

# Shapes in Action 2018

## Sept 11th

*Intro & Symmetries*



# Program schedule for Tue Sept 11<sup>th</sup>

**15:15 Some general instructions**

**15:30 An approach to symmetries**

**16:30 Break**

**16:45 Group work on symmetries**

**17:15 Instructions for Workshop on Fri Sept 14<sup>th</sup> (Laura)  
& Homework**



# Course schedule in a nutshell

- **Sept 11<sup>th</sup> – Oct 16<sup>th</sup>** (period I) **lectures on**  
**Tue 3:15pm - 6pm U9 (U271) Otakaari 1**
- **Sept 14<sup>th</sup>**  
**Fri 1:15pm – 4pm H003 Kokoonpano, Väre**
- **Sept 21<sup>st</sup> - Oct 19<sup>th</sup>**  
**Fri 1:15pm – 4pm T6 (A136) T-building, Konemiehentie 2**

# Lecturer in Mathematics: Kirsi Peltonen

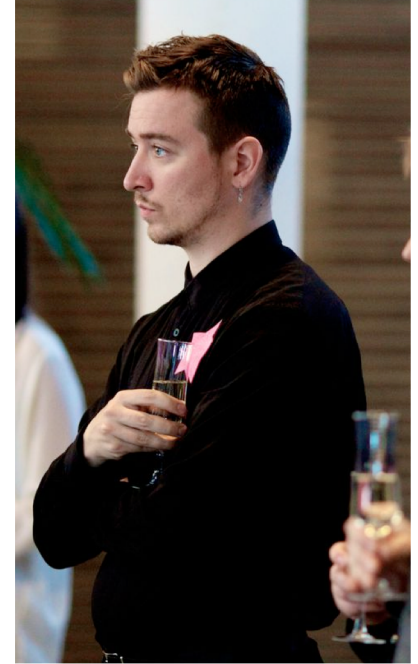
- **Responsible Teacher of the Course**
- **PhD, Docent in Math**
- **Geometric Analysis**
- **Mainstreaming Mathematics**
- **Visualizations**



*Photo: Eero Kaarlehto*

# Lecturer in Arts: Taneli Luotonieni

- **Master of Arts**
- **Doctoral student of Arts at Aalto**
- **Mathematics in Arts education**
- **Artistic research of mathematics**
- **4D, tilings, knots**



*Photo: Eero Kaarlehto*

# Lecturer in Arts: Laura Isoniemi

- **Master of Arts (Textiles)**
- **Art Pedagogy**
- **IDBM Pro studies**
- **Multi-disciplinary freelance designer**
- **Part time teacher at Aalto**
- **Tilings, Patterns, Foldings, Knots**



# Lecturer in Mathematics: Augusto Gerolin

- **Postdoctoral Researcher (Jyväskylä)**
- **Geometric Analysis**
- **Optimal transport**
- **Signs of Mathematics**



# Student groups

## 1. Lari Alakukku (SCI)

Pinja Helasuo (SC))

Taeyoung Kee (BIZ)

Lam Lai (SCI)

Tuukka Mattlar (SCI)

Joonatan Partanen (SCI)

## 2. Felix Bade (SCI)

Aleksei Nagaev (SCI)

Le Nghiem (SCI)

Esa Palosaari (SCI)

Essi Rantanen (SCI)

Anne-Liv Seim (ELEC)

## 3. Luna Ansari (BIZ)

Hà Bui (SCI)

Suha Chowdhury (ELEC)

Aapo Hanski (SCI)

Letizia Iannucci (SCI)

Anna Lebedeff (SCI)

Chao Zhou (ARTS)

## 4. Gomez Grassi (ARTS)

Angeline Jayanegara (SCI)

Khoa Nguyen (SCI)

Sami Nieminen (ELEC)

Emil Nyman (SCI)

Lauri Särkiö (SCI)

Anh Tran (SCI)

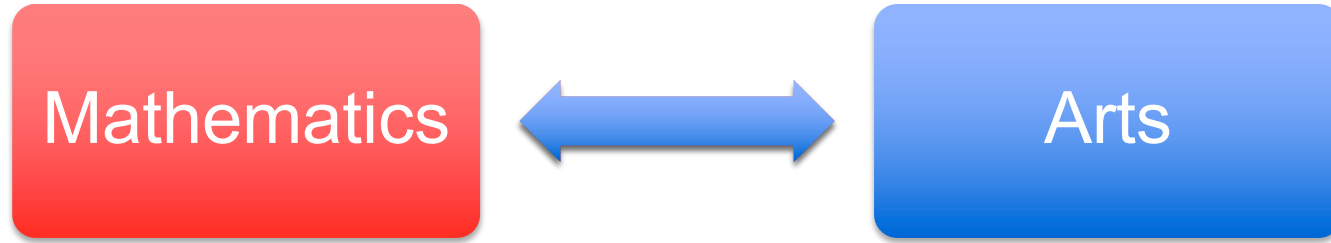
5. Robert Heiska (SCI)  
Aura Kiiskinen (SCI)  
Megan McGlynn (ARTS)  
Trang Nguyen (SCI)  
Darlington Omoifo (SCI)  
Aleksanteri Paakkinen (SCI)  
Gergely Popoluska (BIZ)
6. Olli Kesseli (SCI)  
Huy Nguyen (SCI)  
Jukka Salmijärvi (SCI)  
Daniel Stafford (SCI)  
Markus Tayar (SCI)  
Yi-Chiao Tien (ARTS)  
Pauliina Tomberg (SCI)

7. Minh Toàn Đào (SCI)  
Vinh Nguyen (SCI)  
Roope Ruusuvuori (SCI)  
Jonna Tuupainen (ARTS)  
Saara Vestola (SCI)  
Taige Wang (SCI)
8. Lauri Jokinen (SCI)  
Ida Keskimäki (SCI)  
Dat Nguyen (SCI)  
Yakill Reyes (BIZ)  
Alena Shchevyeva (SCI)  
Perttu Yli-Opas (SCI)  
Minyang Zhang (ARTS)

