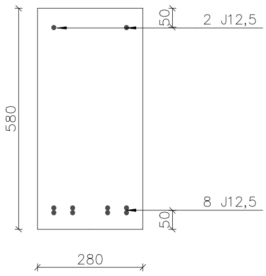
Rak. 43-3111
Prestressed beam with the bonded prestensioned strands



1. Strength values of the materials

The cross-section of the prestressed beam is a rectangle which dimensions are:

Depth: $h := 580 \cdot mm$ width: $b := 280 \cdot mm$

Length (span) of the beam $L := 10 \cdot m$

Conrete class C40/50 $f_{ck} := 40 \cdot MPa$

Average compressive strength $f_{cm} := f_{ck} + 8 \cdot MPa$ $f_{cm} = 48\,MPa$

Average compressive strength at transfer $f_{cmi} := 0.75 \cdot f_{cm}$ $f_{cmi} = 36\,MPa$

Characteristic compressive strength at transfer $f_{cki} := f_{cmi} - 8 \cdot MPa$ $f_{cki} = 28\,MPa$

$$\beta_{cci} := \frac{f_{cmi}}{f_{cm}} \qquad \beta_{cci} = 0.75$$

Average tensile strength of concrete $f_{ctm} := 0.3 \cdot \left(\frac{f_{ck}}{MPa}\right)^{0.667} \cdot MPa$ $f_{ctm} = 3.513\,MPa$

Average tensile strength at transfer; t < 28 days $\alpha := 1$

$$f_{ctmi} := \beta_{cci}{}^{\alpha} \cdot f_{ctm} \qquad f_{ctmi} = 2.635\,MPa$$

Elastic modulus of concrete $E_{cm} := 22000 \cdot MPa \cdot \left(\frac{f_{cm}}{10 \cdot MPa}\right)^{0.3}$ $E_{cm} = 35220\,MPa$

Elastic modulus of concrete at transfer $E_{cmi} := \left(\frac{f_{cmi}}{f_{cm}}\right)^{0.3} \cdot E_{cm}$ $E_{cmi} = 32308\,MPa$

Prestressed strands 8 ϕ_{p7} 12,5, grade St 1600/1800 $n_p := 8$ $\phi_p := 12.5 \cdot mm$

$A_{p1} := 93 \cdot mm^2$

$A_p := n_p \cdot A_{p1}$ $A_p = 744\,mm^2$

Strength values of the strands:

0.1-limit $f_{p0.1k} := 1600 \cdot MPa$

Ultimate strength $f_{pk} := 1800 \cdot MPa$

Elastic modulus $E_p := 195000 \cdot MPa$

Maximum allowable stress of the strands (EC2 5.10.2.1)

$\sigma_{pmax0} := \min\left(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}\right)$ $\sigma_{pmax0} = 1440\,MPa$

Choose the initial stress after the jacking loss $\sigma_{pmax} := 1350 \cdot MPa$

Losses before transfer $\Delta\sigma_{p1} := -33 \cdot MPa$ n. 2.4 % (incl. relaxation +shrinkage+heat treatment)

Stress of the strands **just before the transfer** $\sigma_{po} := \sigma_{pmax} + \Delta\sigma_{p1}$ $\sigma_{po} = 1317\,MPa$

Distance of the strands $c_p := 50 \cdot mm$ from the bottom

Effective depth $d := h - c_p$ $d = 530\,mm$

2. Transfer (release)

2.1. Transformed "ideal" section values at transfer

Reinforcement and also the strands are transformed to concrete in the ratio of the elastic modulie taking into account that there is no conrete at th epint of the reinforcement

Ratio of the elastic modulies of strands and conrete $n_e := \frac{E_p}{E_{cmi}}$ $n_e = 6.036$

Cross-section area $A_m := b \cdot h + (n_e - 1) \cdot A_p$ $A_m = 0.166\,m^2$

First moment about the bottom fibre $S_m := b \cdot h \cdot \frac{h}{2} + (n_e - 1) \cdot A_p \cdot c_p$ $S_m = 0.047\,m^3$

Distance of the centroid from the bottom $pp := \frac{S_m}{A_m}$ $pp = 284.6\,mm$

Second moment $I_m := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - pp\right)^2 + (n_e - 1) \cdot A_p \cdot (c_p - pp)^2$

$I_m = 4.764 \times 10^{-3}\,m^4$

Modulus about the bottom fibre $W_{ma} := \frac{I_m}{pp}$ $W_{ma} = 0.017\,m^3$

Modulus about the top fibre $W_{my} := \frac{I_m}{pp - h}$ $W_{my} = -0.016\,m^3$

2.2 Prestress force

Prestress force just before transfer $P_0 := \sigma_{po} \cdot A_p$ $P_0 = 979.848\,kN$

At transfer the beam is affected by the compressive counterforce of the prestressing force

$-P_0 = -979.848\,kN$

The compressive counterforce is affected at the cetroid of the strands => the force is moved to the centroid of the transferred cross-section and the move is replaced by the moment

$M_p := -P_0 \cdot (pp - c_p)$ $M_p = -229.861\,kNm$

Eccentricity of the prestress force (counter force) $e_p := pp - c_p$ $e_p = 234.588\,mm$

The beam cross-section is affected by the compressive force -P_0 and the negative moment M_P (which induce compressive stressses at the bottom fibre), which induce the stresses:

bottom fibre $\sigma_{cap} := \frac{-P_0}{A_m} + \frac{M_p}{W_{ma}}$ $\sigma_{cap} = -19.63\,MPa$

compressive

at the point of the strands $\sigma_{cpp} := \frac{-P_0}{A_m} + \frac{M_p \cdot (pp - c_p)}{I_m}$ $\sigma_{cpp} = -17.217\,MPa$

top fibre $\sigma_{cyp} := \frac{-P_0}{A_m} + \frac{M_p}{W_{my}}$ $\sigma_{cyp} = 8.357\,MPa$

tensile

Change of the stress in the strands $\Delta\sigma_{pe} := \sigma_{cpp} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pe} = -103.917\,MPa$

Stress in the strands after the transfer $\sigma_{p1} := \sigma_{po} + \Delta\sigma_{pe}$ $\sigma_{p1} = 1213.1\,MPa$

Stress resultant of conrete $N_c := \frac{(\sigma_{cap} + \sigma_{cyp})}{2} \cdot b \cdot h - \sigma_{cpp} \cdot A_p$ $N_c = -902.534\,kN$

Stress resultant of the strands $P_1 := \sigma_{p1} \cdot A_p$ $P_1 = 902.534\,kN$

Equilibrium: $P_1 + N_c = 0\,kN$

Strains:

Bottom fibre $\varepsilon_{cap} := \frac{\sigma_{cap}}{E_{cmi}}$ $\varepsilon_{cap} = -0.608\,‰$

Top fibre $\varepsilon_{cyp} := \frac{\sigma_{cyp}}{E_{cmi}}$ $\varepsilon_{cyp} = 0.259\,‰$

Curvature $\psi_p := \frac{\varepsilon_{cap} - \varepsilon_{cyp}}{h}$ $\psi_p = -1.494\,\frac{‰}{m}$

