ELEC-E8101 Digital and Optimal Control (5 cr), autumn 2022

Lectures

Fridays at 14.15-16.00, lecture hall AS2
Lecturers: 1.Kai Zenger, TuAS-house (Maarintie 8), room 3574, kai.zenger(at)aalto.fi; 2. Ville Kyrki, TuAS house, room 2570, ville.kyrki(at)aalto.fi.

Exercise hours
Wednesdays at 14.15-16.00 lecture hall TU 1
”Laskutupa” (help for homeworks etc); not confirmed yet.

Assistants: Matti Pekkanen, Karol Arndt, Amin Modabberian, Gökhan Alcan. Email: Firstname.Lastname@aalto.fi
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Contents…

- **Introduction: discrete time vs. continuous time control problem**
- **Discretization (state-space, transfer function), ZOH**
- **Properties of a discrete-time system (pulse transfer function, pulse response, weighting function, poles, zeros, mapping of poles from continuous to discrete time systems)**
- **Stability (state stability, BIBO-stability, Jury stability test, frequency response, Bode, Nyquist, gain and phase margins)**
- **Controllability, reachability, observability**
- **Pole placement by state feedback control, regulation and servo problems, static gain**
- **State observer, pole placement of the observer, combining of an observer and state feedback controller**

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…Contents

- **Discrete approximations of continuous-time controllers (Euler, Tustin etc.)**
- **Discrete PID controller, integrator windup and antiwindup**
- **The alias-effect, Nyquist-frequency, choosing the sampling interval, pre-filters**
- **Disturbance models (stochastics, expectation, covariance, white noise, AR, MA, ARMA, ARMAX models, spectral density)**
- **Ideas in optimal control:**
- **Optimal predictor**
- **Minimum variance controller**
- **LQ controller. (Basics of LQG control)**

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- Passing the course: Two intermediate exams. Later a full exam is also available. It corresponds to two intermediate exams together (but does not replace Quizzes and Homework).
- Intermediate exams: maximum 18+18=36 points. Full exam is later arranged (”Rästitentti”): maximum 36 points.
- 12 Quiz problems, max 1 point each. Each problem is given 24 hours before the lecture starts. Solutions must be submitted electronically in MyCourses by the start of the lecture.
- Homework.
  6 homework problems, max 6 x 6 = 36 points. The homework is not compulsory, but it is needed to pass the course with good grade. Homework solutions are submitted by the given time-table in MyCourses.
- Late solutions (Quizz, Homework) are not accepted.

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Evaluation is based on the AS% formula

$$AS\% = \frac{\text{Quiz}}{12}\cdot 20 + \frac{\text{HW}}{36}\cdot 40 + \frac{(\text{IE1}+\text{IE2}) \text{ or full exam}}{36}\cdot 40$$

meaning that the weights of Quizzes, homework and exams are 20%, 40% and 40%, respectively.

Grading: 48%: 1, 60%: 2, 70%: 3, 80%: 4, 90%:5

No minumum requirement of any item. The formula for AS% alone gives the grade.

Exams: Wed 19.10.2022 at 14:15-17:15 **TU1** (intermediate exam 1)
Wed 7.12.2022 at 14:00-17:00 **AS2** (intermediate exam 2)

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- Please register to the course (Sisu), if you have not done that already.
- You do not have to register to intermediate exams.
- You have to register to full exams.

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- **Note:**

- this year the course does not use robot platforms for practical experimenting and project work. The evaluation is based on Quizzes, Homework, and two intermediate exams (or one full exam later). The points stay valid until the course starts again, autumn 2023.

- The course does not have final exam (”Kurssitentti”), but instead two intermediate exams. Later, full exam is arranged (”Rästitentti”), which has the total weight equal to two intermediate exams.

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ELEC-E8101 Introduction

Study material

- Lecture slides (in English, ordinary and print versions) in MyCourses (mycourses.aalto.fi).
- Exercises with solutions (English) in MyCourses.
- Quizz problems with solutions.
- Homework with Solutions.