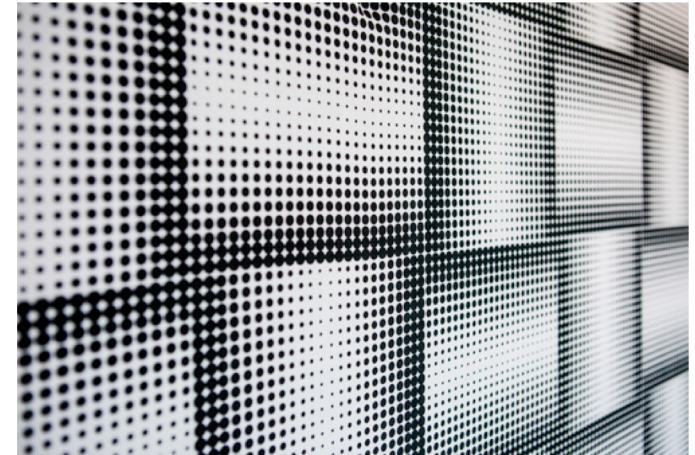
- 9. Duong Cao (ELEC)**
- Hoang Dang (SCI)**
- Elisa Heimo (BIZ)**
- Ahmed Hewidy (SCI)**
- Jeheon Kim (SCI)**
- Mathilda Smith (SCI)**



# Goals



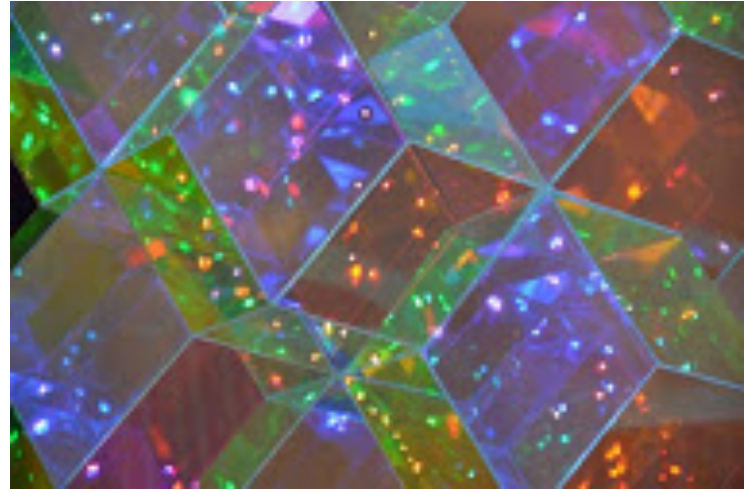
- **Interaction in both ways!**
- **Increase *understanding***
- **Useful tools (or recipes perhaps)**
- **New point of views and connections**
- **Current research perspective**
- **Critical point of view**



*Photo: Päivi Kiuru*

# Math topics include

- Tilings (symmetries, classification)
- Models in geometry (spherical, Euclidean, hyperbolic)
- Surfaces and orbifolds
- Kleinian groups
- Fractals and chaos



*Photo: Viivi Livio*

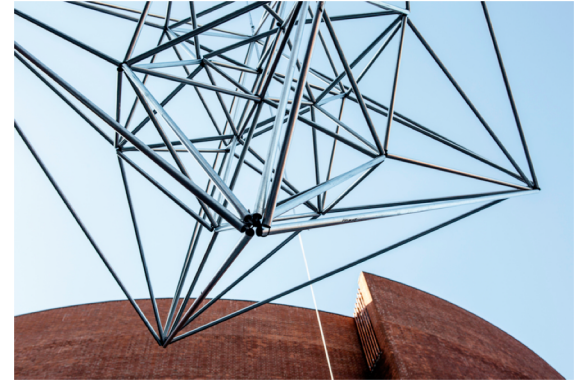
# What are you supposed to do?

**Active participation during contact meetings**

**Portfolio through MyCourses consisting:**

1. **Diary** (=reflections and summary) (1/3)
2. **Weekly exercises** (1/3)
3. **Essay** (1/3)

**Some groupwork tasks**



**Important:** *always consult Kirsi before starting essay/projects*

# Some literature

J.H. Conway, H. Burgiel, C. Goodman-Strauss: **The Symmetries of Things**

D. Mumford, C. Series, D. Wright: **Indra's Pearls**

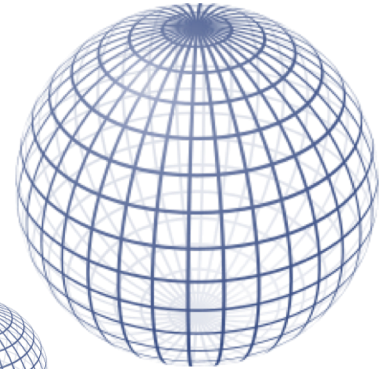
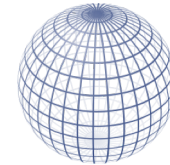
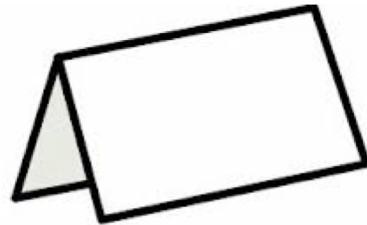
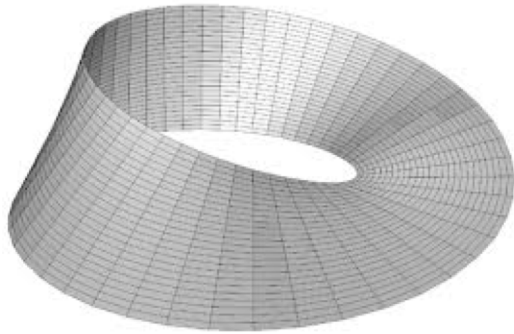
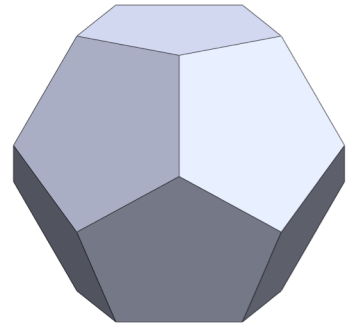
F. A. Farris: **Creating Symmetry**

J. Weeks: **Shape of Space**

# What is *geometry* ?

Γεωμετρία : geo = earth, -metron = measurement

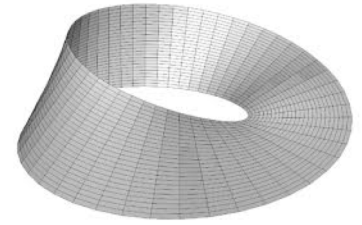
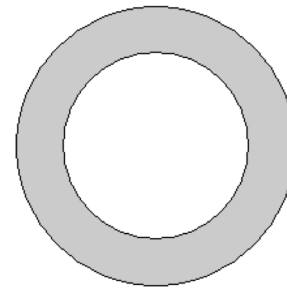
- Operations that preserve distances between points (=isometry)
- Smoothness issues



# What is *topology*?



*Photo: Henry Segerman*



- Continuous operations (in both ways)