The curvature is also obtained from the formula $\psi_p := \frac{M_p}{E_{cmi} \cdot I_m}$ $\psi_p = -1.494\,\frac{‰}{m}$

The prestressing force is equal almost in the whole length of the beam => Mp and the curvature are equal almost in the whole length of the beam

Deflection coefficient $\delta_{ap} := \frac{1}{8}$

Deflection (camber) due to the prestressing $a_p := \delta_{ap} \cdot \psi_p \cdot L^2$ $a_p = -18.669\,mm$

Deflection can also b e obtained from the formula $a_p := \delta_{ap} \cdot \frac{M_p}{E_{cmi} \cdot I_m} \cdot L^2$ $a_p = -18.669\,mm$

(negative => upward deflection => camber)

2.3 Self weight of the beam

Just after the release the beam rises upward from the bed and so the weight of the beam starts affect => the influence of the beam's weight must be taken into account in the stresses of the trans stage

The weight of the beam $g := b \cdot h \cdot 25 \cdot \frac{kN}{m^3}$ $g = 4.06 \frac{kN}{m}$

The bending moment due to beam's weight $M_g := \frac{g \cdot L^2}{8}$ $M_g = 50.75\,kNm$

Stersses due to the weight of the beam

bottom fibre $\sigma_{cag} := \frac{M_g}{W_{ma}}$ $\sigma_{cag} = 3.032\,MPa$

at the location of the strands $\sigma_{cpg} := \frac{M_g \cdot (pp - c_p)}{I_m}$ $\sigma_{cpg} = 2.499\,MPa$

top fibre $\sigma_{cyg} := \frac{M_g}{W_{my}}$ $\sigma_{cyg} = -3.147\,MPa$

The change of the stress of the strands $\Delta\sigma_{pg} := \sigma_{cpg} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pg} = 15.085\,MPa$

Total stress of the strands $\sigma_{p1} := \sigma_{p1} + \Delta\sigma_{pg}$ $\sigma_{p1} = 1228.2\,MPa$

Change of the force of the strands $\Delta P_g := \Delta\sigma_{pg} \cdot A_p$ $\Delta P_g = 11.223\,kN$

The stress resultant of concete $N_{cg} := \frac{(\sigma_{cag} + \sigma_{cyg})}{2} \cdot b \cdot h - \sigma_{cpg} \cdot A_p$ $N_{cg} = -11.223\,kN$

Equilibrium: $\Delta P_g + N_{cg} = -0\,kN$

Strains:

bottom fibre $\varepsilon_{cag} := \frac{\sigma_{cag}}{E_{cmi}}$ $\varepsilon_{cag} = 0.094\,‰$

top fibre $\varepsilon_{cyg} := \frac{\sigma_{cyg}}{E_{cmi}}$ $\varepsilon_{cyg} = -0.097\,‰$

curvature $\psi_g := \frac{\varepsilon_{cag} - \varepsilon_{cyg}}{h}$ $\psi_g = 0.33 \frac{‰}{m}$

The curvature can be obtained also from the formula $\psi_g := \frac{M_g}{E_{cmi} \cdot I_m}$ $\psi_g = 0.33 \frac{‰}{m}$

The self weight of the beam is uniformly distributed load => Mg-area is parabolic

Deflection coefficient $\delta_{ag} := \frac{5}{48}$

Deflection due to self weight of the beam $a_g := \delta_{ag} \cdot \psi_g \cdot L^2$ $a_g = 3.435\,\text{mm}$

The deflectin can be obtained also from the formula $a_g := \delta_{ag} \cdot \frac{M_g}{E_{cmi} \cdot I_m} \cdot L^2$ $a_g = 3.435\,\text{mm}$

(deflection downward)

2.4 The situation just after the release (tarnsfer), when the beam is loaded by the prestress force and the self weight of the beam

For the influencies due to the prestress force and the self weight of the beam the superposition principle is valid if the cross-section remains uncracked.

Stresses:

Bottom fibre $\sigma_{ca} := \sigma_{cap} + \sigma_{cag}$ $\sigma_{ca} = -16.598\,\text{MPa}$

At the position of the strands $\sigma_{cp} := \sigma_{cpp} + \sigma_{cpg}$ $\sigma_{cp} = -14.718\,\text{MPa}$

Top fibre $\sigma_{cy} := \sigma_{cyp} + \sigma_{cyg}$ $\sigma_{cy} = 5.21\,\text{MPa}$

$> f_{ctmi} = 2.635\,\text{MPa}$

Top side is cracked

Strains:

Bottom fibre $\varepsilon_{ca} := \dfrac{\sigma_{ca}}{E_{cmi}}$ $\varepsilon_{ca} = -0.514\,‰$

At the position of the strands $\varepsilon_{cp} := \dfrac{\sigma_{cp}}{E_{cmi}}$ $\varepsilon_{cp} = -0.456\,‰$

Top fibre $\varepsilon_{cy} := \dfrac{\sigma_{cy}}{E_{cmi}}$ $\varepsilon_{cy} = 0.161\,‰$

The stress resultant of concrete $N_c := \dfrac{\sigma_{ca} + \sigma_{cy}}{2} \cdot b \cdot h - \sigma_{cp} \cdot A_p$ $N_c = -913.757\,\text{kN}$

The change of the stress in the strands $\Delta\sigma_{p2} := \Delta\sigma_{pe} + \Delta\sigma_{pg}$ $\Delta\sigma_{p2} = -88.833\,\text{MPa}$

The change of the strain of the strands $\Delta\varepsilon_p := \varepsilon_{cp}$ $\Delta\varepsilon_p = -0.456\,‰$

The change of the stress in the strands is also obtained $\Delta\sigma_{p2} := \Delta\varepsilon_p \cdot E_p$ $\Delta\sigma_{p2} = -88.833\,\text{MPa}$

Total stress in the strands just after the release $\sigma_{p2} := \sigma_{po} + \Delta\sigma_{p2}$

$\sigma_{p2} = 1228.2\,\text{MPa}$

The force of the strands just after release $P_2 := \sigma_{p2} \cdot A_p$ $P_2 = 913.757\,\text{kN}$

Equilibrium $P_2 + N_c = 3.492 \times 10^{-13}\,\text{kN}$

deflection $a := a_p + a_g$ $a = -15.234\,\text{mm}$

upward camber

2.5 Top strands are added

2.5.1 Top strands

Because the tension stress at the top fibre exceeds the tension strength of concrete, the sitaution i
impoved by adding 2 top strands ϕ_{p7} 12,5 mm in the top side of the beam.