- The lectures, lecture slides and exercises with solutions cover the whole course. It is not absolutely necessary to purchase the book.
- Note that the Quizzes must be submitted before the lecture starts. The topics in Quizzes are related mostly to the previous lecture but sometimes also to the next one (lecture material is available in MC).
- Homework assignments (six altogether) are given regularly. They appear in MyCourses.

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ELEC-E8101 Introduction

Textbook:

- **Åström K. J., Wittenmark B.: *Computer Controlled Systems - Theory and Design* (3rd ed.), Prentice-Hall, 1997.**

- Franklin, Powell, Workman: *Digital Control of Dynamic Systems.* Third edition, Addison Wesley, 1998.
- Ogata: *Discrete-Time Control Systems*, 2nd ed., Prentice-Hall, 1994
- Kuo, B. C. : Digital Control Systems (2nd ed.), Oxford University Press, 1992.
- Santina, M. S., Stubberud, A. R., Hostetter, G. H.: Digital Control System Design, Saunders College Publishing, 1994.

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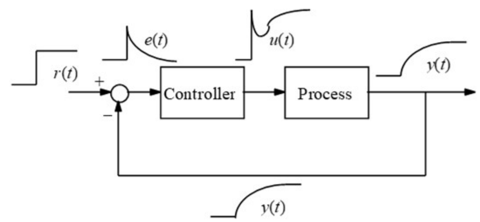
Analog (continuous time) control system

Process controlled by an analog controller



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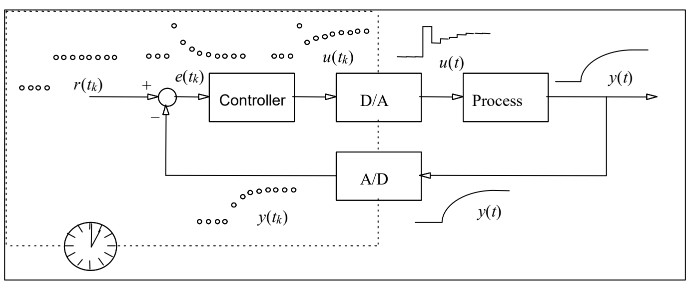
Digital (discrete, discrete-time) control system



In A/D-block the analog signal is *sampled* ; in D/A-block the discrete-time signal is changed into an analog one (hold).

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To think about...

- The system contains both analog and discrete signals (a hybrid system)
- Basic problem already in analog control: how do you treat these kinds of systems analytically?
- Are the traditional time domain and frequency domain methods available now? Can they be modified?
- How do you design digital controllers? What should be taken into account in implementation?

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To think about...

- Is it so that a digital controller only imitates the corresponding analog controller and the result is somewhat worse then (due to loosing information in discretization)?
- Do discrete-time systems have properties that the corresponding analog systems do not have?
- Yes, but this is not so simple: sometimes a discrete controller can perform better than the analog one; on the other hand discrete-time systems have anomalies that do not have a correspondence in the analog world.

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z-transformation

A sequence $\{f_0, f_1, \cdots, f_k, \cdots\}$ for short $\{f_k\}$ or $f(k)$

The *z-transformation* is defined

$$F(z) = Z(f_k) = \sum_{i=0}^{\infty} f_i z^{-i} = f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots$$

Continuous systems: (differential equations/Laplace-transformation)
Discrete systems: (difference equations/Z-transformation)

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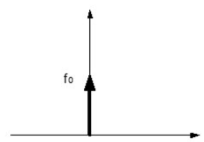
Example. Discrete time impulse

The sequence

$$\{f_0, 0, 0, 0, \cdots\}$$



is an impulse (pulse) with the strength f_0

$f_0\delta(k)$ where $\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$

Z-transformation:

$$F(z) = Z\{f(k)\} = \sum_{k=0}^{\infty} f(k) z^{-k} = \sum_{k=0}^{\infty} f_0 \delta(k) z^{-k} = f_0$$