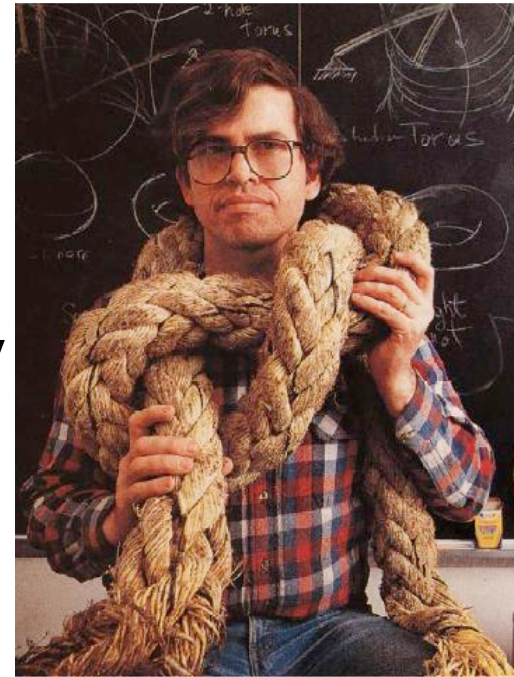




# Symmetries of planar patterns

## Goals for today:

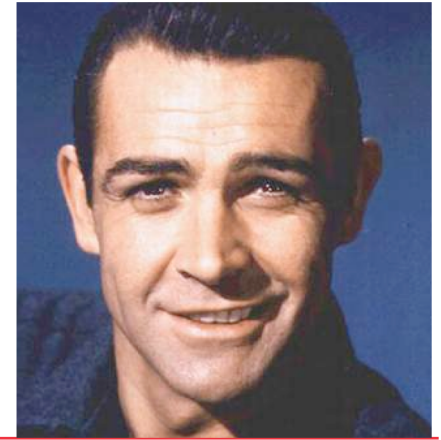
- An introduction to a systematic way to classify (=numerate) patterns
- Focus on concepts
- New insight to an old problem and its earlier solutions



***Signature/Orbifold notation due to  
B. Thurston and J.H. Conway (90)***

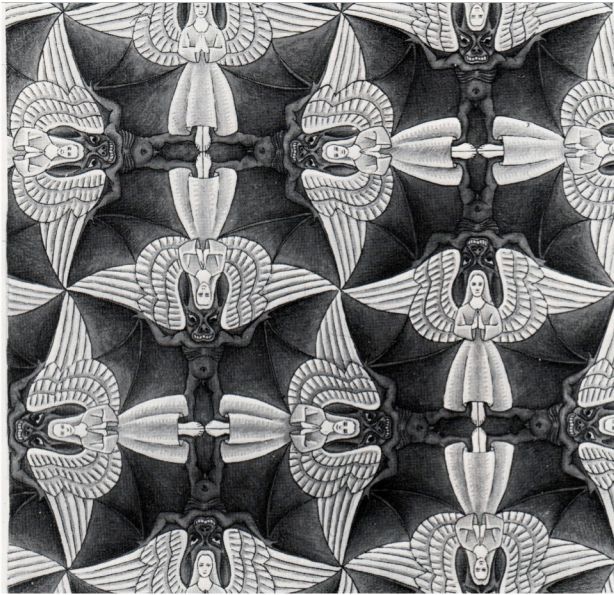
# What is symmetry?

What is beauty?





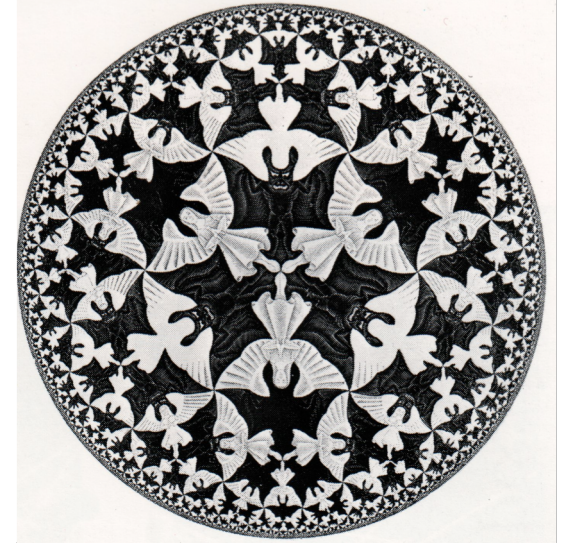
# Euclidean (=flat), spherical and hyperbolic models of 2D geometry



$K = 0$  (17 types)

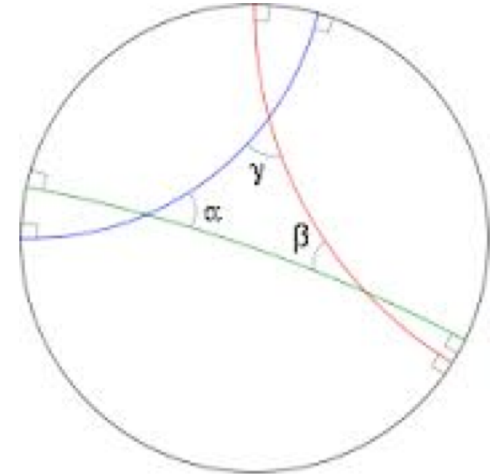
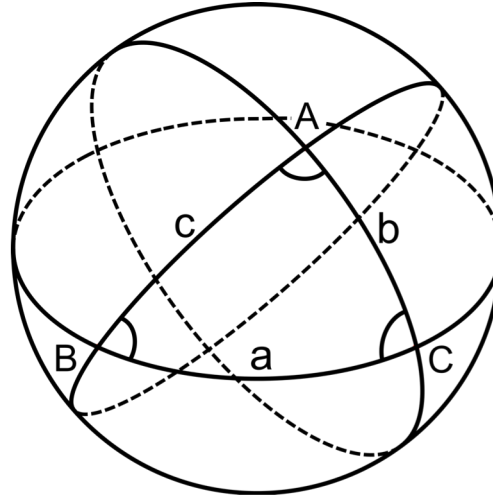
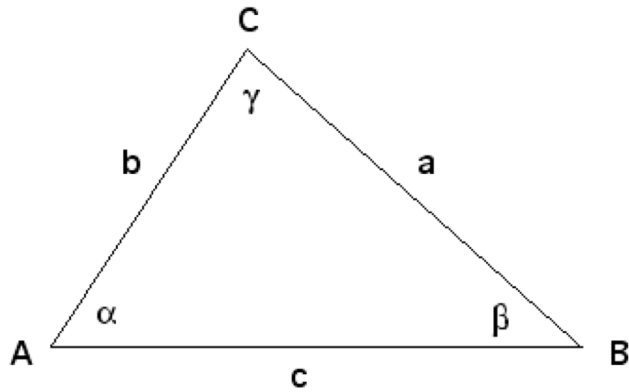