Top strands: $n_{py} := 2$ $A_{py} := n_{py} \cdot A_{p1}$ $A_{py} = 186\,mm^2$

Initial prestress $\sigma_{pymax} := 1200 \cdot MPa$

Losses before transfer $\Delta\sigma_{py} := -25 \cdot MPa$ due to relaxation, shrinkage, heating

Stress just before the transfer $\sigma_{poy} := \sigma_{pymax} + \Delta\sigma_{py}$ $\sigma_{poy} = 1175\,MPa$

Distance from the top side 50 mm => distance from the bottom side $c_{py} := 530 \cdot mm$

Prestress force just before the transfer and the location of the resultant of the prestress force

$P_0 := \sigma_{po} \cdot A_p + \sigma_{poy} \cdot A_{py}$ $P_0 = 1198.4\,kN$

Location of prestress force resultant $c_{res} := \dfrac{\sigma_{po} \cdot A_p \cdot c_p + \sigma_{poy} \cdot A_{py} \cdot c_{py}}{P_0}$ $c_{res} = 137.537\,mm$

2.52 Cross-section values included the top strands

$A_m := b \cdot h + (n_e - 1) \cdot (A_p + A_{py})$ $A_m = 0.167\,m^2$

$S_m := b \cdot h \cdot \dfrac{h}{2} + (n_e - 1) \cdot (A_p \cdot c_p + A_{py} \cdot c_{py})$ $S_m = 0.048\,m^3$

centroid from bottom $pp := \dfrac{S_m}{A_m}$ $pp = 285.964\,mm$

Second moment $I_m := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - pp\right)^2 + (n_e - 1) \cdot \left[A_p \cdot (c_p - pp)^2 + A_{py} \cdot (c_{py} - pp)^2\right]$

$I_m = 0.0048\,m^4$

Bending moulus about the bottom side $W_{ma} := \dfrac{I_m}{pp}$ $W_{ma} = 0.017\,m^3$

Bending modulus about the top side $W_{my} := \dfrac{I_m}{pp - h}$ $W_{my} = -0.016\,m^3$

2.53 Prestress force included the top strands

Prestress force just before the transfer $P_0 := \sigma_{po} \cdot A_p + \sigma_{poy} \cdot A_{py}$ $P_0 = 1198.4\,kN$

During the transfer of the prestress force the reinforced (with the strands), transformed cross-section is loaded the compressive counter-force of the prestress force (palkkia puristava voima) $-P_0 = -1.198 \times 10^3\,kN$

This compressive force is acting to the centroid of the strands =>
This compressive force is moved to the centroid of the transformed cross-section and the movement is compensated by the moment

$M_p := -P_0 \cdot (pp - c_{res})$ $M_p = -177.875\,kNm$

eccentricity of prestress force $e_p := pp - c_{res}$ $e_p = 148.427\,mm$

The beam is loaded by the compressive normal corce $-P_0$ ja the negatiove bending (inducing compressive stress at the bottom fibre) moment Mp, witch induce to the beam the stresses:

bottom fibre $\sigma_{cap} := \frac{-P_0}{A_m} + \frac{M_p}{W_{ma}}$ $\sigma_{cap} = -17.726\,MPa$

compression

at the location of the bottom strands $\sigma_{cpp} := \frac{-P_0}{A_m} + \frac{M_p \cdot (pp - c_p)}{I_m}$ $\sigma_{cpp} = -15.881\,MPa$

at the location of the top strands $\sigma_{cppy} := \frac{-P_0}{A_m} + \frac{M_p \cdot (pp - c_{py})}{I_m}$ $\sigma_{cppy} = 1.834\,MPa$

top fibre $\sigma_{cyp} := \frac{-P_0}{A_m} + \frac{M_p}{W_{my}}$ $\sigma_{cyp} = 3.679\,MPa$

tension

Change of the stress of the prestressing steel - bottom strands $\Delta\sigma_{pe} := \sigma_{cpp} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pe} = -95.852\,MPa$

Stress of the bottom strands $\sigma_{p1} := \sigma_{po} + \Delta\sigma_{pe}$ $\sigma_{p1} = 1221.1\,MPa$

Change of the stress in the top strands $\Delta\sigma_{pye} := \sigma_{cppy} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pye} = 11.069\,MPa$

Force of the top strands increase

Stress of the top strands $\sigma_{py1} := \sigma_{poy} + \Delta\sigma_{pye}$ $\sigma_{py1} = 1186.1\,MPa$

Total force of the strands $P_1 := \sigma_{p1} \cdot A_p + \sigma_{py1} \cdot A_{py}$ $P_1 = 1129.1\,kN$

Stress resultant of concrete $N_c := \frac{(\sigma_{cap} + \sigma_{cyp})}{2} \cdot b \cdot h - \sigma_{cpp} \cdot A_p - \sigma_{cppy} \cdot A_{py}$

$N_c = -1129.1\,kN$

Equilibrium: $P_1 + N_c = 0\,kN$

Strains:

bottom fibre $\varepsilon_{cap} := \frac{\sigma_{cap}}{E_{cmi}}$ $\varepsilon_{cap} = -0.549\,‰$

top fibre $\varepsilon_{cyp} := \frac{\sigma_{cyp}}{E_{cmi}}$ $\varepsilon_{cyp} = 0.114\,‰$

curvature $\psi_p := \frac{\varepsilon_{cap} - \varepsilon_{cyp}}{h}$ $\psi_p = -1.142\,\frac{‰}{m}$

The curvature can be obtained from the equation $\psi_p := \frac{M_p}{E_{cmi} \cdot I_m}$ $\psi_p = -1.142\,\frac{‰}{m}$

Prestressin force is equal almost the whole length of the beam => Mp and curvature are also equa almost the whole length of the beam =>

Deflection coefficient $\delta_{ap} := \frac{1}{8}$

Deflection due to prestressing $a_p := \delta_{ap} \cdot \psi_p \cdot L^2$ $a_p = -14.279\,mm$

Deflection can also be obtained from the equation $a_p := \delta_{ap} \cdot \frac{M_p}{E_{cmi} \cdot I_m} \cdot L^2$ $a_p = -14.279\,mm$

(upward deflection=> camber)

2.5.4 Self weight of the beam

Stresses due to the self weuight of the beam

bottom fibre $\sigma_{cag} := \frac{M_g}{W_{ma}}$ $\sigma_{cag} = 3.011\,MPa$

at the point of the bottom strands $\sigma_{cpg} := \frac{M_g \cdot (pp - c_p)}{I_m}$ $\sigma_{cpg} = 2.485\,MPa$

at the point of top strands $\sigma_{cpyg} := \frac{M_g \cdot (pp - c_{py})}{I_m}$ $\sigma_{cpyg} = -2.57\,MPa$

top fibre $\sigma_{cyg} := \frac{M_g}{W_{my}}$ $\sigma_{cyg} = -3.096\,MPa$

Bottom strands:

Change of the stress of the bottom strands $\Delta\sigma_{pg} := \sigma_{cpg} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pg} = 14.996\,MPa$

Stress of the bottom strands $\sigma_{p1} := \sigma_{p1} + \Delta\sigma_{pg}$ $\sigma_{p1} = 1236.1\,MPa$

Top strands:

Change of the stress of the top strands $\Delta\sigma_{pyg} := \sigma_{cpyg} \cdot \frac{E_p}{E_{cmi}}$ $\Delta\sigma_{pyg} = -15.509\,MPa$

Stress of the top strands $\sigma_{py1} := \sigma_{py1} + \Delta\sigma_{pyg}$ $\sigma_{py1} = 1170.6\,MPa$

Total force of the strands $\Delta P_g := \Delta\sigma_{pg} \cdot A_p + \Delta\sigma_{pyg} \cdot A_{py}$ $\Delta P_g = 8.273\,kN$

