !



Different types of transformers

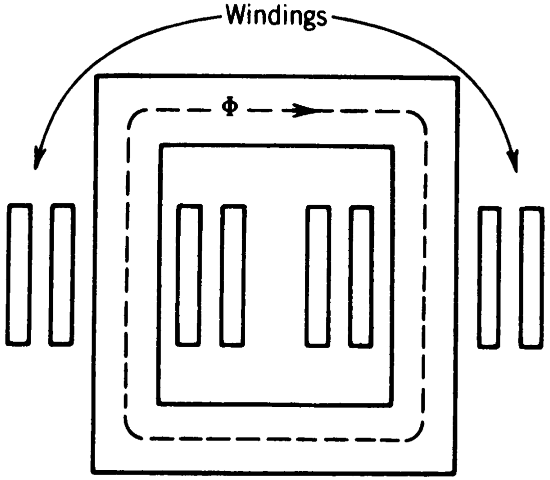
Transmission transformer

Distribution transformer

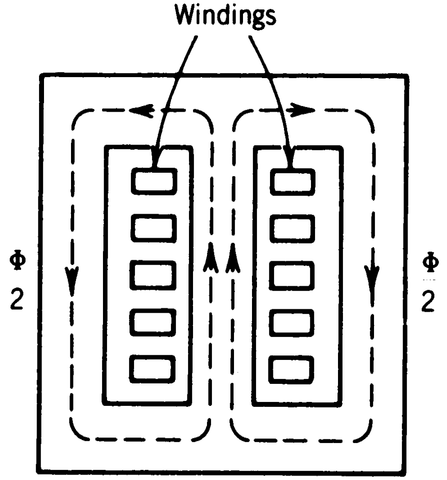
Service transformer

Circuit transformer

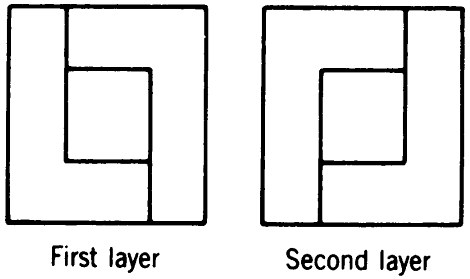
Transformer structure



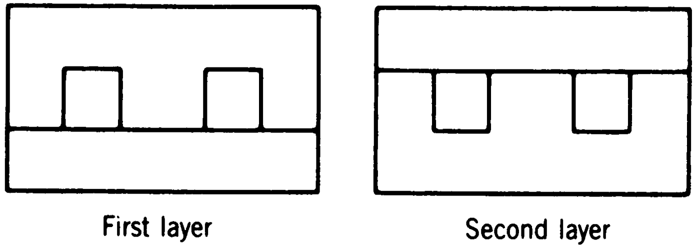
Core-type
(Sydänmuuntaja)



Shell-type
(Vaippamuuntaja)



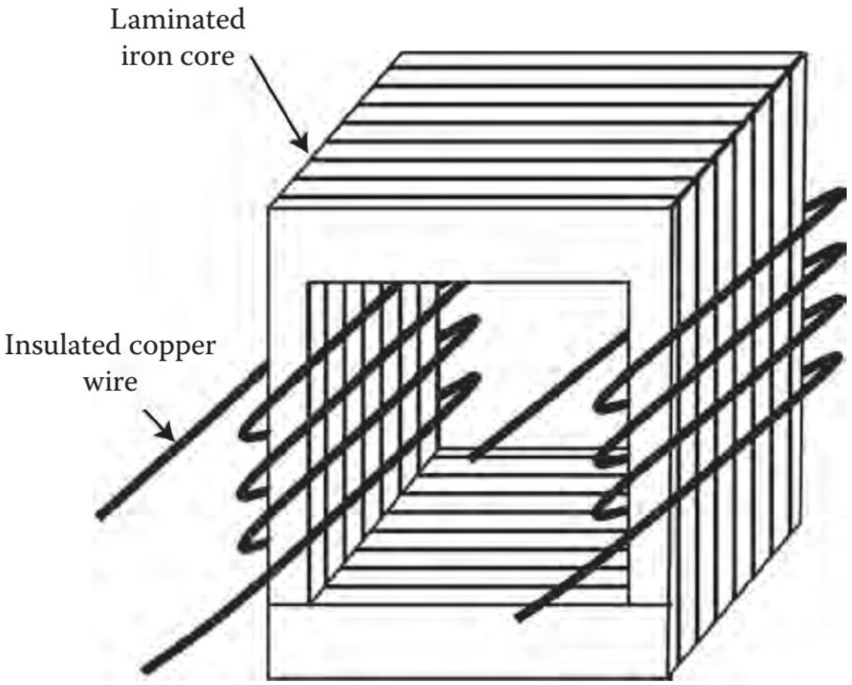
L-shaped lamination

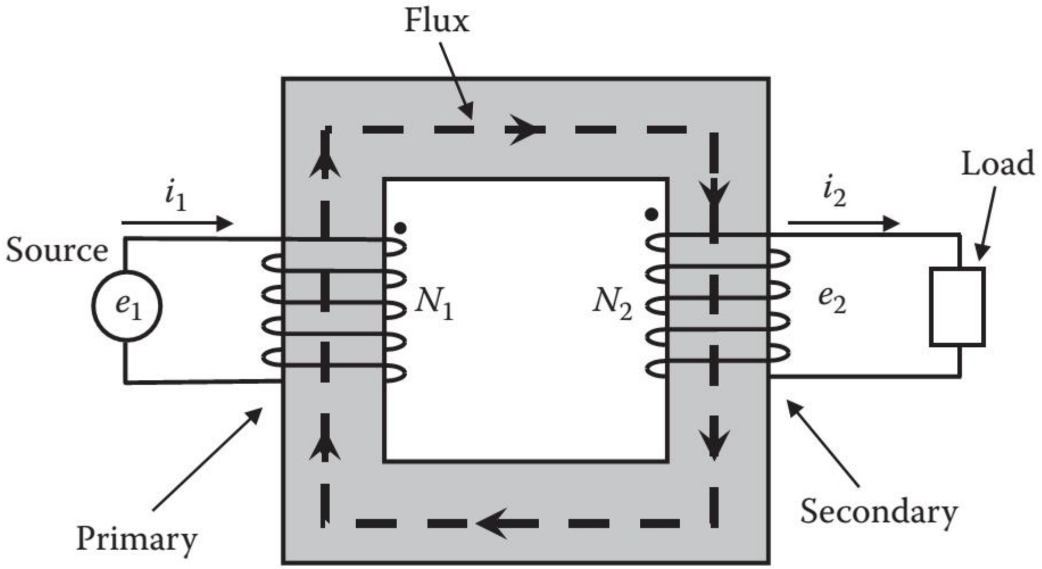


E-shaped lamination

Active components of transformers

- Iron core and at least two windings (coils)



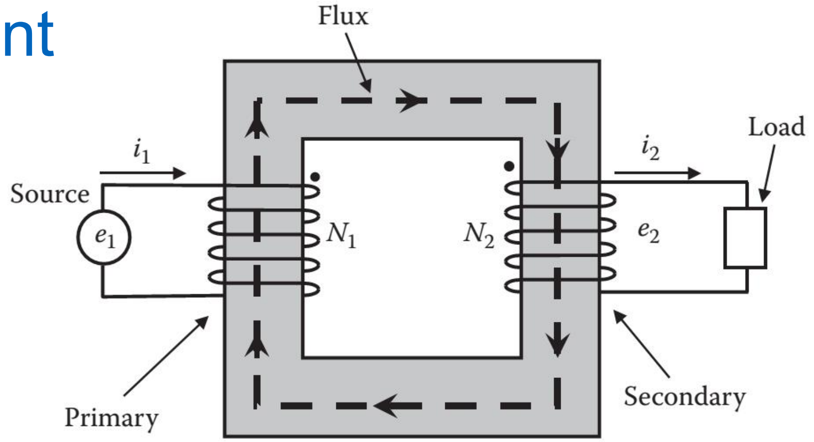


- Alternating current produces flux, which induces voltage

Basic operation of transformers

- Primary coil connected to voltage source ➡ current
- Current magnetizes the core and produces flux
- Flux induces voltage in secondary coil
- Load connected to secondary voltage draws current
- Voltage also induced in core

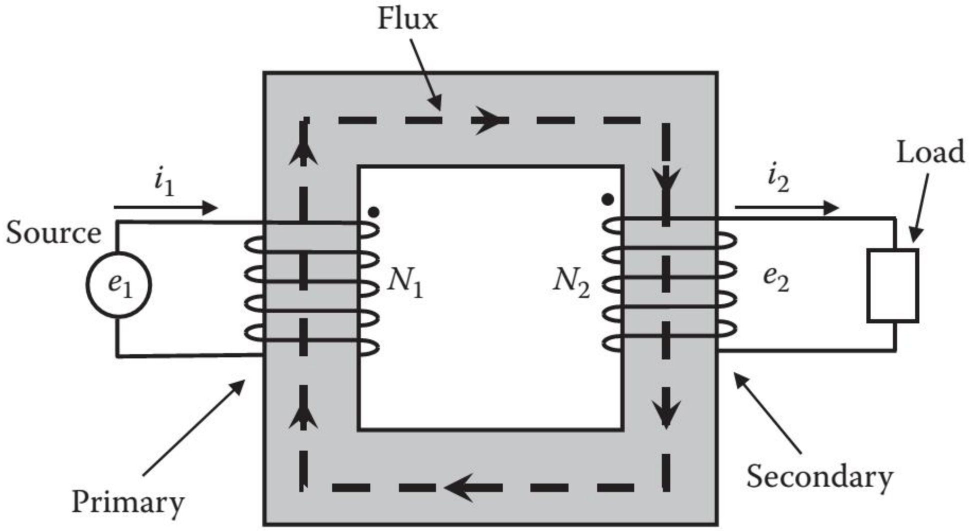
➡ Eddy-current and other losses



Voltage ratio

- Faraday's law:

$$e_1 = -N_1 \frac{\partial \varphi}{\partial t} \Rightarrow \varphi = -\frac{1}{N_1}\int e_1 dt$$



$$e_2 = -N_2 \frac{\partial \varphi}{\partial t} \Rightarrow \frac{e_1}{N_1} = \frac{e_2}{N_2} = \frac{\partial \varphi}{\partial t} \Rightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Current ratio

- Assume ideal transformer (no losses)

$$\bar{S}_1 = \bar{E}_1 \bar{I}_1^*$$

$$\bar{S}_2 = \bar{E}_2 \bar{I}_2^*$$

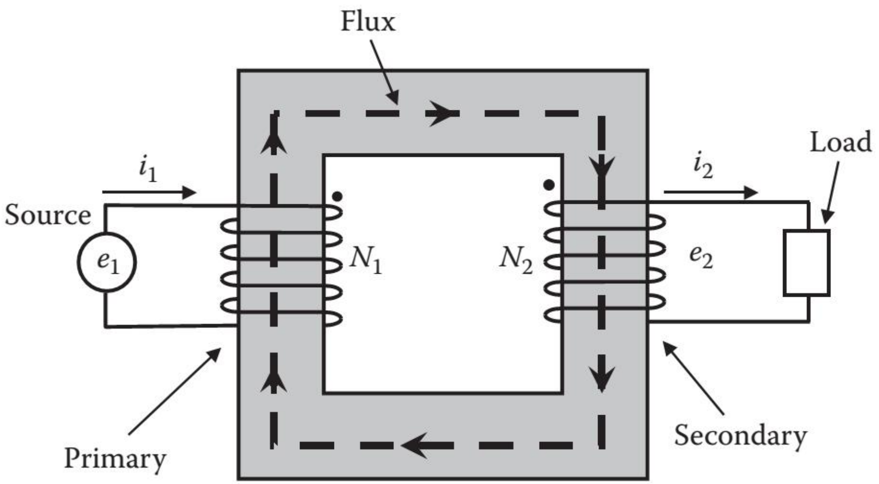
$$\bar{S}_1 = \bar{S}_2 \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