$K > 0$  (14)



$K < 0$  ( $\infty$ )

# Models in terms of angles and triangles



Sum of angles of a triangle =  $180^\circ$

$>180^\circ$

$<180^\circ$

# Some history of Euclidean tilings (=Crystallographic groups )

## 2D Wallpaper groups (17)

- 1890 E.S Federov
- 1924 G. Pólya
- 1992 B. Thurston, J.H. Conway

## 3D Space groups (219/230)

- 1890 W. Barlow, E.S Federov, A. Schönflies
- 1911 L. Bieberbach
- 1948 H. Zassenhaus (algorithmic approach)
- 1983 T.Hahn (ed.) International tables of crystallography
- 2001 B. Thurston, J.H. Conway

# Higher dimensions

**D. Hilbert's 18th Problem (1900 ICM Paris)** *“Is there in  $n$ -dimensional Euclidean space [. . .] only a finite number of essentially different kinds of groups of motions with a [compact] fundamental region”*

- 1910 L. Bieberbach : Only finitely many in all dimensions
- 1948 H. Zassenhaus : Algorithmic approach to all dimensions
- 1980 R. Schwarzenberger

## **4D (4783)**

- 1978 H. Brown, R. Bülow, J. Neubüser (via computers)

## **5D (222 018)**

- 2000 W. Plesken, T. Schultz

## **6D (28 927 922)**

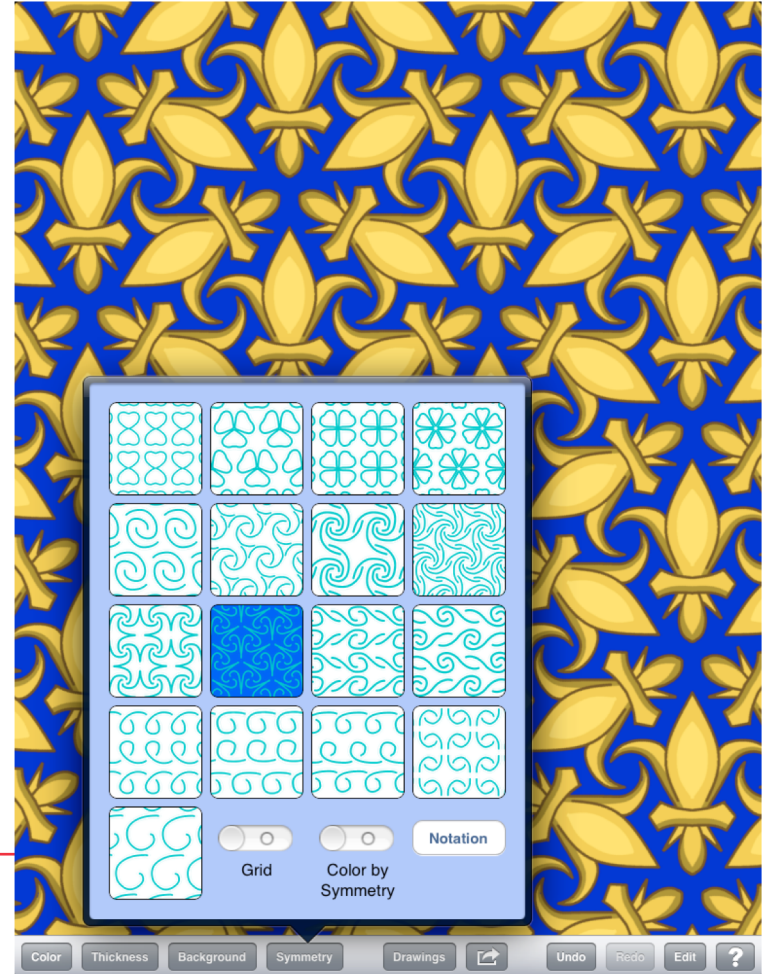
- 2000 W. Plesken, T. Schultz

# Visualizations

**2D** Jeff Weeks: KaleidoPaint application  
for iOS & Android

<http://geometrygames.org/>

**3D** <http://spacegroup.info/>





**Goal: fix *meaning* and  
*notation* for the symmetry**



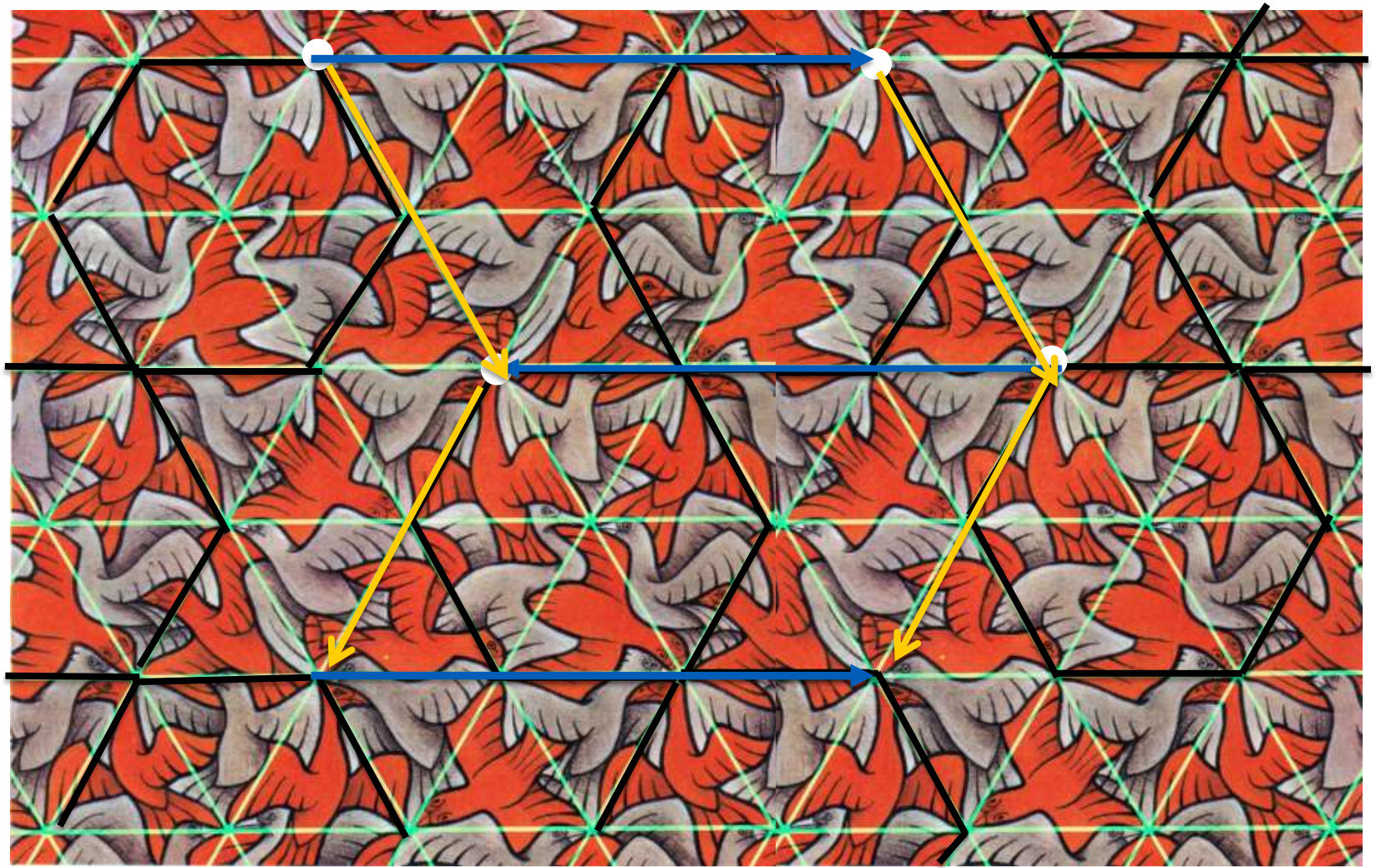
Find a pattern that is repeated  
(up to reflection)