Stress resultant of concrete $N_{cg} := \frac{(\sigma_{cag} + \sigma_{cyg})}{2} \cdot b \cdot h - \sigma_{cpg} \cdot A_p - \sigma_{cpyg} \cdot A_{py}$

$N_{cg} = -8.273\,kN$

Equilibrium: $\Delta P_g + N_{cg} = 0\,kN$

Strains:

bottom fibre $\varepsilon_{cag} := \frac{\sigma_{cag}}{E_{cmi}}$ $\varepsilon_{cag} = 0.093\,‰$

yläpinta $\varepsilon_{cyg} := \frac{\sigma_{cyg}}{E_{cmi}}$ $\varepsilon_{cyg} = -0.096\,‰$

curvature $\psi_g := \frac{\varepsilon_{cag} - \varepsilon_{cyg}}{h}$ $\psi_g = 0.326\,\frac{‰}{m}$

The curvature can be obtained from the equation $\psi_g := \frac{M_g}{E_{cmi} \cdot I_m}$ $\psi_g = 0.326\,\frac{‰}{m}$

Self weight is uniformly distributed => Mg-area is parabolic

Deflection coefficient $\delta_{ag} := \frac{5}{48}$

Deflection due to the self weight $a_g := \delta_{ag} \cdot \psi_g \cdot L^2$ $a_g = 3.395\,\text{mm}$

The deflecgtion can be obtained also from the equation $a_g := \delta_{ag} \cdot \frac{M_g}{E_{cmi} \cdot I_m} \cdot L^2$ $a_g = 3.395\,\text{mm}$

(downward deflection)

2.5.5 Situation just after the transfer, when the beam is loaded by the prestressing force and the self weight of the beam

For the influencies due to the prestress force and the self weight of the beam the superposition principle is valid if the cross-section remains uncracked.

Stresses:

Bottom fibre $\sigma_{ca1} := \sigma_{cap} + \sigma_{cag}$ $\sigma_{ca1} = -14.715\,\text{MPa}$

At the location of the bottom strands $\sigma_{cp1} := \sigma_{cpp} + \sigma_{cpg}$ $\sigma_{cp1} = -13.396\,\text{MPa}$

At the location of the top strands $\sigma_{cppy1} := \sigma_{cppy} + \sigma_{cpyg}$ $\sigma_{cppy1} = -0.736\,\text{MPa}$

Top fibre $\sigma_{cy1} := \sigma_{cyp} + \sigma_{cyg}$ $\sigma_{cy1} = 0.583\,\text{MPa}$

$< \; f_{ctmi} = 2.635\,\text{MPa}$

Top side remains uncracked at the point of the maximum moment

Strains:

Bottom fibre $\varepsilon_{ca1} := \dfrac{\sigma_{ca1}}{E_{cmi}}$ $\varepsilon_{ca1} = -0.455\,‰$

At the location of the bottom strands $\varepsilon_{cp1} := \dfrac{\sigma_{cp1}}{E_{cmi}}$ $\varepsilon_{cp1} = -0.415\,‰$

At the location of the top strands $\varepsilon_{cpy1} := \dfrac{\sigma_{cppy1}}{E_{cmi}}$ $\varepsilon_{cpy1} = -0.023\,‰$

Top fibre $\varepsilon_{cy1} := \dfrac{\sigma_{cy1}}{E_{cmi}}$ $\varepsilon_{cy1} = 0.018\,‰$

Stress resultant of concrete $N_c := \dfrac{\sigma_{ca1} + \sigma_{cy1}}{2} \cdot b \cdot h - \sigma_{cp1} \cdot A_p - \sigma_{cppy1} \cdot A_{py}$

$N_c = -1137.42\,\text{kN}$

Bottom strands:

Change of the stress of the bottom strands $\Delta\sigma_{p2} := \Delta\sigma_{pe} + \Delta\sigma_{pg}$ $\Delta\sigma_{p2} = -80.855\,\text{MPa}$

Change of the strain $\Delta\varepsilon_p := \varepsilon_{cp1}$ $\Delta\varepsilon_p = -0.415\,‰$

Change of the stress can also be obtained $\Delta\sigma_{p2} := \Delta\varepsilon_p \cdot E_p$ $\Delta\sigma_{p2} = -80.855\,\text{MPa}$

Change of the stress can also be obtained $\Delta\sigma_{p2} := \sigma_{cp1} \cdot \dfrac{E_p}{E_{cmi}}$ $\Delta\sigma_{p2} = -80.855\,\text{MPa}$

Change of the stress just after the transfer $\sigma_{p2} := \sigma_{po} + \Delta\sigma_{p2}$

$\sigma_{p2} = 1236.1\,\text{MPa}$

Top strands:

Change of the stress due to self weight $\Delta\sigma_{pyg} := \sigma_{cpyg} \cdot \frac{E_p}{E_{cmi}}$

$\Delta\sigma_{pyg} = -15.509\,\text{MPa}$

Tota change of the stress of the top strands $\Delta\sigma_{py2} := \Delta\sigma_{pye} + \Delta\sigma_{pyg}$ $\Delta\sigma_{py2} = -4.44\,\text{MPa}$

Stress of the top strands just after the transfer $\sigma_{py2} := \sigma_{poy} + \Delta\sigma_{py2}$ $\sigma_{py2} = 1170.6\,\text{MPa}$

Totla force of the strands $P_2 := \sigma_{p2} \cdot A_p + \sigma_{py2} \cdot A_{py}$ $P_2 = 1137.42\,\text{kN}$

Equilibrium $P_2 + N_c = 0\,\text{kN}$

Deflection $a_1 := a_p + a_g$ $a = -15.234\,\text{mm}$

upward (camber)

3. Final stage after all losses

3.1. Prestess force after the losses

Final stage affter all prestress losses

Suppose the long-term losses (due to shrinkage, creep and relaxation) about 20 % of σpmax-stress; the loss-% is supposed the same also for the top strands (reality the lossses of the top strands may be different). This calculation is apporximate. Later the losses will calculate more precise.

The loss value does not include the elastic shortening because it is reversible

The prestress losses at the final stage

bottom strands $\Delta\sigma_p := -0.2\cdot\sigma_{pmax}$ $\Delta\sigma_p = -270\,\text{MPa}$

top strands $\Delta\sigma_{py} := -0.2\cdot\sigma_{pymax}$ $\Delta\sigma_{py} = -240\,\text{MPa}$

Change of the prestress force $\Delta P := \Delta\sigma_p\cdot A_p + \Delta\sigma_{py}\cdot A_{py}$ $\Delta P = -245.52\,\text{kN}$

Location of the resultant of the change of the prestress loss

$$c_{res} := \frac{A_p\cdot\Delta\sigma_p\cdot c_p + A_{py}\cdot\Delta\sigma_{py}\cdot c_{py}}{\Delta P} \qquad c_{res} = 137.5\,\text{mm}$$

3.2 Section value, when the strength of concrete has developed it's nominal value

Concrete strength is it's nominal value $f_{ck} = 40\,\text{MPa}$ and the elastic modulus $E_{cm} = 35220\,\text{MPa}$