- The quantity $F = I_1 N_1 - I_2 N_2$ is called magnetomotive force
- Magnetomotive force is responsible of producing the flux

Load Impedance seen by the primary

- Impedance connected to secondary is seen at the primary side.

$$Z'_{load} = \frac{E_1}{I_1}$$



$$Z_{laod} = \frac{E_2}{I_2}$$

$$Z'_{load} = Z_{load}\left(\frac{N_1}{N_2}\right)^2$$

Transformer ratings

- Mechanical ratings:
  - Thermal limits, dimensions, weight, volume, etc…
- Electrical ratings
  - Voltages, currents, and power
- Exceeding ratings can damage the transformer
  - Excessive voltages cause insulation failure
  - Excessive currents cause overheating, which results in melting
  - In both cases the result is internal short circuit
- Nameplate “S, V1 / V2” e.g. “10kVA, 8kV / 240V)

Main parameters from nameplate

- Nameplate: “10kVA, 8kV/240V)
  - Voltage ratio: $$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{8000}{240} = 33.33$$
  - Rated primary current: $$I_1 = \frac{S}{E_1} = \frac{10000}{8000} = 1.25\,A$$
  - Rated secondary current: $$I_2 = \frac{S}{E_2} = \frac{10000}{240} = 41.67\,A$$
  - Rated load impedance: $$Z_{load} = \frac{E_2}{I_2} = \frac{240}{41.67} = 5.76\,\Omega$$

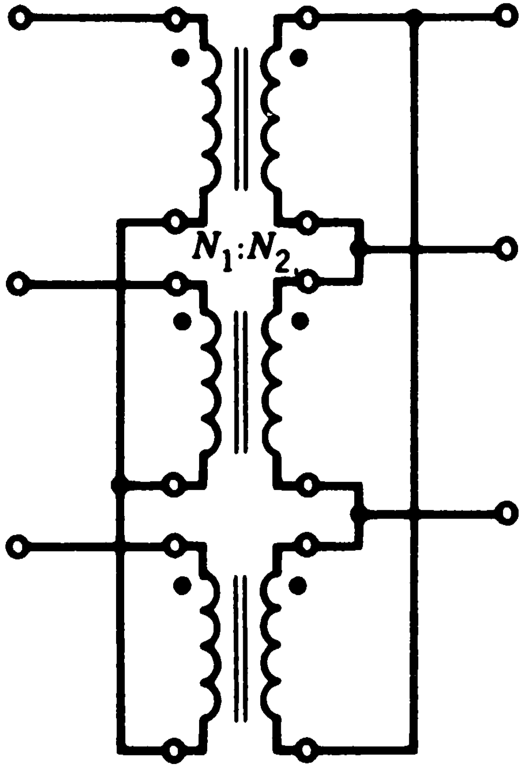
Three-phase transformer

- Three similar single-phase transformers connected to form a three-phase transformer

- Four possible connection:

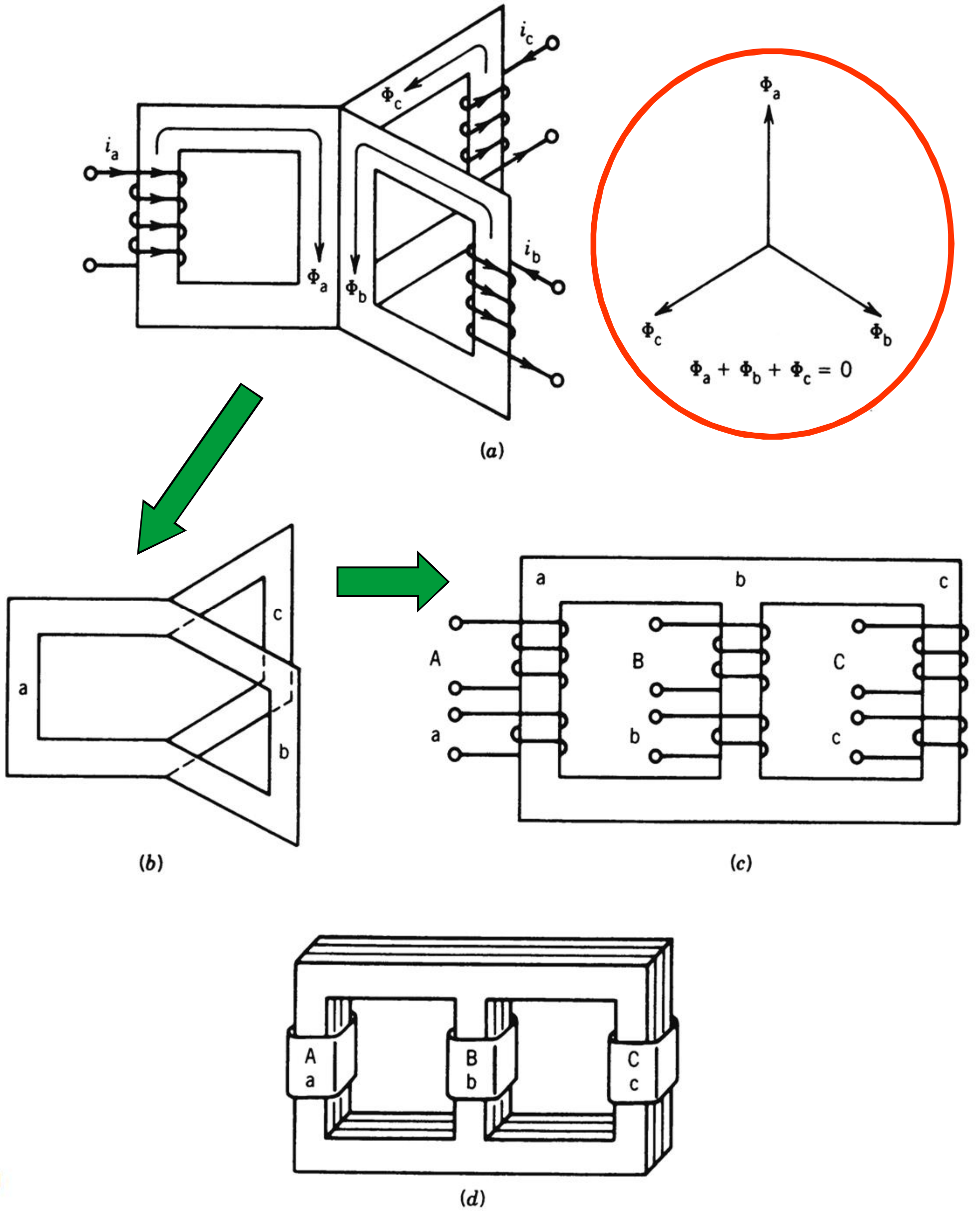
  Y-Δ Δ-Y Δ-Δ Y-Y

- Some connections result in phase shift



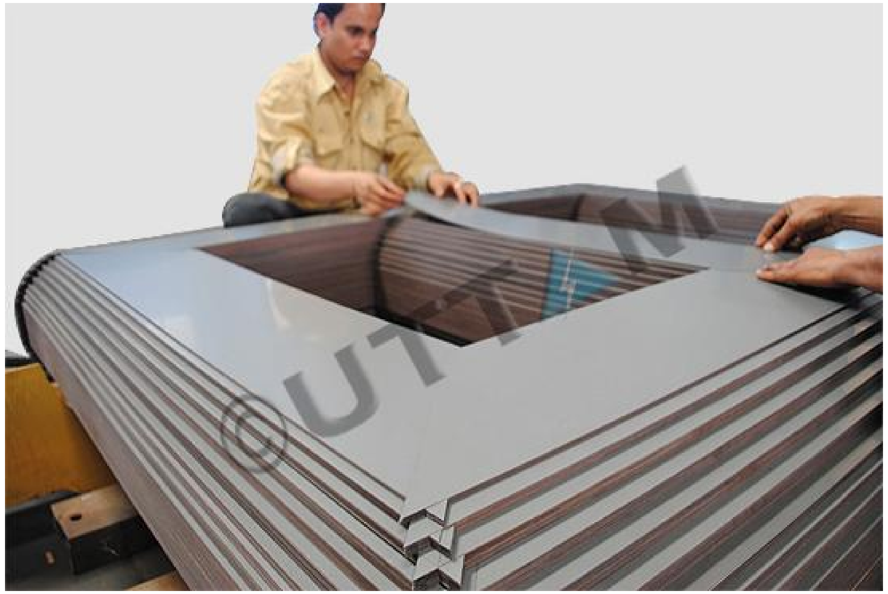
Three-phase transformer unit

- Balanced three-phase voltage
- Balanced three-phase flux
- Return leg can be removed
- In-plan construction easy to manufacture