# Would like to know

- **What are the essentially different ways to subdivide plane into patterns that are repeated ?**
- **What is a ‘pattern’ and a ‘repetition’?**
- **How to distinguish different subdivisions?**
- **How to get them all?**
- **How to describe the topological type of ‘patterns’?**
- **Does one topological type correspond only one subdivision?**

# First we need some definitions.....

**Symmetry** (group) := **actions** that preserve the given pattern in the given **geometry**.

Consider today only **patterns in the plane**.

Given **geometry** = standard Euclidean plane.

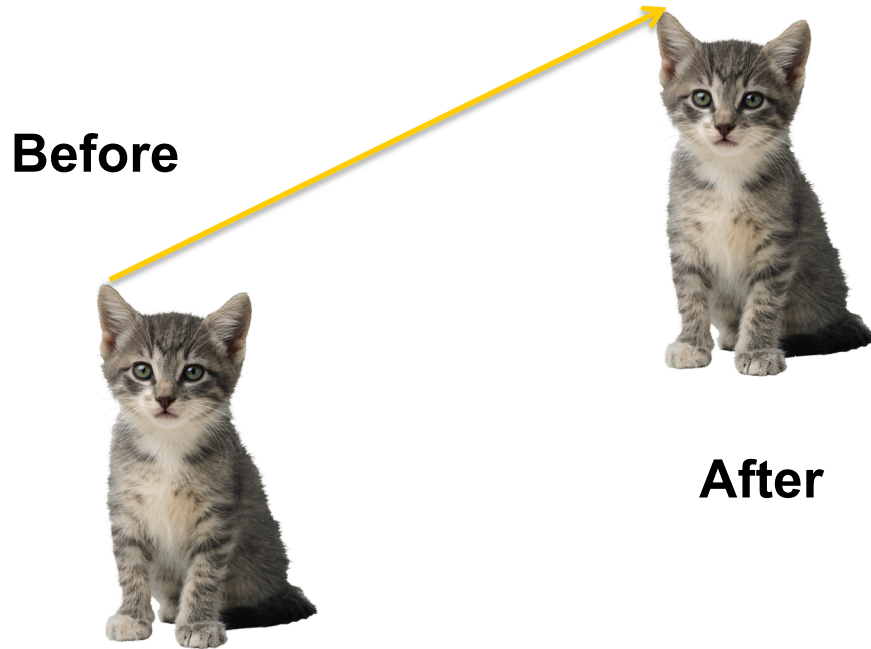
**Actions** := rigid motions (= *isometries*) of the plane:

- Translations
- Rotations with respect to a point
- Reflections with respect to a line

And all possible combinations of those.

(Proof: second year linear algebra)

# A (pure) translation



**Note: only 'before' and 'after' situations are taken into account. Not the actual way of performing the motion.**

# A (pure) rotation of 90 degrees (to the positive direction) wrt the top of the right ear



**After**



**Before**

# A (pure) reflection wrt a line

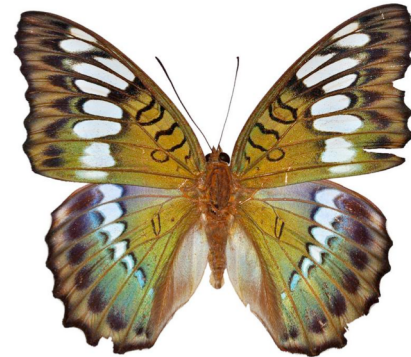
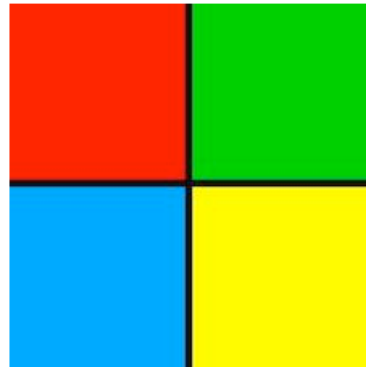
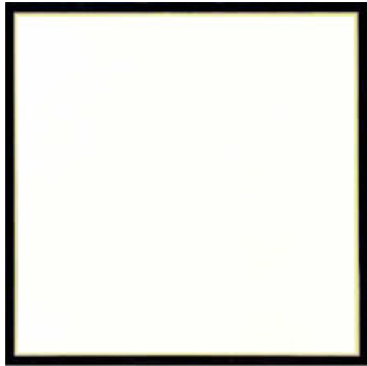
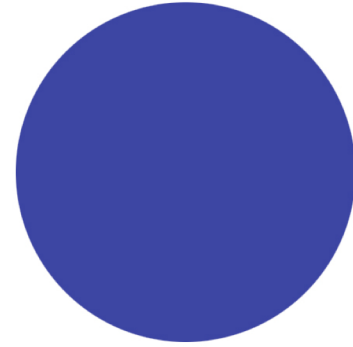
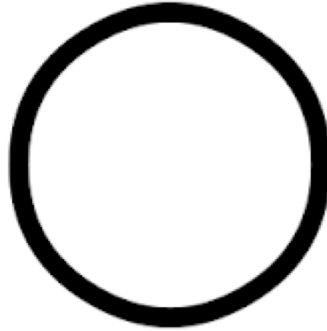
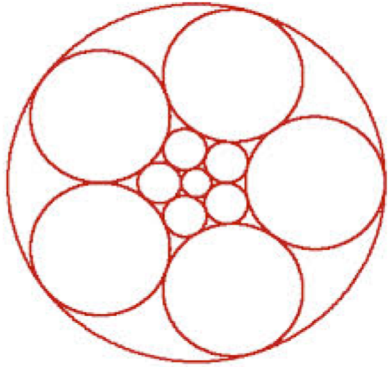