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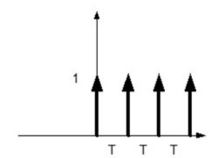
Example. Discrete (unit) step function

Sequence $\{1, 1, 1, 1, \cdots\}$

In the terms f_k



k is a time index (0,1,2,...) . The absolute time at time instant *k* is *kT*, where *T* is *the sampling interval*

Z-transformation:

$$Z\{f(k)\} = 1 + 1\cdot z^{-1} + \cdots + 1\cdot z^{-k} + \cdots$$
$$= \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

(sum of a geometric series)

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Example. $f(k) = a^k, \quad |a| < 1$

Pulse sequence $\{1, a, a^2, a^3, \cdots\}$

$$F(z) = \sum_{k=0}^{\infty} a^k z^{-k} = 1 + az^{-1} + a^2 z^{-2} + \cdots$$
$$= \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$

The sum converges in the complex plane region $|az^{-1}| < 1$ or $|a| < |z|$

It is not necessary to consider the convergence regions in what follows.

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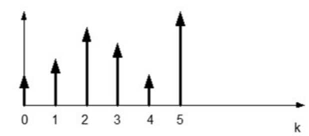
Movement in time ("right"=delay, "left"=prediction)

Pulse sequence f(k):



$$F(z) = Z\{f(k)\} = f_0 + f_1 z^{-1} + f_2 z^{-2} + f_3 z^{-3} + \cdots$$

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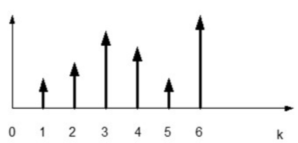
Delay with one sampling interval:



$$Z\{f(k-1)\} = 0 + f_0 z^{-1} + f_1 z^{-2} + f_2 z^{-3} + \cdots$$
$$= z^{-1}\left(f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots\right)$$
$$= z^{-1} F(z)$$

One sample delay thus means multiplication by z^{-1}; correspondingly by z^{-n} when the delay is *n* steps

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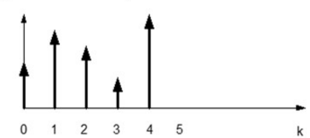
Prediction by one sample interval:



$$Z\{f(k+1)\} = f_1 + f_2 z^{-1} + f_3 z^{-2} + \cdots$$
$$= z \cdot \left(f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots\right) - z f_0$$
$$= zF(z) - z f_0$$

Compare to the derivative of Laplace-transformation in the continuous time case.

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Z-muunnosteoreemoja ja muunnospareja

Määritelmä: $F(z) = Z\{f(k)\}(z) = \sum_{k=0}^{\infty} f(k)z^{-k}$

Z-muunnos	Diskreetin ajan funktio	
$F(z)$	$f(k)$	T1
$C_1F_1(z)+C_2F_2(z)$	$C_1f_1(k)+C_2f_2(k)$	T2
$F(az)$	$a^{-k}f(k)$	T3
$z^{-a}F(z)$	$\begin{cases} 0; & k \le a-1 \\ f(k-a); & k \ge a \end{cases}, a>0$	T4
$z^{a}F(z)-\left[z^{a}f(0)+z^{a-1}f(1)+\cdots zf(a-1)\right]$	$f(k+a)$, $a>0$	T5

Z-muunnos	Diskreetin ajan funktio	
1	$\delta_k(k) = \begin{cases} 1; & k=0 \\ 0; & k \neq 0 \end{cases}$	M1
$\frac{z}{z-1}$	1	M2
$\frac{z}{(z-1)^2}$	k	M3
$\frac{z}{z-a}$	a^k	M4
$\frac{az}{(z-a)^2}$	ka^k	M5
$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\sin(ak)$	M6
$\frac{z(z-\cos(a))}{z^2-2z\cos(a)+1}$	$\cos(ak)$	M7
$\frac{bz\sin(a)}{z^2-2bz\cos(a)+b^2}$	$b^k\sin(ak)$	M8
$\frac{z(z-b\cos(a))}{z^2-2bz\cos(a)+b^2}$	$b^k\cos(ak)$	M9

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The final value theorem

Consider a pulse sequence *f(k)* and its z-transform *F(z)*.