Ratio of elastic modulies of the strands and concrete $n_e := \frac{E_p}{E_{cm}}$ $n_e = 5.537$

The elsatic modulus of conrete has been changed, so the section values are calculated again

The section values included the top strands

$$A_m := b\cdot h + (n_e - 1)\cdot(A_p + A_{py}) \qquad A_m = 0.167\,\text{m}^2$$

$$S_m := b\cdot h\cdot\frac{h}{2} + (n_e - 1)\cdot(A_p\cdot c_p + A_{py}\cdot c_{py}) \qquad S_m = 0.048\,\text{m}^3$$

cenroid from the bottom face $pp := \frac{S_m}{A_m}$ $pp = 286.354\,\text{mm}$

Second moment of area

$$I_m := \frac{b\cdot h^3}{12} + b\cdot h\cdot\left(\frac{h}{2} - pp\right)^2 + (n_e - 1)\cdot\left[A_p\cdot(c_p - pp)^2 + A_{py}\cdot(c_{py} - pp)^2\right]$$

$$I_m = 0.0048\,\text{m}^4$$

Section modulus about the bottom face $W_{ma} := \frac{I_m}{pp}$ $W_{ma} = 0.017\,m^3$

Section modulus about the top face $W_{my} := \frac{I_m}{pp - h}$ $W_{my} = -0.016\,m^3$

3.3 Effect of the prestress losses

The reinforced (ordinary and the prestressing reinforcement) transformed cross-section is loaded by the counterforce of the change of the prestress loss force (tension force)

$-\Delta P = 245.52\,kN$ at the point of the prestress resultant force

Siirretään -ΔP is moved to the centroid of the transformed cross-section and this movement is compenssated by the moment

$\Delta M_p := -\Delta P \cdot (pp - c_{res})$ $\Delta M_p = 36.602\,kNm$

Eccebntricity of the prestress force $e_p := pp - c_{res}$ $e_p = 149.081\,mm$

Stresses due to the prestress losses

bottom fibre $\Delta\sigma_{cap} := \frac{-\Delta P}{A_m} + \frac{\Delta M_p}{W_{ma}}$ $\Delta\sigma_{cap} = 3.66\,MPa$

tension

at the location of the bottom strands $\Delta\sigma_{cpp} := \frac{-\Delta P}{A_m} + \frac{\Delta M_p \cdot (pp - c_p)}{I_m}$ $\Delta\sigma_{cpp} = 3.278\,MPa$

tension

at the location of the top strands $\Delta\sigma_{cppy} := \frac{-\Delta P}{A_m} + \frac{\Delta M_p \cdot (pp - c_{py})}{I_m}$ $\Delta\sigma_{cppy} = -0.387\,MPa$

top fibre $\Delta\sigma_{cyp} := \frac{-\Delta P}{A_m} + \frac{\Delta M_p}{W_{my}}$ $\Delta\sigma_{cyp} = -0.769\,MPa$

tension

Stress resultant of concrete $\Delta N_c := \frac{(\Delta\sigma_{cap} + \Delta\sigma_{cyp})}{2} \cdot b \cdot h - \Delta\sigma_{cpp} \cdot A_p - \Delta\sigma_{cppy} \cdot A_{py}$

$\Delta N_c = 232.414\,kN$

<u>Bottom strands</u>:

Change of the stress due to the prestress losses $\Delta\sigma_{p\Delta} := \Delta\sigma_{cpp} \cdot \frac{E_p}{E_{cm}}$ $\Delta\sigma_{p\Delta} = 18.151\,MPa$

tension

Total stress change $\Delta\sigma_{p3} := \Delta\sigma_p + \Delta\sigma_{p\Delta}$ $\Delta\sigma_{p3} = -251.849\,MPa$

lesser than $\Delta\sigma_p = -270\,MPa$

The difference is a consequence of the fact that the elastic shortening affect as the same way to ΔP than to Po, so a part of the original elastic shortening is reversible

Stress of the bottom strand after the losses $\sigma_{p3} := \sigma_{p2} + \Delta\sigma_p + \Delta\sigma_{p\Delta}$ $\sigma_{p3} = 984.3\,\text{MPa}$

Top strands

Change of the stress due to the prestress losses $\Delta\sigma_{py\Delta} := \Delta\sigma_{cppy} \cdot \frac{E_p}{E_{cm}}$ $\Delta\sigma_{py\Delta} = -2.142\,\text{MPa}$

Stress of the top strands after the losses $\sigma_{py3} := \sigma_{py2} + \Delta\sigma_{py} + \Delta\sigma_{py\Delta}$ $\sigma_{py3} = 928.4\,\text{MPa}$

Total force of the strands $P_3 := \sigma_{p3} \cdot A_p + \sigma_{py3} \cdot A_{py}$ $P_2 = 1.137 \times 10^3\,\text{kN}$

Change of the force due to the losses $\Delta P_3 := \Delta\sigma_{p3} \cdot A_p + \left(\Delta\sigma_{py} + \Delta\sigma_{py\Delta}\right) \cdot A_{py}$

$\Delta P_3 = -232.414\,\text{kN}$

Equilibrium: $\Delta P_3 + \Delta N_c = -0\,\text{kN}$

Strains:

bottom fibre $\Delta\varepsilon_{cap} := \frac{\Delta\sigma_{cap}}{E_{cm}}$ $\Delta\varepsilon_{cap} = 0.104\,‰$

top $\Delta\varepsilon_{cyp} := \frac{\Delta\sigma_{cyp}}{E_{cm}}$ $\Delta\varepsilon_{cyp} = -0.022\,‰$

change of the curvature due to the losses $\Delta\psi_p := \frac{\Delta\varepsilon_{cap} - \Delta\varepsilon_{cyp}}{h}$ $\Delta\psi_p = 0.217\,\frac{‰}{m}$

or also from the equation $\Delta\psi_p := \frac{\Delta M_p}{E_{cm} \cdot I_m}$ $\Delta\psi_p = 0.217\,\frac{‰}{m}$

The prestress force is equal at almost the whole length of the beam =>
ΔMp ja the curvature can be asuumed to be equal at the wole length of the beam

Deflection coefficien $\delta_{ap} := \frac{1}{8}$

Change of the deflection $\Delta a_p := \delta_{ap} \cdot \Delta\psi_p \cdot L^2$ $\Delta a_p = 2.71\,\text{mm}$

Deflectiion can be obtained also from the equation $\Delta a_p := \delta_{ap} \cdot \frac{\Delta M_p}{E_{cm} \cdot I_m} \cdot L^2$ $\Delta a_p = 2.71\,\text{mm}$

Uopward deflection (camber) decrease due to the losses

3.4. Imposed dead load

Imposed dead load $g_2 := 9\cdot\frac{kN}{m}$ $M_{g2} := \frac{g_2\cdot L^2}{8}$ $M_{g2} = 112.5\,kNm$