**Before (or after)**



**After (or before)**

# Which pattern is most (least) symmetric?





# What about infinite (repeating) patterns?





# Group work

1. What symmetries can you find in the provided pictures?
2. Are there reflection lines or rotation points ?
3. Can you find the smallest possible piece that gives you the whole tessellation when all symmetries are taken into account ?

## 4. Stamping Videos:

<https://www.youtube.com/watch?v=Gpb6nxvabkM>

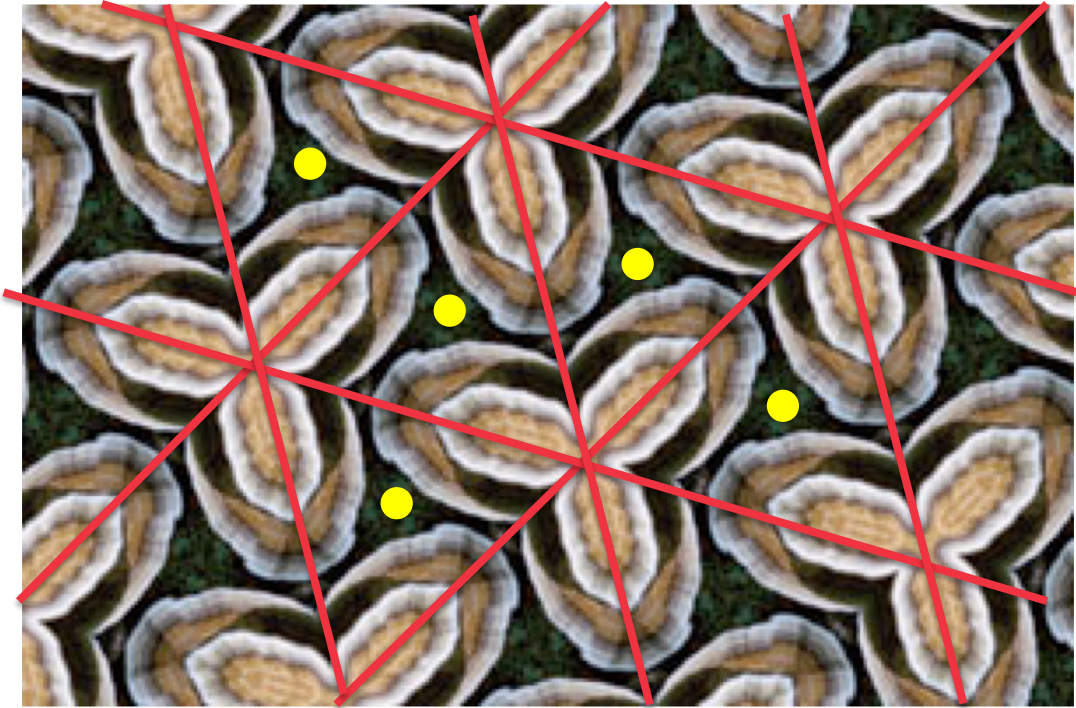
<https://www.youtube.com/watch?v=E69KrNrj4KQ> (2222)

[https://www.youtube.com/watch?v=GlzwVB3\\_6Y8](https://www.youtube.com/watch?v=GlzwVB3_6Y8) (4\*2)

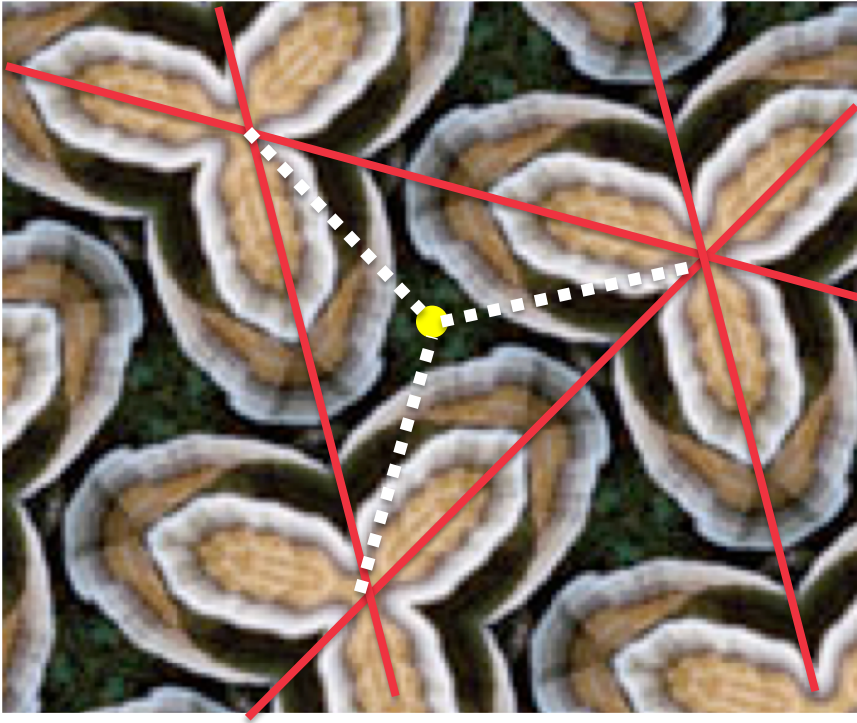
# Find reflection lines first (if any)



