Experiment 2: Noise Measurements

PHYS-C0258: Quantum Labs

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Objectives: Measurements of noise spectra emitted by a resistor. Analysis of different noise types, temperature dependent noise levels, and noise mitigation.

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1 Introduction

Noise is a ubiquitous occurrence in all physics experiments and every measured quantity is subjected to a noise background. At the same time, noise is a key contributor to measurement uncertainties and errors. In the quantum world the measured signals often have very small amplitudes, requiring high-gain amplification. For such small signals, the signal-to-noise ratio is a key parameter to determine the significance and reliability of measurement results.

In this experiment we will measure the noise levels and spectrum of an ohmic resistor at different temperatures. It is our aim to understand where noise comes from, how it behaves, and most importantly, how it can be minimized.

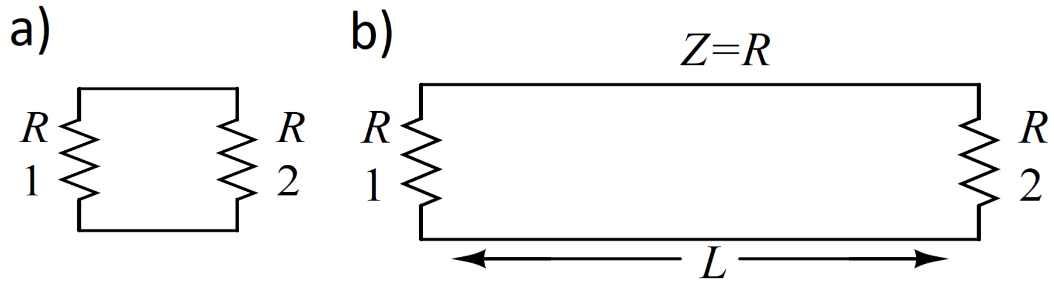

Figure 1: Circuits for the derivation of Nyquist theory. a) two identical resistors in a loop and b) two resistors separated by line distance L forming a lossless transmission line.

2 Noise Theory

On the most fundamental level, noise is the disturbance or fluctuation of an otherwise constant physical measure or signal. In this experiment we will focus on electronic noise, especially on power fluctuations of a constant electric signal.

Electric noise can be caused by a number of different sources including measurement instruments, amplifiers, external electromagnetic fields, voltage or signal generators etc. However, even when eliminating these "artificial" noise sources in an experiment, more fundamental noise sources remain, most importantly thermal (Johnson) noise. This type of noise can be observed even in simple devices, for example in an ohmic resistor.

Johnson-Nyquist noise occurs due to thermal agitation of the electro-magnetic modes in a conductor which are coupled to a thermal environment. The Nyquist theory gives a quantitative expression for the Johnson-Nyquist noise generated by a system in thermal equilibrium and is typically applied to estimate the limiting signal-to-noise ratio of an experimental apparatus.

In a simple system of two conductors, see fig.1 a) with equal resistance R at a given temperature T, conductor 1 produces a current I in the circuit equal to the electro- motive force due to thermal agitation divided by the total resistance 2R. This current delivers power to conductor 2. Conductor 2 produces a similar current which delivers power to conductor 1 until reaching an equilibrium state. Because the two conductors are at the same temperature and the ciruit is symmetric, the second law of thermodynamics dictates that the power in both directions is equal. Note that no assumption about the nature of conductors has been made at this point.

Starting from equilibrium, power transfer from conductor 1 to 2 in any frequency range would violate the second law of thermodynamics. Hence, the electromotive force due to thermal agitation in conductors are universal functions of the frequency f, the resistance R and the Temperature T. This was experimentally confirmed by Dr. J. B. Johnson's experiments in 1928.

For the derivation of the mean-square voltage $\langle V^2 \rangle$ across a conductor we consider a lossless one-dimensional transmission line of length L terminated at both ends by conductors with resistance R and characteristic impedance Z = R, see fig. 1 b). Any voltage wave propagating along the transmission line is completely absorbed by the terminating resistor without reexcitations. Voltage oscillations $V = V_0 exp(i(k_x x + \omega t))$ propagate along the transmission line at velocity $v = \frac{1}{k_x}$. With the periodic boundary condition $V(0) = V(L)$ and the wave vector $k_x L = 2\pi n$ where n is any integer, the density of modes is

$$D(\omega) = \frac{1}{L}\frac{dn}{d\omega} = \frac{1}{L}\frac{dn}{dk_x}\frac{dk_x}{d\omega} = \frac{1}{2\pi v} \tag{1}$$

The mean energy per mode is given by the Planck formula,

$$\langle \epsilon(\omega) \rangle = \frac{\hbar\omega}{exp(\frac{\hbar\omega}{k_B T} - 1)} \approx k_B T \tag{2}$$

where we approximate the classical limit $\hbar\omega << k_B T$ according to the equipartition theorem.

The frequency-dependent energy density can then be expressed with the energy density per mode and density of modes:

$$U(\omega) = D(\omega)\langle \epsilon(\omega) \rangle = \frac{k_B T}{2\pi v} \tag{3}$$

The power follows as

$$P(\omega) = vU(\omega) = \frac{k_B T}{2\pi} \to P(f) = k_B T \tag{4}$$

This thermal electromotive force generated by the resistor 1 generates a current $I = \frac{V}{2R}$ in the transmission line. Thus, the power absorbed by resistor 2 is

$$P(f) = \langle I^2(f) \rangle R = \left\langle \frac{V^2(f)}{4R^2} \right\rangle R = \frac{\langle V^2(f) \rangle}{4R} \tag{5}$$

With equation 2 we can equate

$$\langle V^2(f) \rangle = 4Rk_B T \tag{6}$$

This is the mean-square voltage per unit frequency. To calculate the mean-square voltage over a given frequency range Δf we integrate this formula over f within Δf and find the Nyquist theorem:

$$\langle V^2 \rangle = 4Rk_B T\Delta f \tag{7}$$

In our experiment Δf is given by the measurement bandwidth.