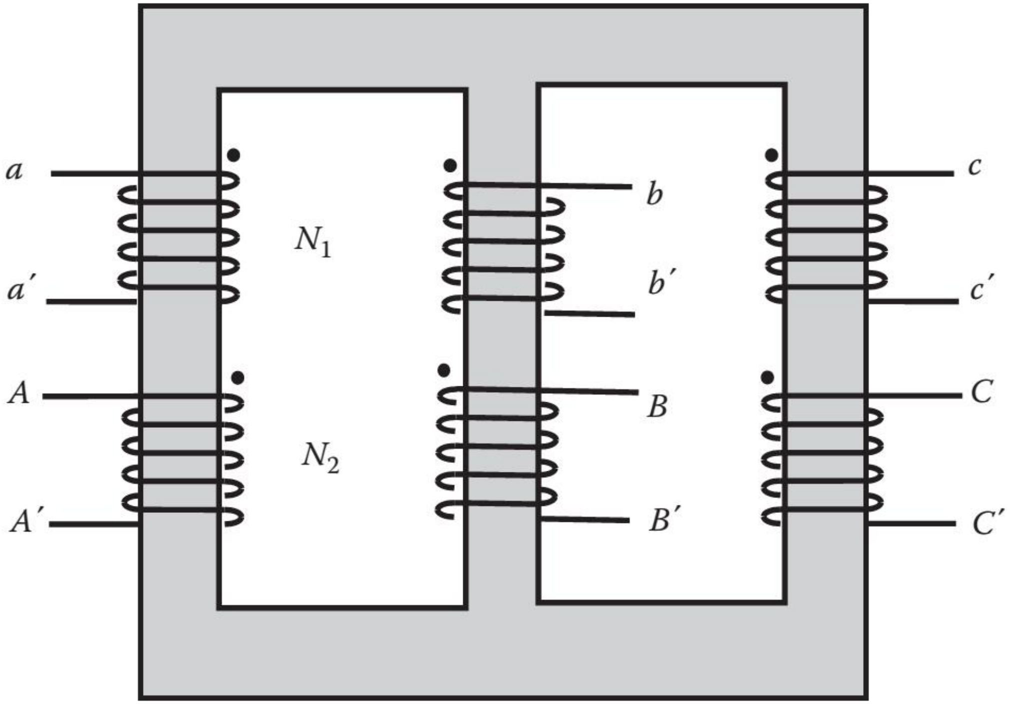
Anouar Belahcen

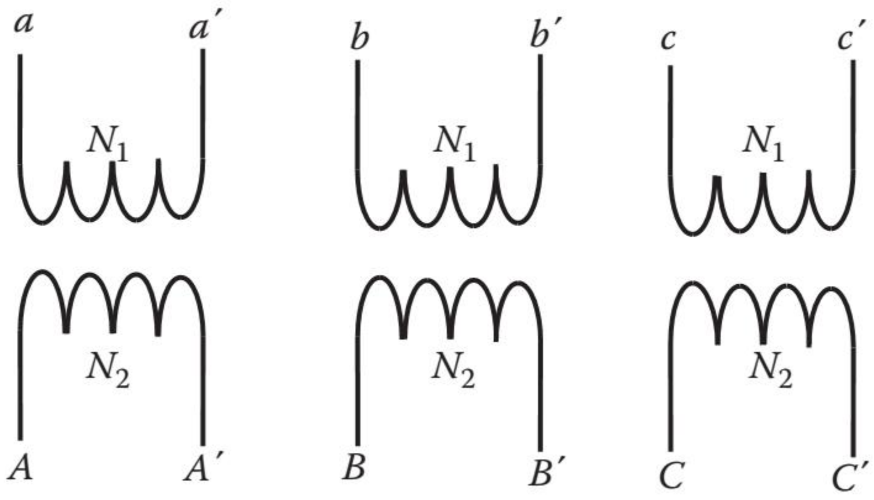
Transformer structure



Three-phase transformer

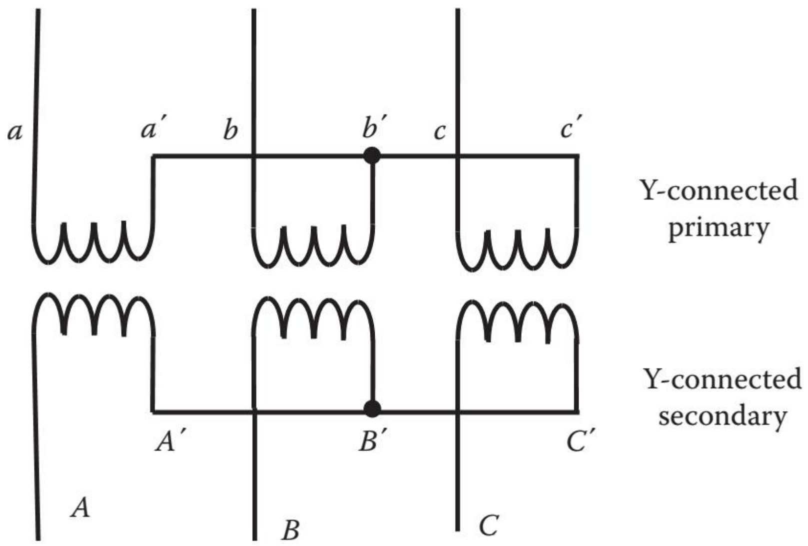
- Constructed from three legged core
- Nameplate: e.g. “60kVA, 8kV(Δ)/416(Y)”

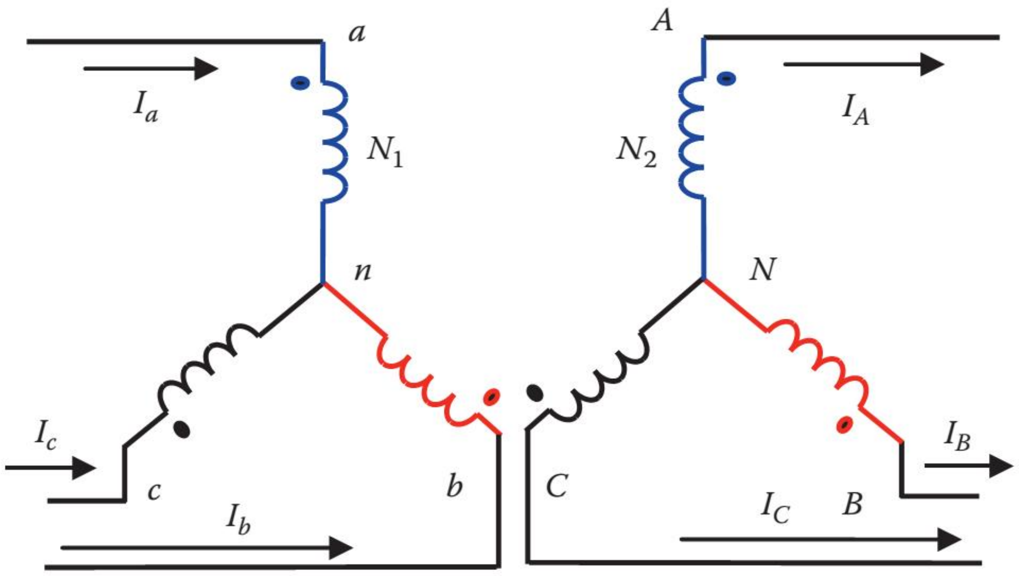




Y-Y connection

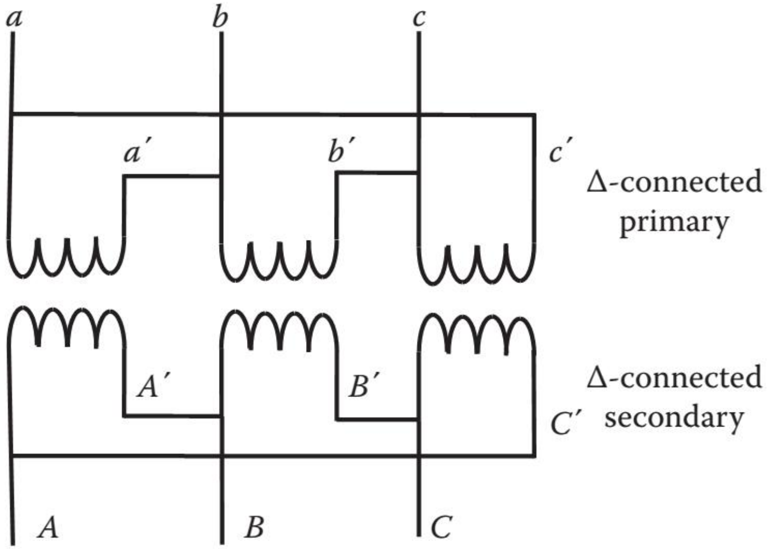
- a’, b’ and c’ connected to the same point (n)
- A’, B’, and C’ connected to the same point (N)
- Turn ratio equals voltage ratio equals current ratio

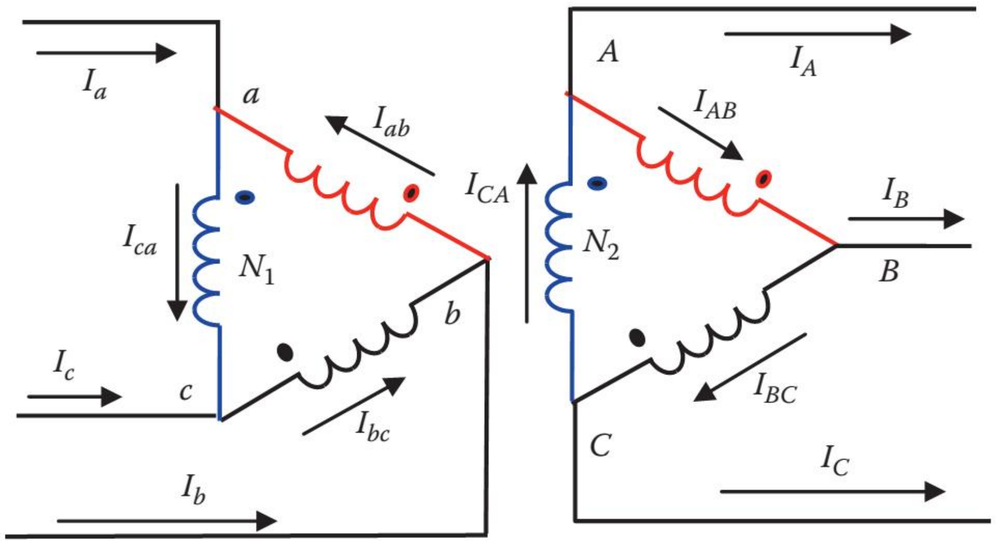




Δ- Δ connection

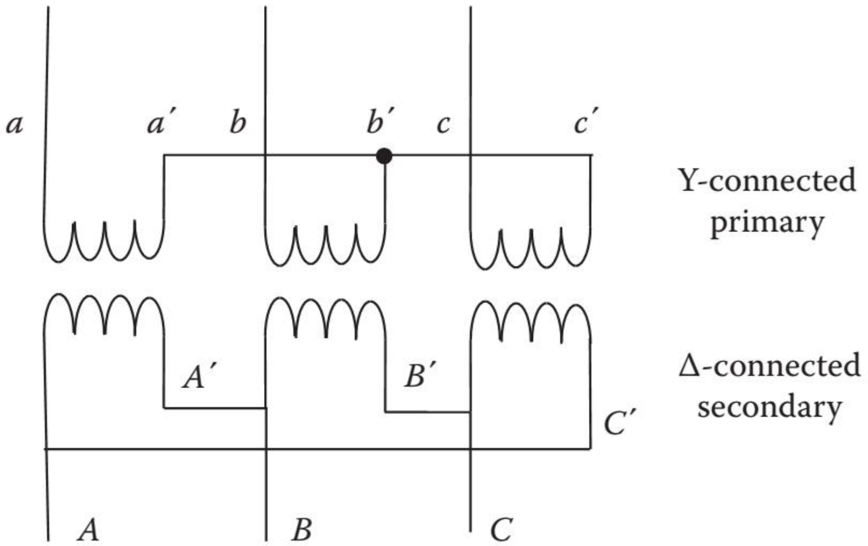
- a'-b, b'-c and c'-a connection. No neutral point
- A'-B, B'-C, and C'-A connection. No neutral point
- Turn ratio equals voltage ratio equals current ratio