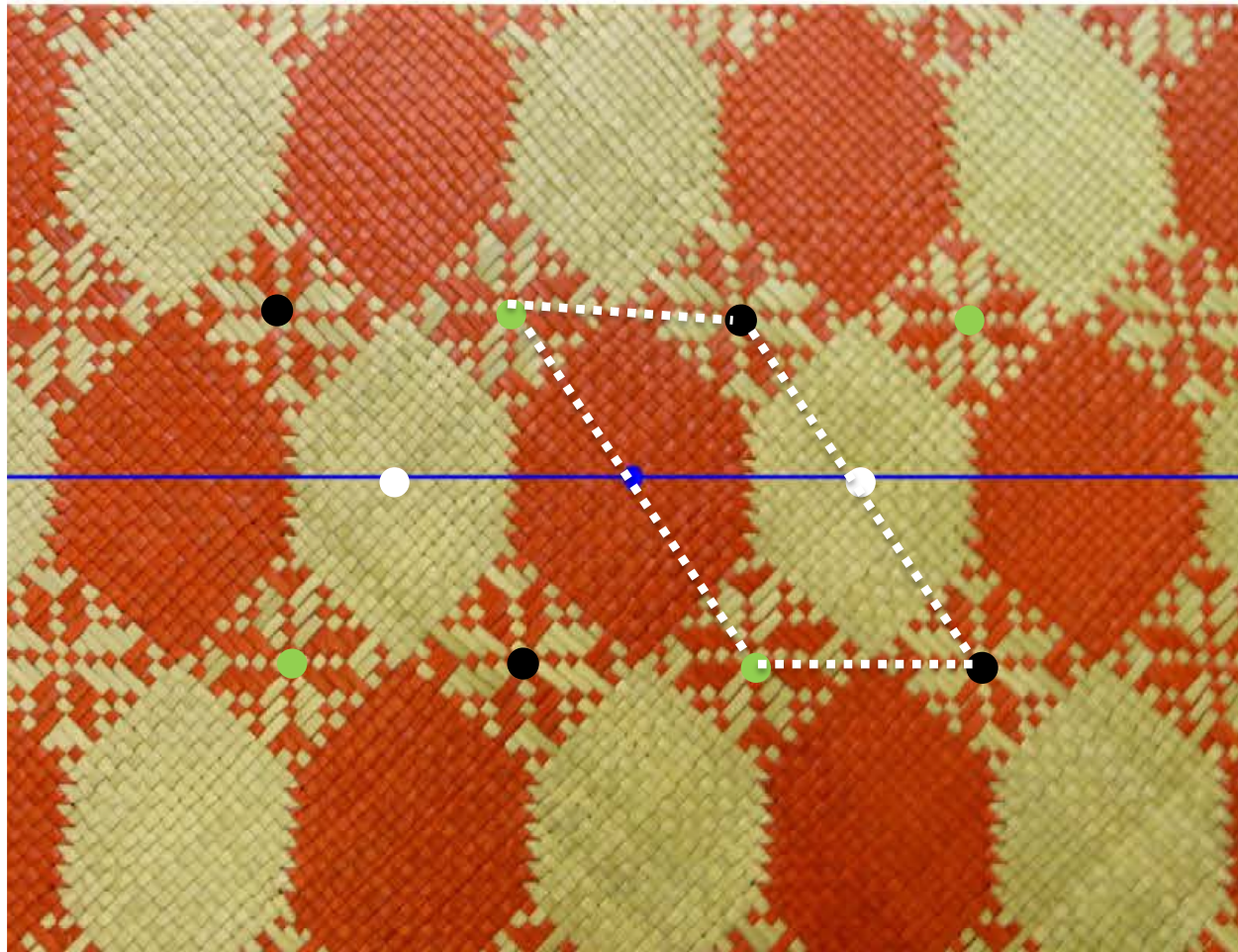
# Then rotation points (if any)