Stresses due to the imposed dead load

bottom fibre $\sigma_{cag2} := \frac{M_{g2}}{W_{ma}}$ $\sigma_{cag2} = 6.721\,MPa$

at the loaction of the bottom strands $\sigma_{cpg2} := \frac{M_{g2}\cdot(pp - c_p)}{I_m}$ $\sigma_{cpg2} = 5.547\,MPa$

at the loacation of the top strands $\sigma_{cpyg2} := \frac{M_{g2}\cdot(pp - c_{py})}{I_m}$ $\sigma_{cpyg2} = -5.718\,MPa$

top fibre $\sigma_{cyg2} := \frac{M_{g2}}{W_{my}}$ $\sigma_{cyg2} = -6.892\,MPa$

Alajänteet:

Change of the stress in the bottom strands $\Delta\sigma_{pg2} := \sigma_{cpg2}\cdot\frac{E_p}{E_{cm}}$ $\Delta\sigma_{pg2} = 30.712\,MPa$

Top strands:

Change of the stress in the top strandsJänneteräksen jännityk $\Delta\sigma_{pyg2} := \sigma_{cpyg2}\cdot\frac{E_p}{E_{cm}}$ $\Delta\sigma_{pyg2} = -31.66\,MPa$

Strains

bottom fibre $\varepsilon_{cag2} := \frac{\sigma_{cag2}}{E_{cm}}$ $\varepsilon_{cag2} = 0.191\,‰$

top fibre $\varepsilon_{cyg2} := \frac{\sigma_{cyg2}}{E_{cm}}$ $\varepsilon_{cyg2} = -0.196\,‰$

curvature due to the imposed dead load $\psi_{g2} := \frac{\varepsilon_{cag2} - \varepsilon_{cyg2}}{h}$ $\psi_{g2} = 0.666\,\frac{‰}{m}$

Curvature cab be obtained also from the equation $\psi_{g2} := \frac{M_{g2}}{E_{cm}\cdot I_m}$ $\psi_{g2} = 0.666\,\frac{‰}{m}$

Impsed dead load is uniformly distributed => Mg2-are is prabolic

Deflection coefficient $\delta_{ag2} := \frac{5}{48}$

Deflection due to the imosed dead load $a_{g2} := \delta_{ag2} \cdot \psi_{g2} \cdot L^2$ $a_{g2} = 6.941\,mm$

Deflection can be obtained also form the equation $a_{g2} := \delta_{ag2} \cdot \frac{M_{g2}}{E_{cm} \cdot I_m} \cdot L^2$ $a_{g2} = 6.941\,mm$

(downward deflection)

3.5. Imposed live load

Imposed live load $q := 10 \cdot \frac{kN}{m}$

combination factor for the frequent load combin, $\psi_1 := 0.7$

combination factor for the quasi-permanent combin. $\psi_2 := 0.6$

$M_q := \frac{q \cdot L^2}{8}$ $M_q = 125\,kNm$

Stresses due to the imposed live load

bottom fibre $\sigma_{caq} := \frac{M_q}{W_{ma}}$ $\sigma_{caq} = 7.467\,MPa$

at the loaction of the bottom strands $\sigma_{cpq} := \frac{M_q \cdot (pp - c_p)}{I_m}$ $\sigma_{cpq} = 6.164\,MPa$

at the loacation of the top strands $\sigma_{cpyq} := \frac{M_q \cdot (pp - c_{py})}{I_m}$ $\sigma_{cpyq} = -6.354\,MPa$

top fibre $\sigma_{cyq} := \frac{M_q}{W_{my}}$ $\sigma_{cyq} = -7.658\,MPa$

Bottom strands:

Change of the stress in the bottom strands $\Delta\sigma_{pq} := \sigma_{cpq} \cdot \frac{E_p}{E_{cm}}$ $\Delta\sigma_{pq} = 34.125\,MPa$

Top strands:

Change of the stress in the top strands $\Delta\sigma_{pyq} := \sigma_{cpyq} \cdot \frac{E_p}{E_{cm}}$ $\Delta\sigma_{pyq} = -35.177\,MPa$

Strains:

bottom fibre $\varepsilon_{caq} := \frac{\sigma_{caq}}{E_{cm}}$ $\varepsilon_{caq} = 0.212\,\text{‰}$

top fibre $\varepsilon_{cyq} := \frac{\sigma_{cyq}}{E_{cm}}$ $\varepsilon_{cyq} = -0.217\,\text{‰}$

curvature $\psi_q := \dfrac{\varepsilon_{caq} - \varepsilon_{cyq}}{h}$ $\psi_q = 0.74 \dfrac{‰}{m}$

curvature also from the equation $\psi_q := \dfrac{M_q}{E_{cm} \cdot I_m}$ $\psi_q = 0.74 \dfrac{‰}{m}$

Imposed live load is unifiormly distributed => Mg2-are is parabolic

Deflection coefficient $\delta_{aq} := \dfrac{5}{48}$

Deflection due to the live load $a_q := \delta_{aq} \cdot \psi_q \cdot L^2$ $a_q = 7.713\,mm$

deflection also from the equation $a_q := \delta_{aq} \cdot \dfrac{M_q}{E_{cm} \cdot I_m} \cdot L^2$ $a_q = 7.713\,mm$

(downward deflection)

3.6 Total stresses and strains at the final stage after the losses

3.6.1 Quasi-permanent load combination

Concrete stresses under the long-term (quasi-permanent) load combination

bottom fibre $\sigma_{ca} := \sigma_{cap} + \sigma_{cag} + \Delta\sigma_{cap} + \sigma_{cag2} + \psi_2 \cdot \sigma_{caq}$ $\quad \sigma_{ca} = 0.146\,\text{MPa}$

$< \; f_{ctm} = 3.513\,\text{MPa}$

at the location of the bottom strands $\quad \sigma_{cp} := \sigma_{cpp} + \sigma_{cpg} + \Delta\sigma_{cpp} + \sigma_{cpg2} + \psi_2 \cdot \sigma_{cpq}$ $\quad \sigma_{cp} = -0.873\,\text{MPa}$

at the location of the top strands $\quad \sigma_{cpy} := \sigma_{cppy} + \sigma_{cpyg} + \Delta\sigma_{cppy} + \sigma_{cpyg2} + \psi_2 \cdot \sigma_{cpyq}$

$\sigma_{cpy} = -10.653\,\text{MPa}$

top fibre $\quad \sigma_{cy} := \sigma_{cyp} + \sigma_{cyg} + \Delta\sigma_{cyp} + \sigma_{cyg2} + \psi_2 \cdot \sigma_{cyq}$ $\quad \sigma_{cy} = -11.672\,\text{MPa}$

$< \; 0.45 \cdot f_{ck} = 18\,\text{MPa}$

Concrete strains

bottom fibre $\quad \varepsilon_{ca} := \dfrac{\sigma_{cap} + \sigma_{cag}}{E_{cmi}} + \dfrac{\Delta\sigma_{cap} + \sigma_{cag2} + \psi_2 \cdot \sigma_{caq}}{E_{cm}}$ $\quad \varepsilon_{ca} = -0.034\,‰$

at the location of the bottom strands $\quad \varepsilon_{cp} := \dfrac{\sigma_{cpp} + \sigma_{cpg}}{E_{cmi}} + \dfrac{\Delta\sigma_{cpp} + \sigma_{cpg2} + \psi_2 \cdot \sigma_{cpq}}{E_{cm}}$