3 Experiment

The experiment is conducted at QCD labs (Micronova, Tietotie 3). In the lab we have a spectrum analyzer, an amplifier, a voltage source, cables, and the noise source resistor ready for the

Table 1: Good starting parameters of the spectrum analyzer.

Parameters	set Values
Mode	Spectrum
Ref. Level	173 pW
Att	0 dB
SWT	100 ms
RBW	2 MHz
VBW	2 kHz
FREQ	6 GHz
SPAN	0 Hz

experiment. All data is recorded with the spectrum analyzer, which requires a USB-stick for data transfer, please bring your own.

The key parameters of the spectrum analyzer will be set to usable values before the experiment. You are free to modify them, but keep track of your measurement parameters. In case you get lost, table1 contains a set of usable parameters to get started.

Assemble the experimental setup according to figure 2. At first, keep the resistor outside of the nitrogen bath to check your setup and to record a baseline measurement at room temperature. Now it is time to check the noise contributions from the different elements of your circuit. What noise power is generated from the resistor, and what power is generated by the rest of the circuit? Disconnect the resistor and check the circuit noise level. Also disconnect the cable and record an amplifier baseline noise measurement.

You can use the spectrum analyzer to get a quick analysis of your noise level. By inserting a time-scale marker in you trace, you can press the "MKR FUNCT" button and select "NOISE MEAS" -> on. This gives a noise level measurement at the marker position. Unfortunately we do not know the measurement uncertainty of this value, so it is good to monitor the noise level, but not suitable for proper analysis.

When the room temperature trace has been recorded and saved, place the resistor and cable in the nitrogen can. You will have to wait for a few minutes for the resistor to reach the temperature of the N_2 bath (77 K). Then repeat the measurement with the same parameters as in your room temperature measurement. After you have finished the low temperature measurements, take the resistor out of the nitrogen bath and let it warm up. Make sure that the noise level returns to its original value.

A comment on using the spectrum analyzer and saving traces: In normal operation you can use the "continuous sweep" option, which allows you to see any changes in the signal in real time. If everything is set up and you want to save a trace, change the setting to "single sweep" and then, under the "trace" menu, select "Export ASCII". Select your USB stick and save the .DAT file there. The file contains the instrument parameters and the trace data. For analysis you can

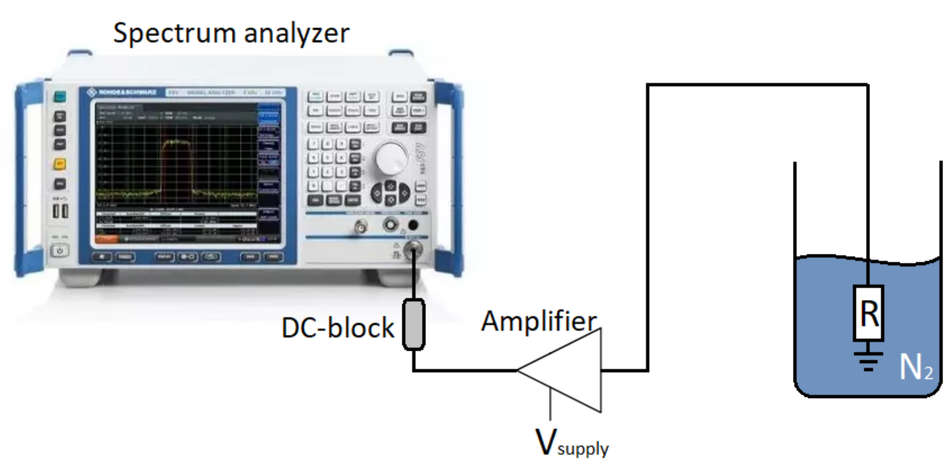

Figure 2: Wiring diagram of a resistor-noise measurement in a liquid nitrogen bath. Noise is generated by the resistor R and reaches an amplifier through a coaxial cable. The amplifier output is connected to a spectrum analyzer through a DC-block, which protects the spectrum analyzer from DC currents. The overall signall is then recorded with the spectrum analyzer.

read this file e.g. using python or matlab code.

4 Tips for data analysis

- Plot the data you have collected (time-traces, spectrum) in clear figures with proper labels. Clearly indicate which plot refers to which measurement conditions (with/without resistor, temperature).
- Calculate the power average of each trace. Subtract the average from your signal and calculate the noise level using RMS and Peak-to-peak technique. Comment on which is a better representation of the actual noise level. Also calculate the signal-to-noise ratio by comparing the average signal power to noise power.
- Compare the noise levels at room temperature to those at 77K. Is the decrease in noise consistent with theory? What uncertainties do we have to account for?
- Comment on the frequency spectrum. What changes in amplitude can we see and why? You don't have to do any statistical analysis here, an explanation based on the plot is sufficient.
- Summarize the results and conclusions you can draw from your measurements. Was the goal of the experiment met? If not, what went wrong?