# Further subdivisions into identical pieces







**No reflection lines !  
Four different rotation  
points of order 2**

**2222**



\*\*

2\*22

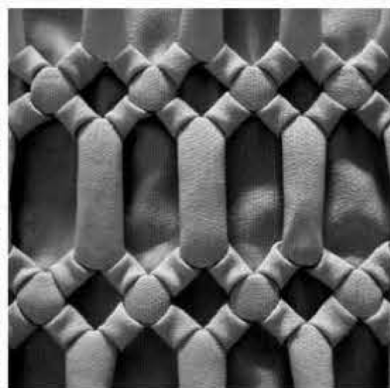
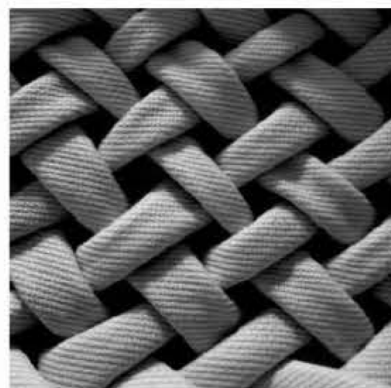
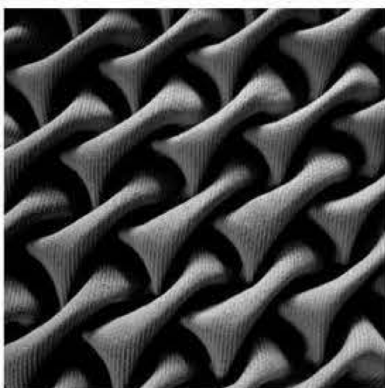
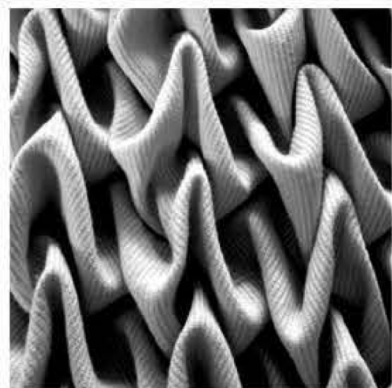
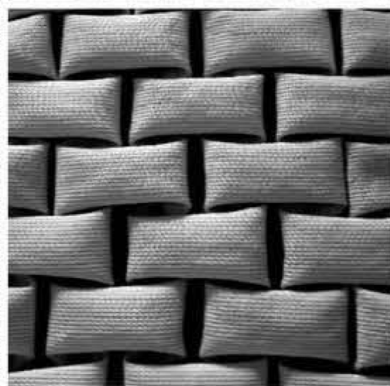
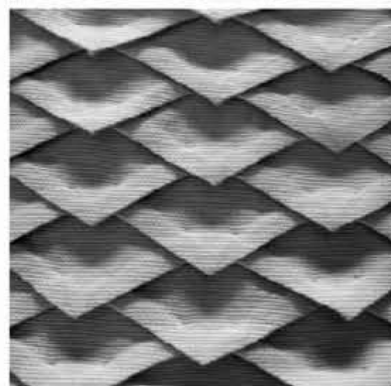
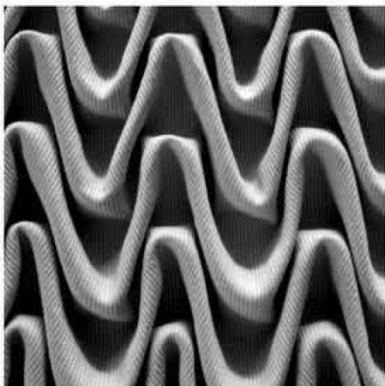
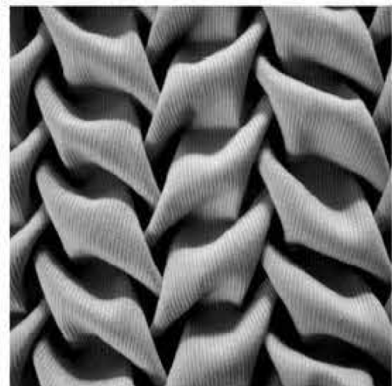
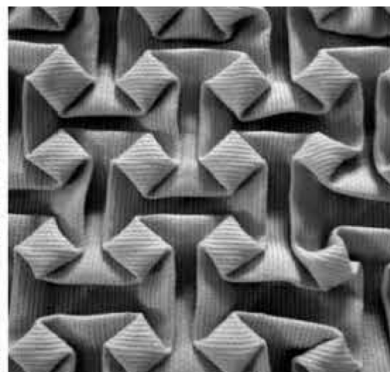
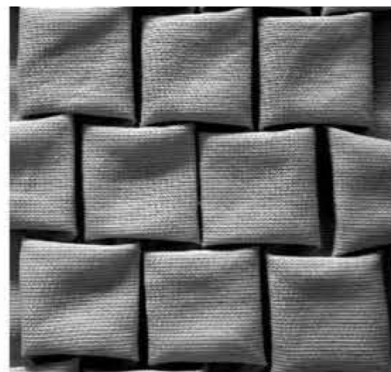
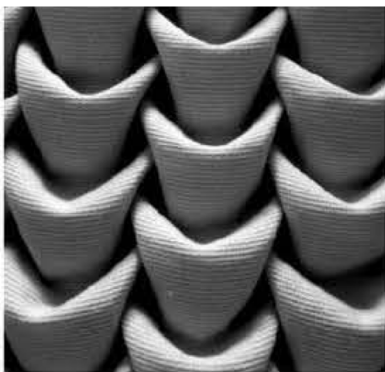
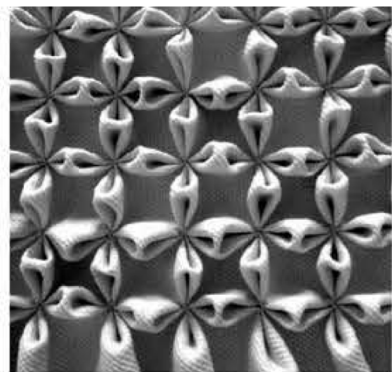
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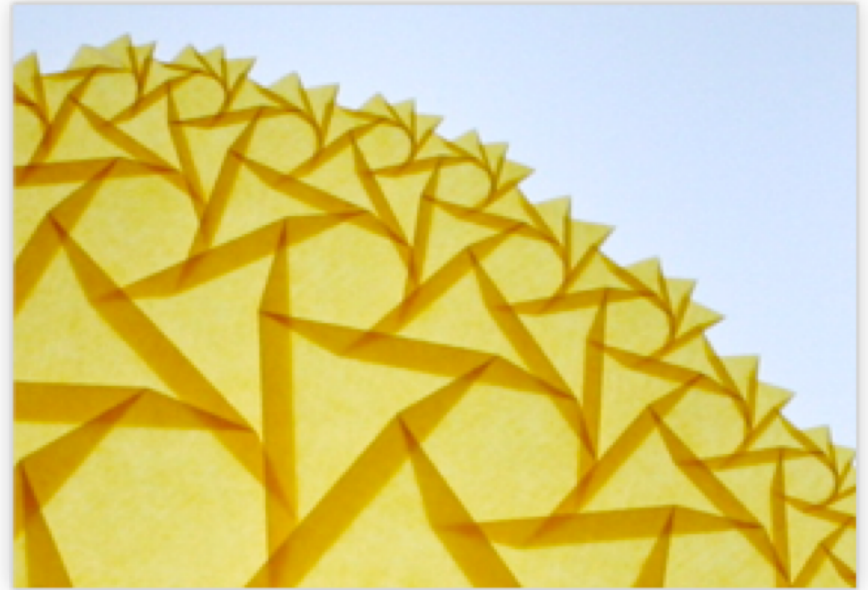
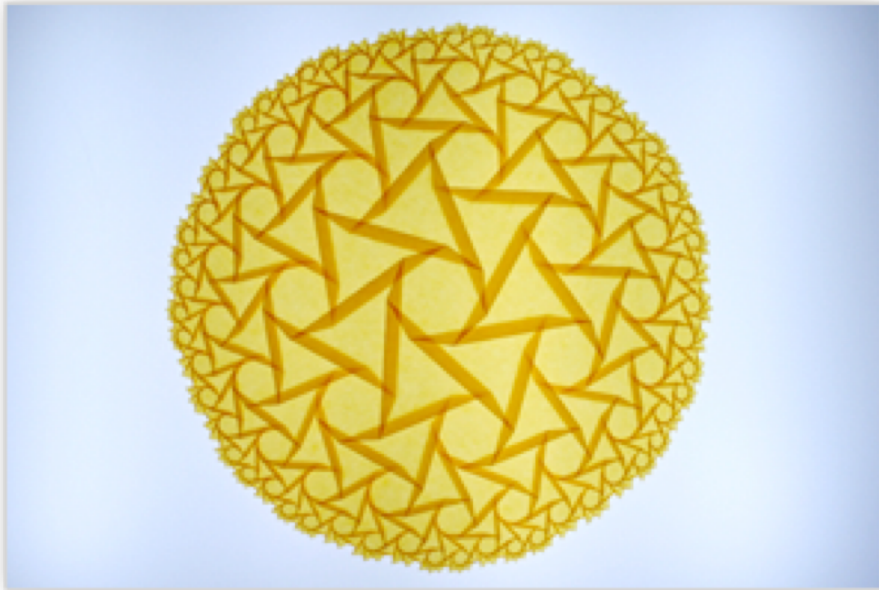


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**A!**









# Homework

**1. Reflections through MyCourses by Sept 14<sup>th</sup>**

**2. By Sept 18<sup>th</sup> through MyCourses**

**Analyse at least 10 images. Are there**

- reflection lines ? How many different ?**
- rotation points ? How many different ?**
- mirror images without reflection lines ?**