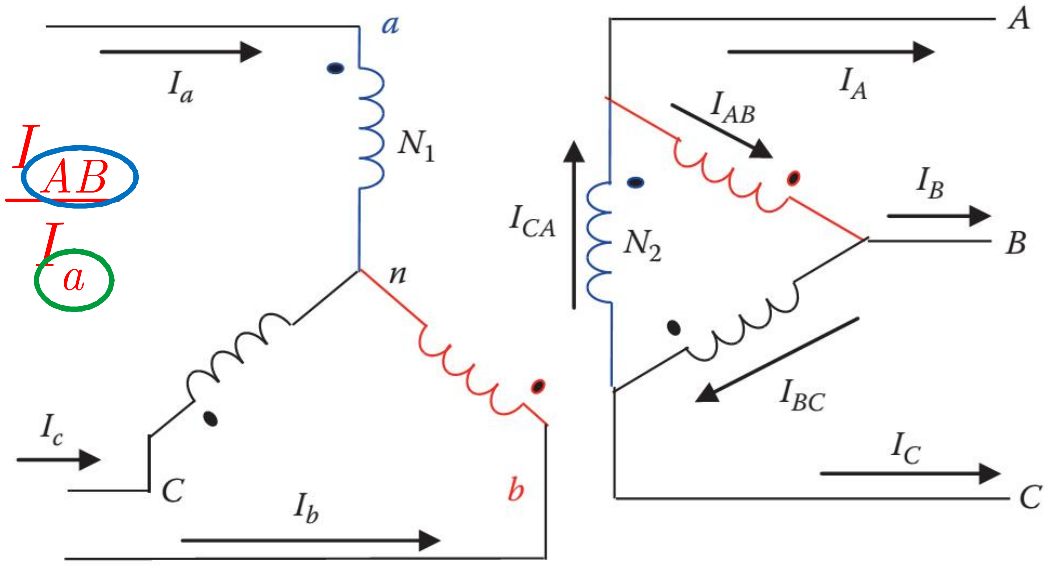


Y- Δ connection (Δ-Y connection)

- a’, b’ and c’ connected to the same point (n)
- A’-B, B’-C, and C’-A connection. No neutral point
- Pay attention to phase versus line-to-line quantities

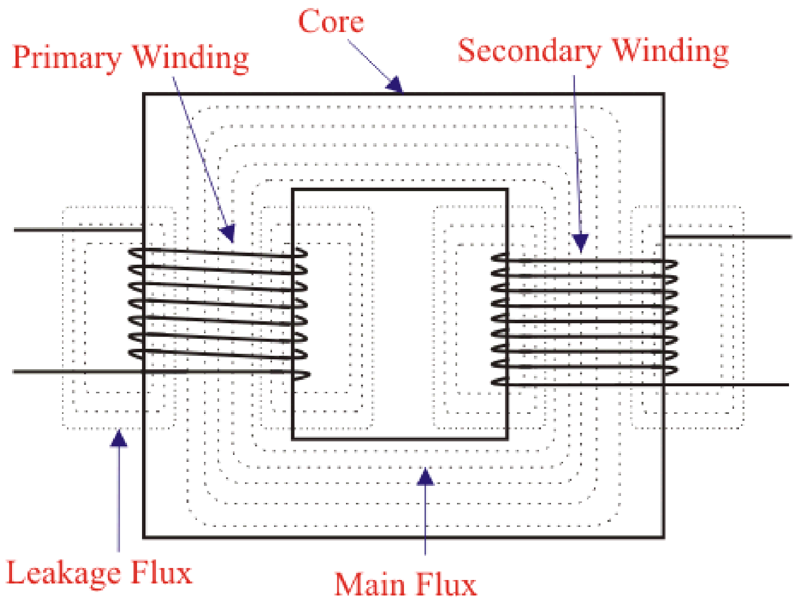


$$\frac{V_{an}}{V_{AB}} = \frac{N_1}{N_2} = \frac{I_{AB}}{I_a}$$



Actual transformer

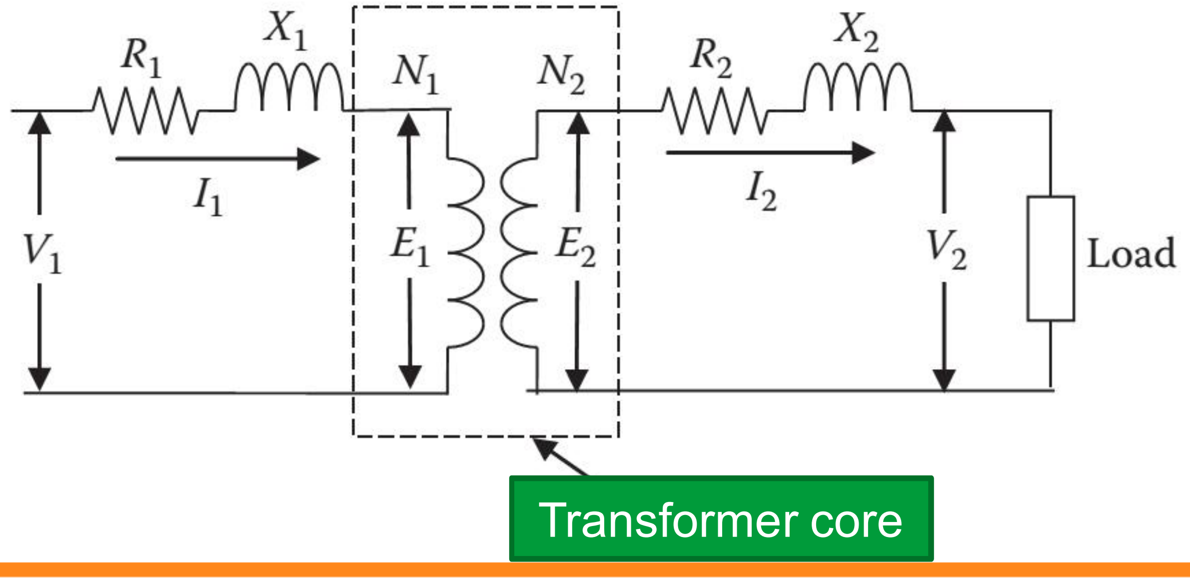
- In the previous slides we discussed ideal transformers
- Actual transformers present internal losses:
  - Resistive losses in the windings
  - Iron losses in the iron core
- Windings have inductances



- Iron core has finite permeability, thus requires magnetization current

Actual transformer

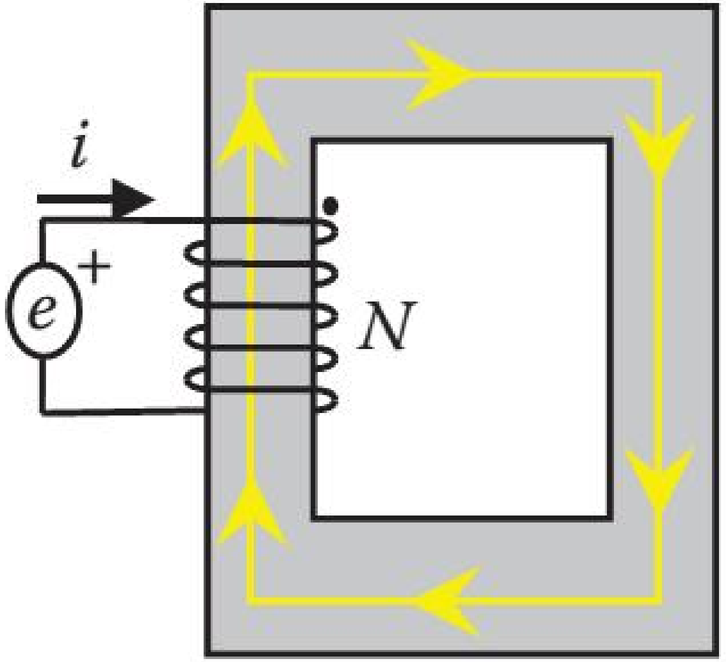
- Model of windings resistances and inductances
- Calculations with the ideal transformer are not handy
- Electromotive force different from terminal voltage



$$\frac{N_1}{N_2} = \frac{E_1}{E_2} \neq \frac{V_1}{V_2}$$

Some concepts of magnetism

- Ampere and Faraday laws

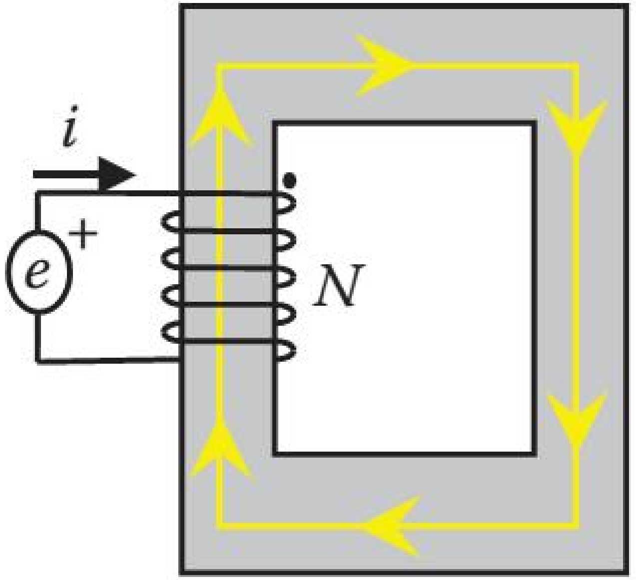


$$\oint H dl = \int J ds = Ni$$

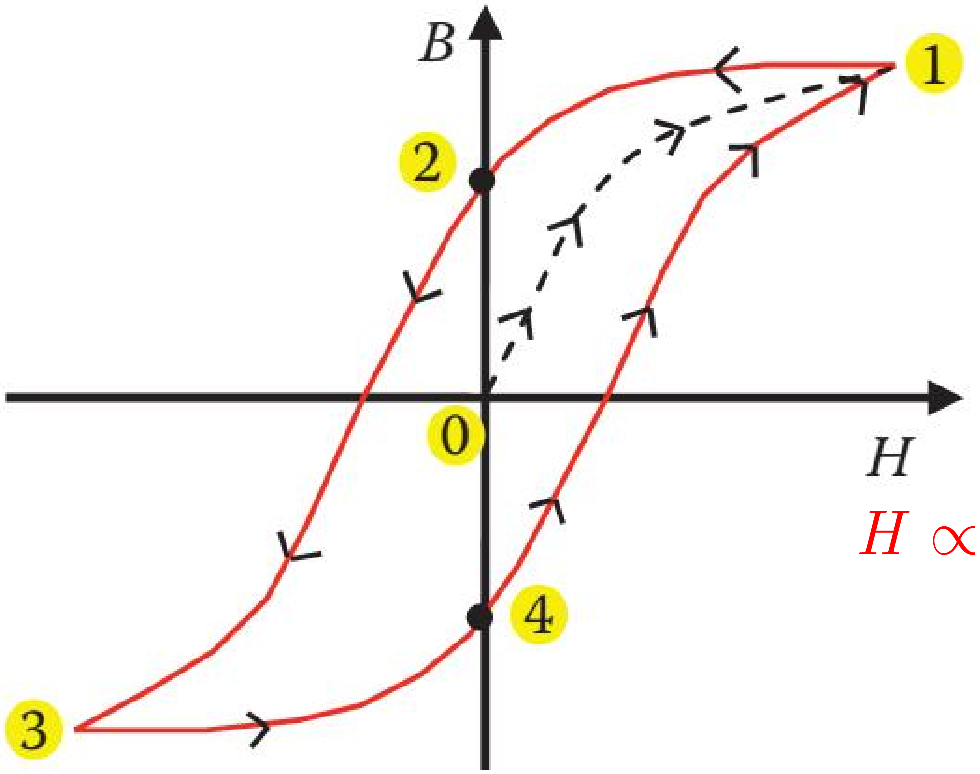
$$e = -N\frac{\partial \psi}{\partial t} = -NA\frac{\partial B}{\partial t}$$