$\varepsilon_{cp} = -0.059\,‰$

top fibre $\quad \varepsilon_{cy} := \dfrac{\sigma_{cyp} + \sigma_{cyg}}{E_{cmi}} + \dfrac{\Delta\sigma_{cyp} + \sigma_{cyg2} + \psi_2 \cdot \sigma_{cyq}}{E_{cm}}$ $\quad \varepsilon_{cy} = -0.33\,‰$

Stresses of the strands

Bottom strands $\quad \sigma_p := \sigma_{p3} + \Delta\sigma_{pg2} + \psi_2 \cdot \Delta\sigma_{pq}$ $\quad \sigma_p = 1035.5\,\text{MPa}$

$< \; \min(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}) = 1440\,\text{MPa}$

Top strands $\quad \sigma_{py} := \sigma_{py3} + \Delta\sigma_{pyg2} + \psi_2 \cdot \Delta\sigma_{pyq}$ $\quad \sigma_{py} = 875.651\,\text{MPa}$

Deflection

$a_{longterm} := a_p + a_g + \Delta a_p + a_{g2} + \psi_2 \cdot a_q$ $\quad a_{longterm} = 3.395\,\text{mm}$

3.6.2 Frequent load combination

Concrete stresses under the frequent load combination

bottom fibre $\sigma_{ca} := \sigma_{cap} + \sigma_{cag} + \Delta\sigma_{cap} + \sigma_{cag2} + \psi_1 \cdot \sigma_{caq}$ $\sigma_{ca} = 0.893\,MPa$

$< \; f_{ctm} = 3.513\,MPa$

at the location of the bottom strands $\sigma_{cp} := \sigma_{cpp} + \sigma_{cpg} + \Delta\sigma_{cpp} + \sigma_{cpg2} + \psi_1 \cdot \sigma_{cpq}$ $\sigma_{cp} = -0.256\,MPa$

at the location of the top strands $\sigma_{cpy} := \sigma_{cppy} + \sigma_{cpyg} + \Delta\sigma_{cppy} + \sigma_{cpyg2} + \psi_1 \cdot \sigma_{cpyq}$

$\sigma_{cpy} = -11.288\,MPa$

top fibre $\sigma_{cy} := \sigma_{cyp} + \sigma_{cyg} + \Delta\sigma_{cyp} + \sigma_{cyg2} + \psi_1 \cdot \sigma_{cyq}$ $\sigma_{cy} = -12.438\,MPa$

$< \; 0.45 \cdot f_{ck} = 18\,MPa$

Concrete strains

bottom fibre $$\varepsilon_{ca} := \frac{\sigma_{cap} + \sigma_{cag}}{E_{cmi}} + \frac{\Delta\sigma_{cap} + \sigma_{cag2} + \psi_1 \cdot \sigma_{caq}}{E_{cm}}$$ $\varepsilon_{ca} = -0.034\,‰$

at the location of the bottom strands $$\varepsilon_{cp} := \frac{\sigma_{cpp} + \sigma_{cpg}}{E_{cmi}} + \frac{\Delta\sigma_{cpp} + \sigma_{cpg2} + \psi_1 \cdot \sigma_{cpq}}{E_{cm}}$$ $\varepsilon_{cp} = -0.042\,‰$

top fibre $$\varepsilon_{cy} := \frac{\sigma_{cyp} + \sigma_{cyg}}{E_{cmi}} + \frac{\Delta\sigma_{cyp} + \sigma_{cyg2} + \psi_1 \cdot \sigma_{cyq}}{E_{cm}}$$ $\varepsilon_{cy} = -0.352\,‰$

Stresses in the strands

Bottom strands $\sigma_p := \sigma_{p3} + \Delta\sigma_{pg2} + \psi_1 \cdot \Delta\sigma_{pq}$ $\sigma_p = 1038.9\,MPa$

$< \; \min\left(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}\right) = 1440\,MPa$

Concete strain at the location of the bottom strands $\varepsilon_{cp} = -0.042\,‰$

A major part of the elastic shortening is reversed

For the elastic shortening is left only $\Delta\sigma_{pe\infty} := \varepsilon_{cp} \cdot E_p$ $\Delta\sigma_{pe\infty} = -8.105\,MPa$

Long-term unreversible prestress losses the elastic shortening excluded:

before the transfer $\Delta\sigma_{p1} = -33\,MPa$

The long-term losses after the transfer $\Delta\sigma_p = -270\,MPa$

The sum of the unreversible losses $\Delta\sigma_{p\infty} := \Delta\sigma_{p1} + \Delta\sigma_p$ $\Delta\sigma_{p\infty} = -303\,MPa$

about 22 % of initial prestress

The efffective prerstress after the long-term losses

$\sigma_{p\infty} := \sigma_{pmax} + \Delta\sigma_{p\infty}$ $\sigma_{p\infty} = 1047\,MPa$

$\sigma_{p\infty} + \Delta\sigma_{pe\infty} = 1038.9\,\text{MPa}$ $\quad = \quad$ $\sigma_p = 1038.9\,\text{MPa}$

Top strands $\sigma_{py} := \sigma_{py3} + \Delta\sigma_{pyg2} + \psi_1 \cdot \Delta\sigma_{pyq}$ $\qquad \sigma_{py} = 872.134\,\text{MPa}$

Deflection

$a := a_p + a_g + \Delta a_p + a_{g2} + \psi_1 \cdot a_q$ $\qquad a = 4.166\,\text{mm}$

$< \quad \dfrac{L}{250} = 40\,\text{mm}$

3.7. Approximative estimate of stresses at the final stage
Simpified calculation for preliminary purposes

An approximative calculation of the stresses at the final stage by calculating straight from the estimated prestress force after the losses by using the section values corresponding the nominal strength

Suppose the long-term losses due to shrinkage, creep and relaxation about 20 % of stress σpma loss-% is taken the same also for the top strands (in reality the losses may be different for top strands than the bottom strands)

The value does not include the effect of the elastic shortening because it is reversible.

$r := 1 - 0.2 \qquad r = 0.8$

Prestress at the final stage after the long-term losses at the time t=∞:

bottom strands $\sigma_{p\infty} := r \cdot \sigma_{pmax}$ $\qquad \sigma_{p\infty} = 1080\,\mathrm{MPa}$

top strands $\sigma_{py\infty} := r \cdot \sigma_{pymax}$ $\qquad \sigma_{py\infty} = 960\,\mathrm{MPa}$

Effective prestress force at the final stage $P_\infty := A_p \cdot \sigma_{p\infty} + A_{py} \cdot \sigma_{py\infty}$ $\qquad P_\infty = 982.1\,\mathrm{kN}$

Location of the prestress force resultant $c_{res} := \dfrac{A_p \cdot \sigma_{p\infty} \cdot c_p + A_{py} \cdot \sigma_{py\infty} \cdot c_{py}}{P_\infty}$ $\qquad c_{res} = 137.273\,\mathrm{mm}$

Nominal strength of concrete $f_{ck} = 40\,\mathrm{MPa}$ and elastic modulus $E_{cm} = 35220.5\,\mathrm{MPa}$