If *f(k)* approaches a limit value as *k* approaches infinity, it holds

$$\underbrace{\lim}_{k\to\infty} f(k) = \underbrace{\lim}_{z\to 1} (1-z^{-1})F(z)$$

This is the *final value theorem*. Compare to the continuous time case

$$\underbrace{\lim}_{t\to\infty} y(t) = \underbrace{\lim}_{s\to 0} sY(s)$$

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Discrete-time system

Example. A first order difference equation

$$y(k+1)-ay(k)=u(k)$$

Initial conditions e.g. *y*(0)=0, *u*(0)=1, *u*(k)=0, *k >0*

Calculation directly from the equation (which is directly an algorithm)

$$y(1)=u(0)=1$$
$$y(2)=ay(1)+u(1)=a$$
$$y(3)=ay(2)+u(2)=a^2$$

Result is the pulse sequence

$$\{a^{k-1}\}, k=1,2,\ldots$$

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Note that the difference equation

$$y(k+1)-ay(k)=u(k)$$

means in absolute time

$$y[(k+1)T]-ay(kT)=u(kT)$$

Take the Z-transformation

$$zY(z)-zy(0)-aY(z)=U(z)$$

Initial conditions are considered to be zero when deriving the *pulse transfer function*, leading to

$$(z-a)Y(z)=U(z)$$

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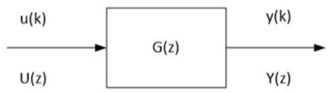
The pulse transfer function



$$G(z) = \frac{Y(z)}{U(z)} = \frac{1}{z-a}$$

With a unit impulse as input $\delta(k)$

the output (pulse response) is the inverse transformation of the pulse transfer function.

$$Y(z) = \frac{1}{z-a} \cdot 1 = z^{-1}\frac{z}{z-a} \Rightarrow y(k) = \begin{cases} 0, & k = 0 \\ a^{k-1}, & k > 0 \end{cases}$$

Compare to the continuous time case.

z-transformation is used to make the analysis and calculations of discrete-time systems more tractable. Compare again to the continuous time case.

The results are meaningful only at **sampling instants**

$$t_k = kT, \quad k = 0,1,2,\dots$$

Note the following transformation pairs (the latter deviates somewhat from the continuous time analogue)

1. $z^{-a}F(z)$; $\begin{cases} 0, & k \le a-1, a > 0 \\ f(k-a), & k \ge a \end{cases}$

2. $z^{a}F(z) - \left[z^{a}f(0) + z^{a-1}f(1) + \cdots + zf(a-1)\right]$; $f(k+a), a > 0$

In the inverse transformation similar ideas as in the continuous time case are applied (dividing terms into sums of smaller entities and then using tables)

Example. $\frac{1}{(z-1)(z-2)} = \frac{A}{z-1} + \frac{B}{z-2}$

No inverse transformation is found from the tables; so let us modify

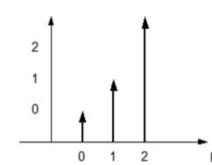
$$\frac{1}{(z-1)(z-2)} = z^{-1}z\frac{1}{(z-1)(z-2)} = z^{-1}z\left(\frac{A}{z-1} + \frac{B}{z-2}\right)$$

$$A = \underbrace{\lim}_{z\to 1} \frac{1}{z-2} = -1; \quad B = \underbrace{\lim}_{z\to 2} \frac{1}{z-1} = 1$$

$$\frac{1}{(z-1)(z-2)} = z^{-1}z\left[\frac{1}{z-2} - \frac{1}{z-1}\right] = z^{-1}\left[\frac{z}{z-2} - \frac{z}{z-1}\right]$$