Some concepts of magnetism

- Hysteresis



$$B \propto \int edt$$



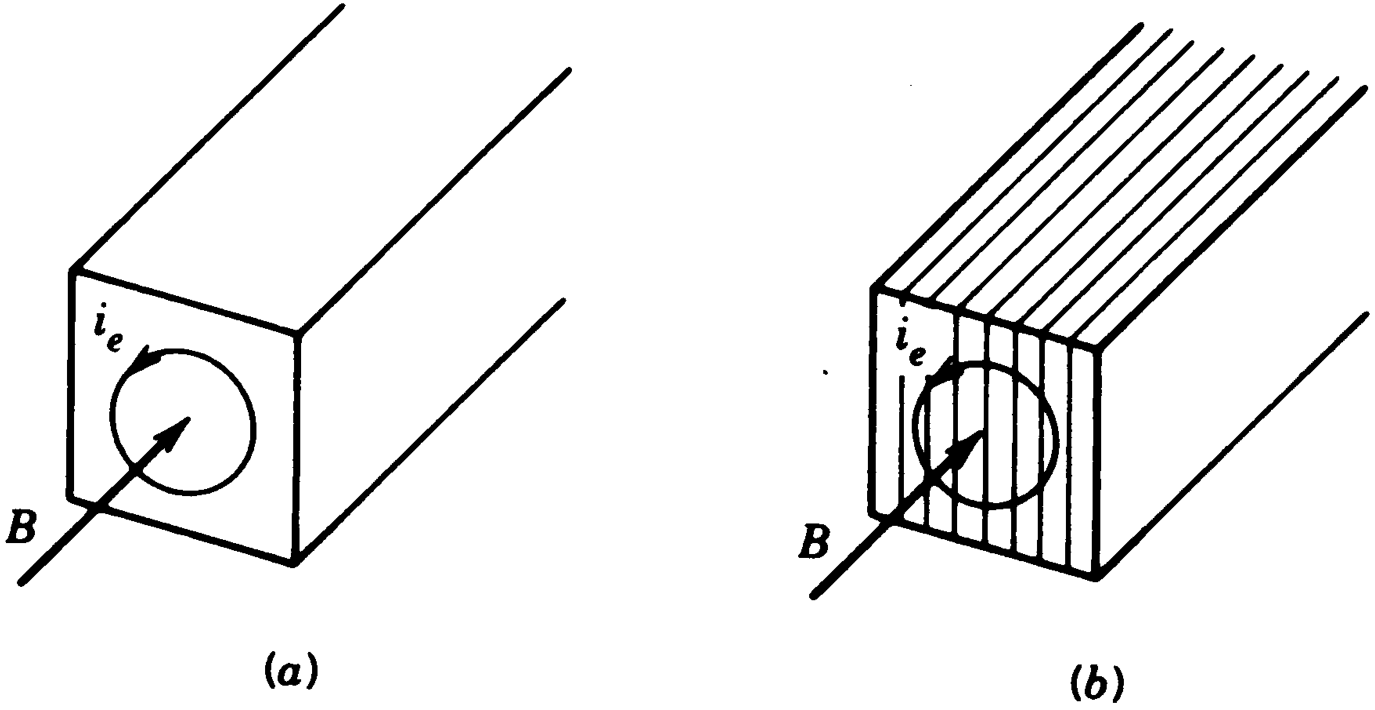
$$H \propto NI$$

- Magnetic energy

$$W = \int HdB \Rightarrow P_{hys} \propto \hat{B}^2 f$$

Some concepts of magnetism

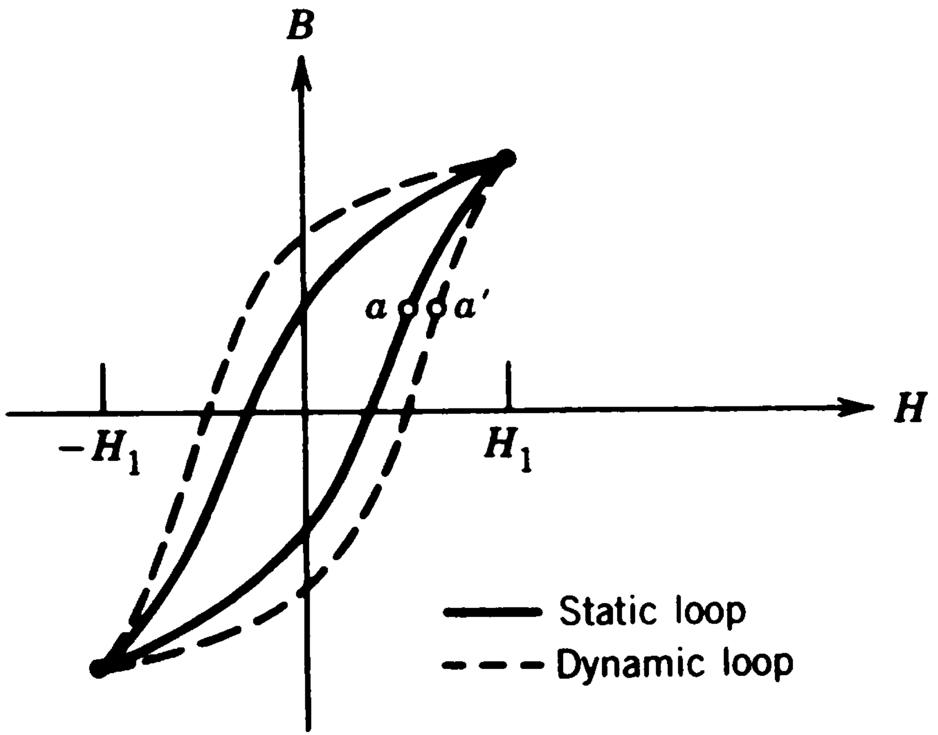
- Eddy-current



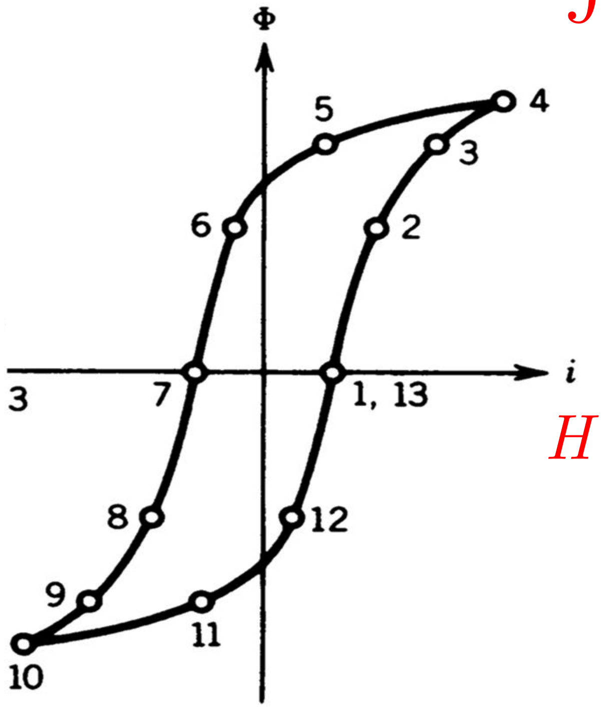
$$i_e \propto e \propto \frac{dB}{dt} \Rightarrow P_{eddy} \propto \hat{B}^2 f^2$$

Core losses

- Lumping of hysteresis and Eddy-current losses
- Shifting from BH to voltage and current



$$B \propto \phi \propto \int e dt$$

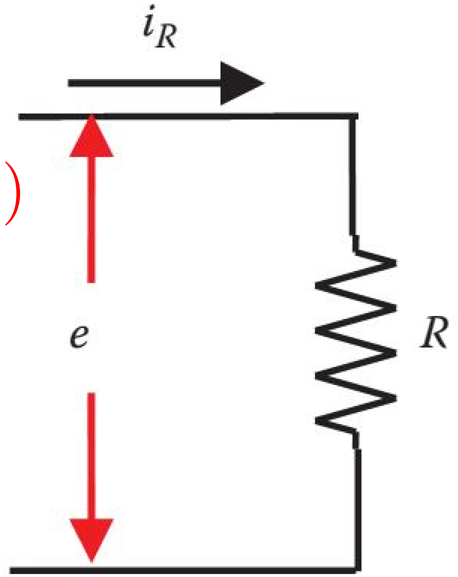


$$H \propto NI$$

Iron core as circuit element

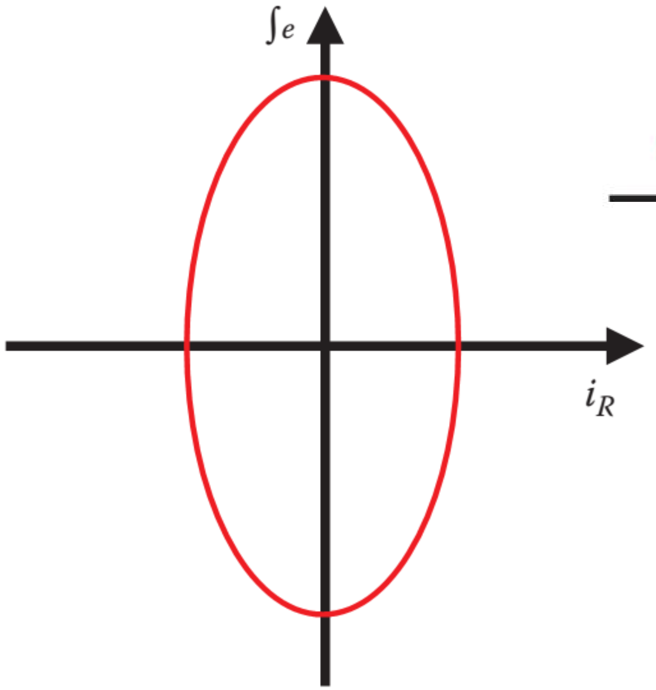
- The behavior of iron core can be obtained by two circuit element:
  - Resistor to simulate loss

$$e = E_{max} \sin(\omega t)$$



$$\int e dt = -\frac{E_{max}}{\omega} \cos(\omega t)$$

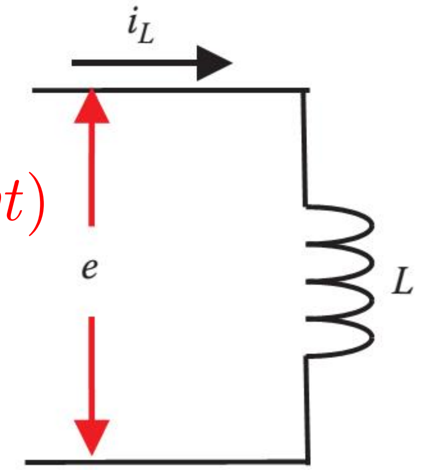
$$i_R = \frac{e}{R} = \frac{E_{max}}{R} \sin(\omega t)$$