Ratio of the elastic modulies $n_e := \dfrac{E_p}{E_{cm}}$ $\qquad n_e = 5.537$

The elastic modulus of concrete is changed, so the section values are calculated again

Section values the top strands included

$A_m := b \cdot h + (n_e - 1) \cdot (A_p + A_{py})$ $\qquad A_m = 0.167\,\mathrm{m}^2$

$S_m := b \cdot h \cdot \dfrac{h}{2} + (n_e - 1) \cdot (A_p \cdot c_p + A_{py} \cdot c_{py})$ $\qquad S_m = 0.048\,\mathrm{m}^3$

centroid from the bottom $pp := \dfrac{S_m}{A_m}$ $\qquad pp = 286.354\,\mathrm{mm}$

Second moment of area $I_m := \dfrac{b \cdot h^3}{12} + b \cdot h \cdot \left(\dfrac{h}{2} - pp\right)^2 + (n_e - 1) \cdot \left[A_p \cdot (c_p - pp)^2 + A_{py} \cdot (c_{py} - pp)^2\right]$

$I_m = 0.0048\,\mathrm{m}^4$

Section modulus about the bottom side $W_{ma} := \dfrac{I_m}{pp}$ $\qquad W_{ma} = 0.017\,\mathrm{m}^3$

Section modulus about the top side $W_{my} := \dfrac{I_m}{pp - h}$ $\qquad W_{my} = -0.016\,\mathrm{m}^3$

A reinforced cross-section (transformed cross-section) is loaded by the compressive counterforce $-P_\infty$ of the prestressing force at the location of the prestress force resultant

$-P_\infty = -982.08\,kN$

$-P_\infty$ is moved to the centroid of this transformed cross-sectio and the movement is compensated the negative moment

$M_{p\infty} := -P_\infty \cdot (pp - c_{res})$ $\qquad M_{p\infty} = -146.409\,kNm$

Eccentricity of the prestress force resultant $\quad e_p := pp - c_{res}$ $\qquad e_p = 149.081\,mm$

Stresses due to the prestressing:

bottom fibre $\quad \sigma_{cap} := \frac{-P_\infty}{A_m} + \frac{M_{p\infty}}{W_{ma}}$ $\qquad \sigma_{cap} = -14.641\,MPa$

compression

at the location of the bottom strands $\quad \sigma_{cpp} := \frac{-P_\infty}{A_m} + \frac{M_{p\infty} \cdot (pp - c_p)}{I_m}$ $\qquad \sigma_{cpp} = -13.113\,MPa$

at the loacation of the top strands $\quad \sigma_{cppy} := \frac{-P_\infty}{A_m} + \frac{M_{p\infty} \cdot (pp - c_{py})}{I_m}$ $\qquad \sigma_{cppy} = 1.548\,MPa$

top fibre $\quad \sigma_{cyp} := \frac{-P_\infty}{A_m} + \frac{M_{p\infty}}{W_{my}}$ $\qquad \sigma_{cyp} = 3.075\,MPa$

tension

Stress resultant of concrete $\quad N_c := \frac{(\sigma_{cap} + \sigma_{cyp})}{2} \cdot b \cdot h - \sigma_{cpp} \cdot A_p - \sigma_{cppy} \cdot A_{py}$

$N_c = -929.657\,kN$

Bottom strands:

Stress change due to prestress $\quad \Delta\sigma_{pe} := \sigma_{cpp} \cdot \frac{E_p}{E_{cm}}$ $\qquad \Delta\sigma_{pe} = -72.603\,MPa$

Stress in the bottom strands $\quad \sigma_{p4} := \sigma_{p\infty} + \Delta\sigma_{pe}$ $\qquad \sigma_{p4} = 1007.4\,MPa$

Top strands

Stress change due to prestress $\quad \Delta\sigma_{pye} := \sigma_{cppy} \cdot \frac{E_p}{E_{cm}}$ $\qquad \Delta\sigma_{pye} = 8.569\,MPa$

Stress in the top strands $\quad \sigma_{py4} := \sigma_{py\infty} + \Delta\sigma_{pye}$ $\qquad \sigma_{py4} = 968.6\,MPa$

Resultant force of the strands $\quad P_4 := \sigma_{p4} \cdot A_p + \sigma_{py4} \cdot A_{py}$ $\qquad P_4 = 929.657\,kN$

Equilibrium: $\quad P_4 + N_c = 0\,kN$

Strains

bottom fibre $\varepsilon_{cap} := \frac{\sigma_{cap}}{E_{cm}}$ $\varepsilon_{cap} = -0.416\,‰$

top $\varepsilon_{cyp} := \frac{\sigma_{cyp}}{E_{cm}}$ $\varepsilon_{cyp} = 0.087\,‰$

curvature $\psi_p := \frac{\varepsilon_{cap} - \varepsilon_{cyp}}{h}$ $\psi_p = -0.867\,\frac{‰}{m}$

Curvature can be obtained also from the equation $\psi_p := \frac{M_{p\infty}}{E_{cm} \cdot I_m}$ $\psi_p = -0.867\,\frac{‰}{m}$

Prestress force and it's eccentricity are equal almost the whole length of the beam =>
Mp and the curvature are equal almost the whole lenght of the beam

Deflection coefficien $\delta_{ap} := \frac{1}{8}$

Deflection due to prestress after the losses $a_{p\infty} := \delta_{ap} \cdot \psi_p \cdot L^2$ $a_{p\infty} = -10.84\,mm$

Deflection can be obtained also from the equation $a_{p\infty} := \delta_{ap} \cdot \frac{M_{p\infty}}{E_{cm} \cdot I_m} \cdot L^2$ $a_{p\infty} = -10.84\,mm$
(upward deflection)

Prestress losses reduce the upward deflection by amount $a_{p\infty} - a_p = 3.439\,mm$

The difference compared to the previous calculations is due to the fact that in this approximative calculation the cross-section in which the concrete is reached the nominal strength is loaded by tl prestress force after the long-term losses P∞= Po-ΔP

In the previous calculations the initial prestress force (Po) is loaded the cross-section in which the concrete strength is not reached it's nominal value and when calculating the losses (ΔP the cross-section with the nominal strength of concrete is used.

In the previous calculations at the transfer stage the concrete stresses due to prestressing are (page 9):

bottom fibre $\sigma_{cap1} := -17.726MPa$

top fibre $\sigma_{cyp1} := 3.715 \cdot MPa$

Due to prestress losses the concrete stresses are (page.16):

bottom fibre $\Delta\sigma_{cap1} := 3.66MPa$

tiop fibre $\Delta\sigma_{cyp1} := -0.769 \cdot MPa$

At the final stage the concrete stresses due to the effective prestressing are:

bottom fibre $\sigma_{cap\infty} := \sigma_{cap1} + \Delta\sigma_{cap1}$ $\sigma_{cap\infty} = -14.066\,MPa$

top fibre $\sigma_{cyp\infty} := \sigma_{cyp1} + \Delta\sigma_{cyp1}$ $\sigma_{cyp\infty} = 2.946\,MPa$

The diffrence to the result in the page 25 is small; results are almoust the same in spite of the approximative method