Iron core as circuit element

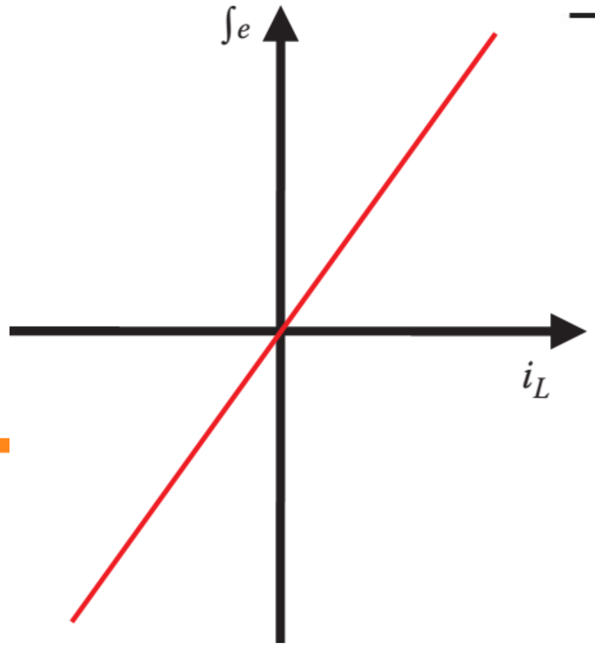
- The behavior of iron core can be obtained by two circuit element:
  - Inductance to simulate phase shift

$$e = E_{\max}\sin(\omega t)$$



$$\int e\,dt = -\frac{E_{\max}}{\omega}\cos(\omega t)$$

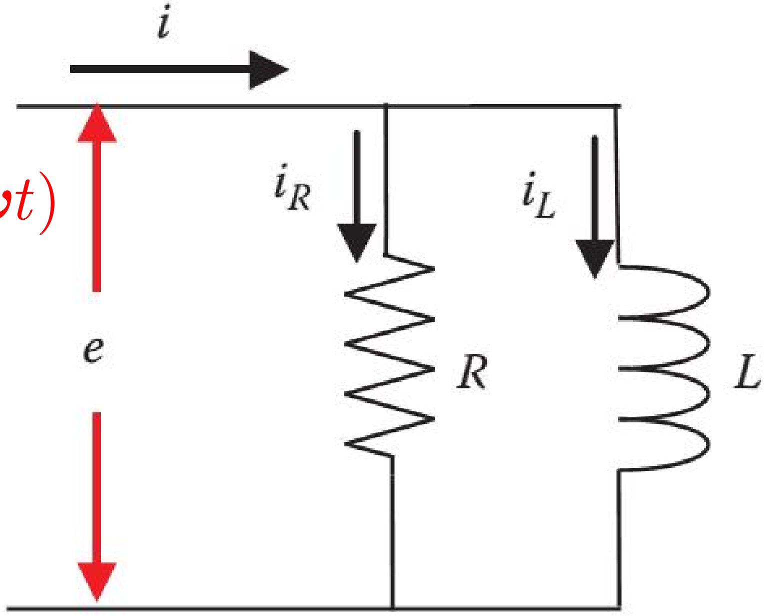
$$i_L = \frac{1}{L}\int e\,dt = -\frac{E_{\max}}{\omega L}\cos(\omega t)$$

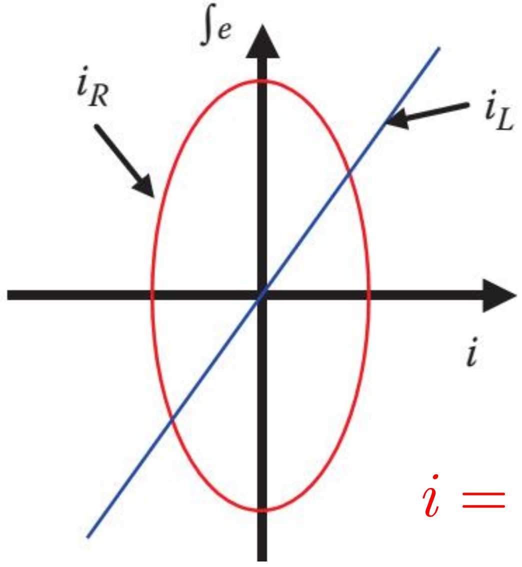


Iron core as circuit element

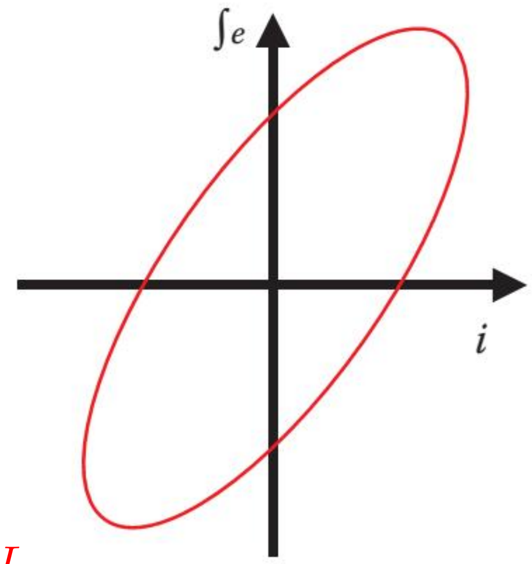
- The behavior of iron core can be obtained by two circuit element:
  - Combine resistor and inductor

$$e = E_{\max} \sin(\omega t)$$



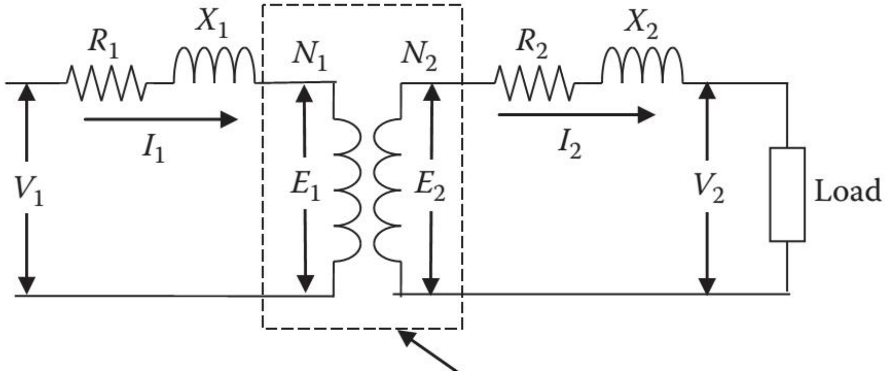


$$i = i_R + i_L$$

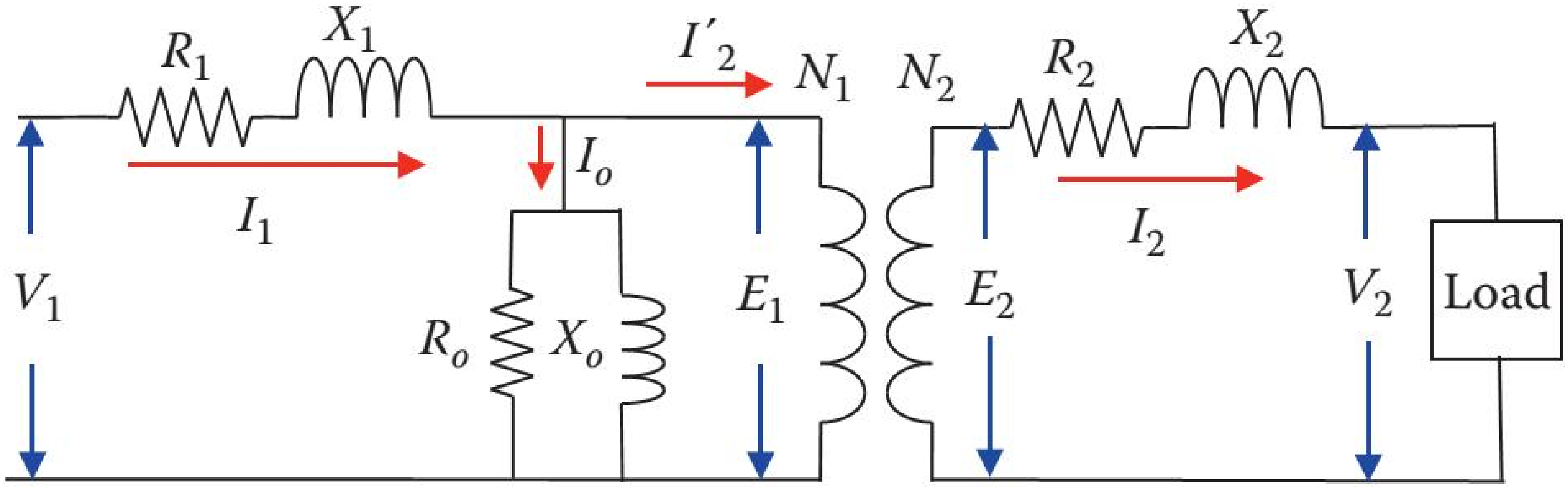


Equivalent circuit of transformer

- Introduce the core on the primary side

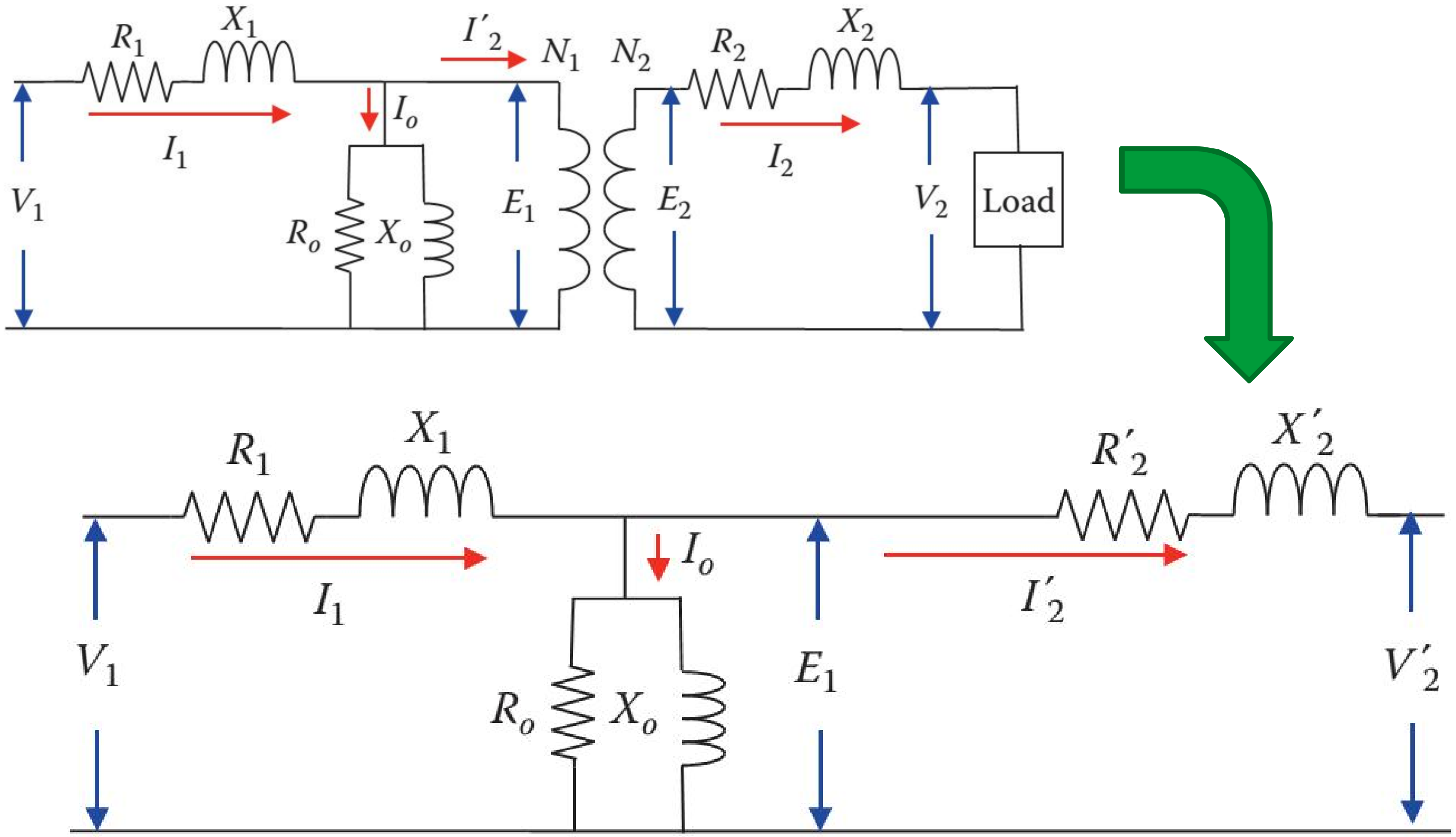


$$\overline{I}_2' = \frac{N_2}{N_1}\overline{I}_2$$



Equivalent circuit of transformer

- Reduce the secondary side to the primary



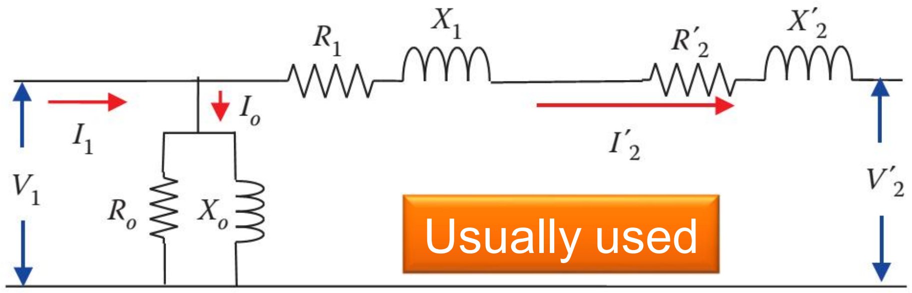
$$\bar{V}_2' = \frac{N_1}{N_2}\bar{V}_2$$

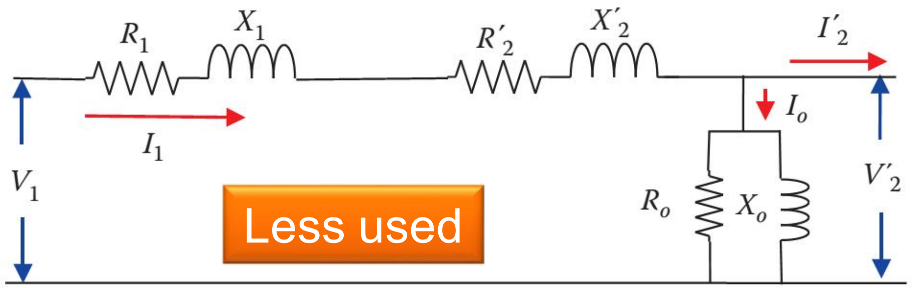
$$R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2$$

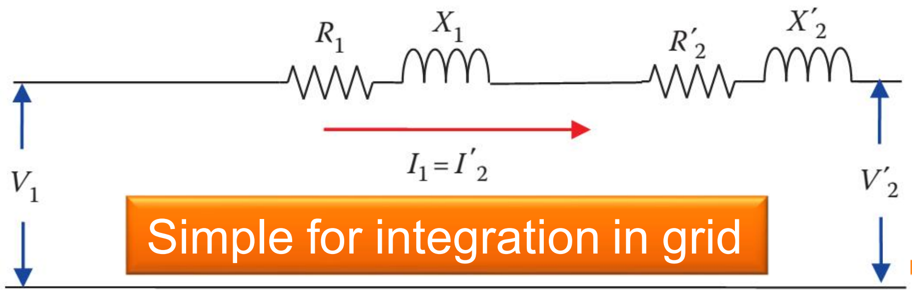
$$X_2' = \left(\frac{N_1}{N_2}\right)^2 X_2$$

Approximate equivalent circuit

- The magnetizing branch can be shifted to either side or removed!

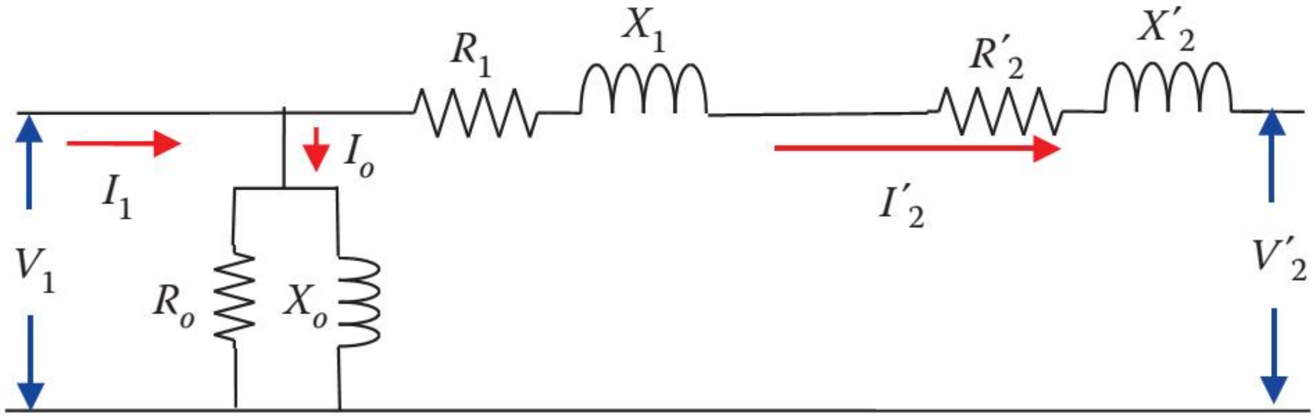






Parameters of equivalent circuit

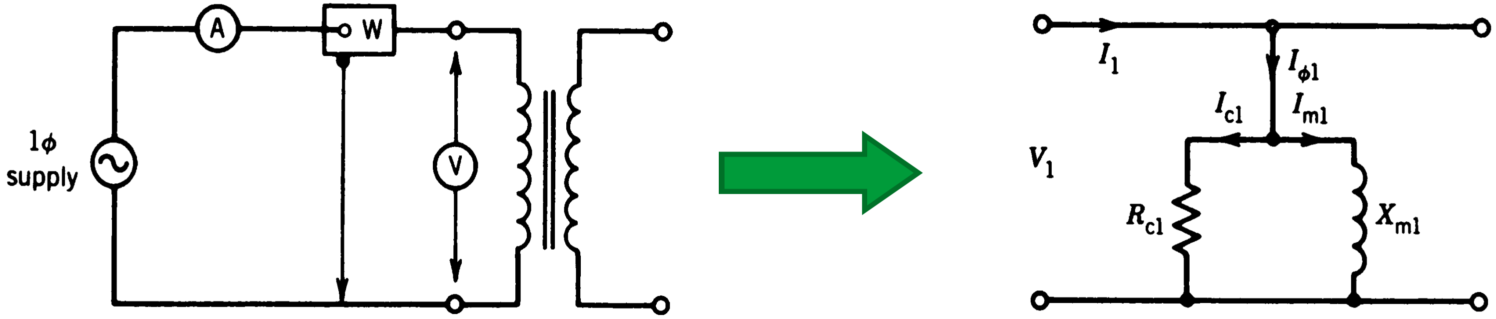
- Some measurements are needed for the extraction of the equivalent circuit parameters



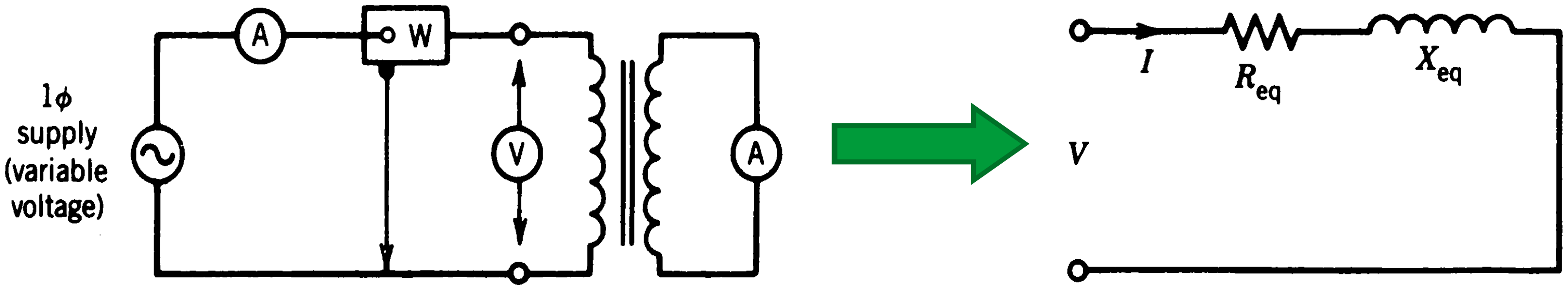
- Standard measurements:
  - No-load test: gives magnetization characteristic
  - Short-circuit test: gives winding characteristics

Equivalent Circuit Parameters

- No-load test: rated voltage on one side, the other side is open



- Short-Circuit test: rated current on one side, the other side is short-circuited



Equivalent Circuit Parameters

- Calculation example: 10 kVA, 2200/220 V, 60 Hz
- Open-circuit done with rated voltage applied to low-voltage side of transformer
- Short-circuit test done with low voltage side shorted and the transformer is fed from high-voltage side

	Open-circuit test H-voltage side open	Short-circuit test L-voltage side shorted
Voltmeter	220 V	150 V
Ammeter	2,5 A	4,55 A
Wattmeter	100 W	215 W

Equivalent Circuit Parameters

- Open-circuit results

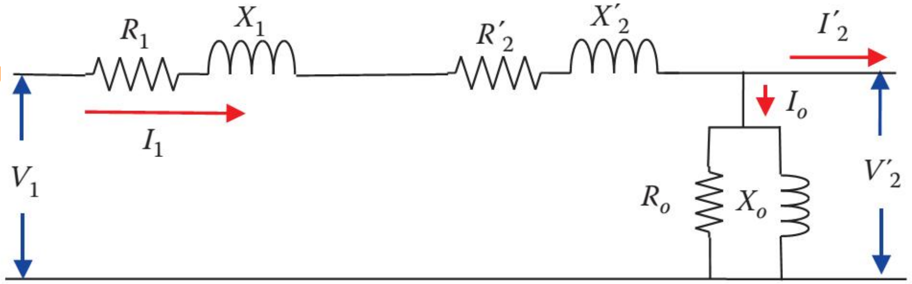
$$R_{0L} = \frac{V_{oc}^2}{P_{oc}} = \frac{220^2}{100} = 484\,\Omega \qquad I_0 = \frac{V_{oc}}{R_{0L}} = \frac{220}{484} = 0.45\,A$$

$$I_{X_0} = \sqrt{(I_{oc}^2 - I_{R_0}^2)} = \sqrt{(2.5^2 - 0.45^2)} = 2.46\,A \qquad X_{0L} = \frac{V_{oc}}{I_{X_0}} = \frac{220}{2.46} = 89.4\,\Omega$$

- Short-circuit results

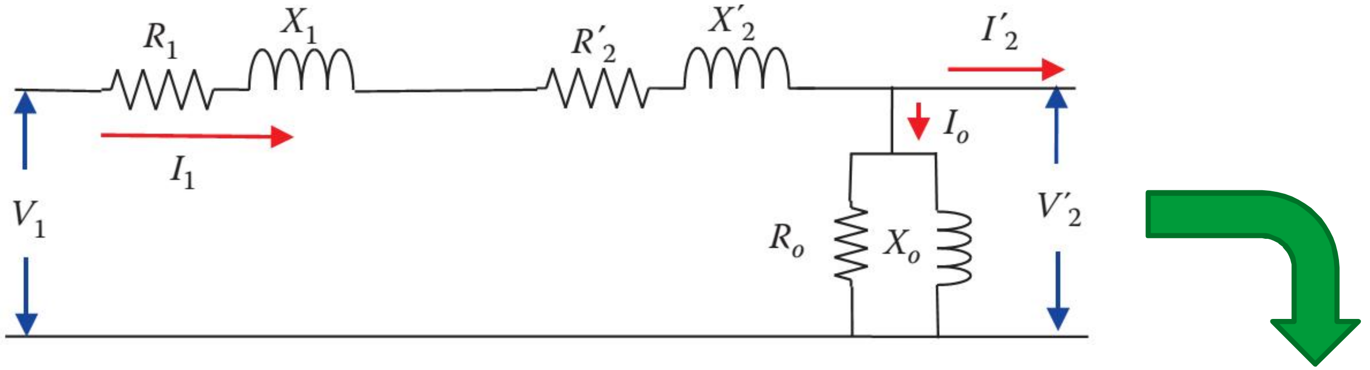
$$R_{eqH} = \frac{P_{sc}}{I_{sc}^2} = \frac{215}{4.55^2} = 10.4\,\Omega \qquad Z_{eqH} = \frac{V_{sc}}{I_{sc}} = \frac{150}{4.55} = 32.97\,\Omega$$

$$X_{eqH} = \sqrt{\left(Z_{eqH}^2 - R_{eqH}^2\right)} = \sqrt{\left(32.97^2 - 10.4^2\right)} = 31.3\,\Omega$$



Equivalent Circuit Parameters

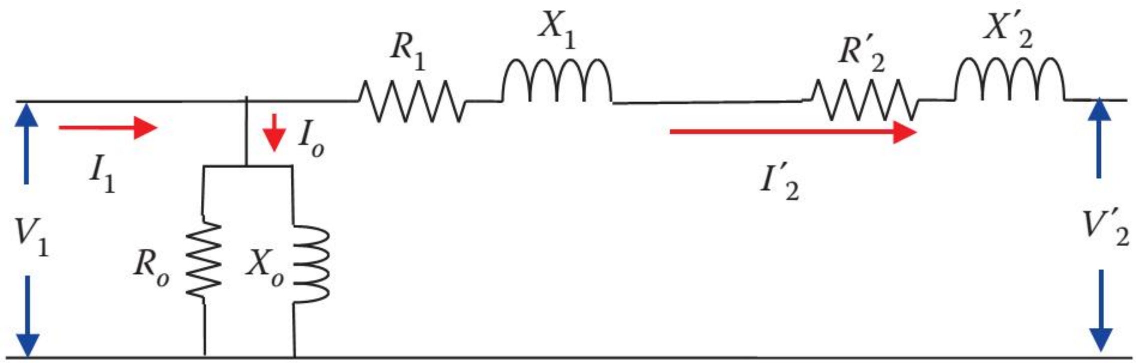
- Move the magnetizing branch from low voltage side to high voltage side



$$a = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{2200}{220} = 10$$

$$R_{0H} = a^2 R_{0L} = 10^2 \times 484 = 48400\,\Omega$$

$$X_{0H} = a^2 X_{0L} = 10^2 \times 89.4 = 8940\,\Omega$$



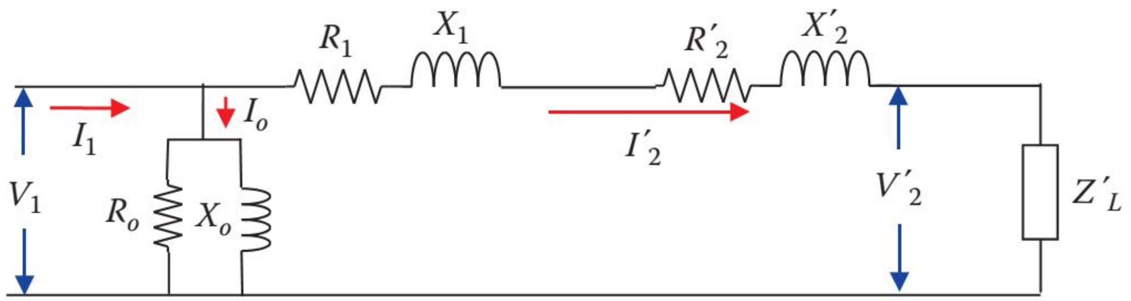
Transformer analysis

- Transformer and load parameters:

$$\frac{N_1}{N_2} = 10 \qquad \mathrm{R}_{eq} = R_1 + R_2^{'} = 1\,\Omega \qquad \mathrm{R}_0 = 1000\,\Omega$$

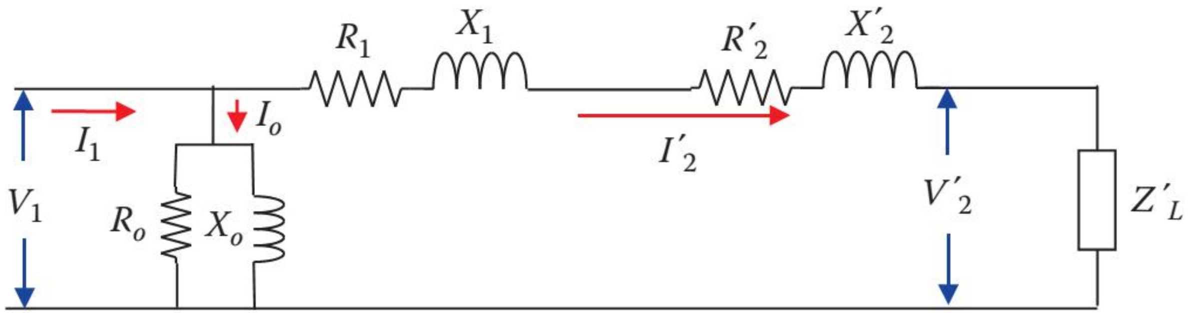
$$X_{eq} = X_1 + X_2^{'} = 10\,\Omega \qquad X_0 = 5000\,\Omega$$

$$\bar{Z}_L = 0.5\angle 30^\circ\,\Omega$$



- The primary voltage is 1000 V, Compute load voltage

Transformer analysis



- Load

$$Z_L^{'} = \left(\frac{N_1}{N_2}\right)^2 Z_L = 100 \times 0.5 = 50\,\Omega$$

- Secondary current

$$\bar{I}_2^{'} = \frac{\bar{V}_1}{\left(\mathrm{R}_{eq} + jX_{eq}\right) + \bar{Z}_L^{'}} = \frac{1000\angle 0^\circ}{\left(1 + j10\right) + 50\angle 30^\circ} = 17.7\angle -38.31^\circ\,A$$

- Load voltage

$$\bar{V}_2^{'} = \bar{I}_2^{'}\bar{Z}_L^{'}\left(17.7\angle -38.31^\circ\right)\left(50\angle 30^\circ\right) = 885\angle -8.31^\circ\,V$$

$$V_2 = \frac{N_2}{N_1} V_2^{'} = 88.5\,V$$

Transformer Efficiency

- Efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Ouput power+losses}} = \frac{\text{Intput power-losses}}{\text{Input power}}$$

- Output power

$$P_{out} = V_2 I_2 \cos(\theta_2) = V_2^{'} I_2^{'} \cos(\theta_2)$$

- Input power

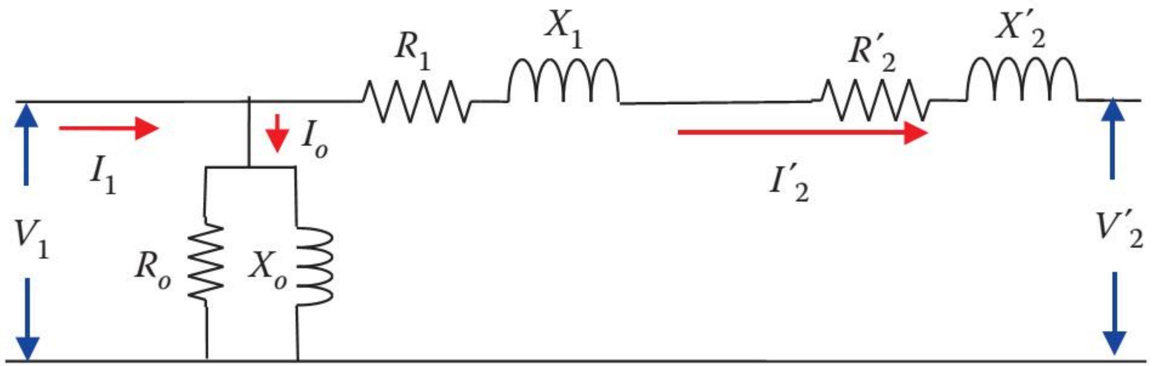
$$P_{in} = V_1 I_1 \cos(\theta_1)$$

- Losses
  - Copper losses

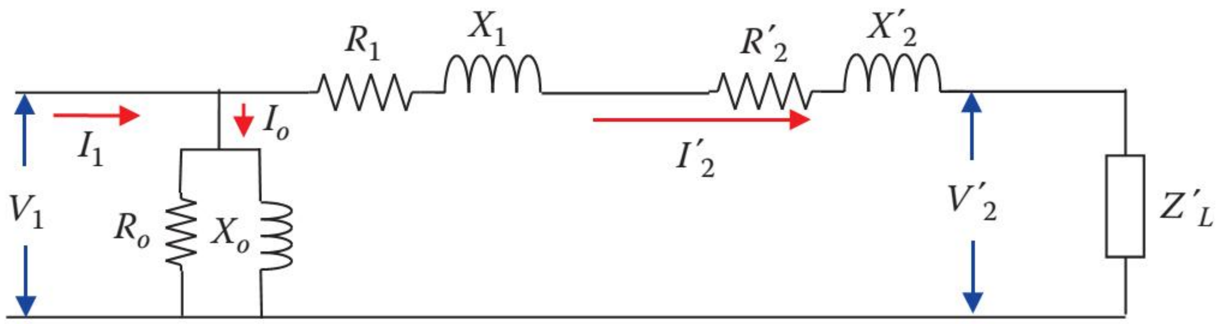
$$P_{Cu} = (I_2^{'})^2 (R_1 + R_2^{'})$$

  - Iron losses

$$P_{iron} = \frac{V_1^2}{R_0}$$



Transformer Efficiency



$$P_{Cu} = \left(I_2^{'}\right)^2 R_{eq} = 17.7^2 \times 1 = 313.29\,W$$

$$P_{iron} = \frac{V_1^2}{R_0} = \frac{1000^2}{1000} = 1\,\text{kW}$$

$$P_{out} = V_2^{'} I_2^{'} \cos\left(\theta_2\right) = 885 \times 17.7 \times \cos 30 = 13.57\,\text{kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{Cu} + P_{iron}} = \frac{13570}{13570 + 313.29 + 1000} = 91.2\%$$