The inverse transformation of the bracketed term is

$$f(k) = 2^k - 1, \quad k = 0,1,2,\dots$$



Taking the delay z^{-1} into account gives finally

$$f(k-1), \quad k \ge 1 \quad \text{or} \quad \begin{cases} 2^{k-1} - 1, & k = 1,2,3,\dots \\ 0, & k = 0 \end{cases}$$

Stability

We can operate with pulse transfer functions as with transfer functions. E.g. the closed loop pulse transfer function can be calculated, the denominator of which is the *characteristic polynomial*. Its zeros are the system *poles*. The zeros of the numerator are the system *zeros*.

The pulse transfer function can be divided into a sum of terms like

$\frac{z}{z-a}$;the inverse transformation a^k stays bounded when $|a|<1$

If even one term "explodes", the system is *unstable*.

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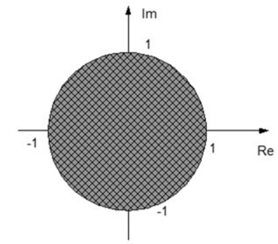
Stability region:

$$G_K(z) = \frac{z-a}{(z-b)(z-c)(z-d)}$$



A discrete-time system is stable , when the poles (*b*, *c*, *d*) are located inside the unit disc. Compare to the left half plane for continuous time systems.

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Shift-operators

A *forward shift* operator q (in time)

$$qy(k) = y(k+1)$$

A *backward shift operator* q^{-1} (in time)

$$q^{-1}y(k) = y(k-1)$$

Correspondingly $q^n y(k) = y(k+n)$

$$q^{-n} y(k) = y(k-n)$$

(Compare to the continuous time differential operator *p*)

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Example. A difference equation (input-output representation of a system)

$$y(k+2) + a_1 y(k+1) + a_2 y(k) = b_0 u(k+1) + b_1 u(k)$$

can be written

$$A(q)y(k) = B(q)u(k)$$

in which

$$A(q) = q^2 + a_1 q + a_2$$
$$B(q) = b_0 q + b_1$$

are *operator polynomials*. A similar equation can also be written as a function of q^{-1}

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Note the formal relationship

$$p \Leftrightarrow s \quad \text{continuous time}$$

$$q \Leftrightarrow z \quad \text{discrete time}$$

However, when using *p* or *q*, we operate in time domain. When using *s* or *z* a transformation to Laplace or Z-domain has been done to deal with transfer functions.

This can sound theoretical. However, in control engineering there are many design methods, in which either time domain or frequency domain (transfer functions) are used.

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From state-space-representation to pulse transfer function

A discrete state-space-representation is defined in an analog manner to the continuous time case; derivatives are replaced with a shift in time.

$$x(k+1) = Fx(k) + Gu(k)$$
$$y(k) = Hx(k)$$

Let us try to find the pulse transfer function. Take the Z-transformation and eliminate *x*. The derivation is quite analogous to the continuous time case.

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$$zX(z) = FX(z) + GU(z)$$
$$(zI - F)X(z) = GU(z)$$
$$X(z) = (zI - F)^{-1}GU(z)$$
$$Y(z) = HX(z) = H(zI - F)^{-1}GU(z)$$

The pulse transfer function is obtained as

$$\Sigma(z) = \frac{Y(z)}{U(z)} = H(zI - F)^{-1}G$$

compare to the continuous time case:

$$C(sI - A)^{-1}B$$

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From input-output representation to state-space equations

A similar method as in the continuous time case is available

Example.

$$y(k+2) - 1.3y(k+1) + 0.4y(k) = u(k+1) - 0.4u(k)$$

-state-space?
-pulse transfer function?

Write the equation as a function of the *q*-operator

$$q^2y(k) - 1.3qy(k) + 0.4y(k) = qu(k) - 0.4u(k)$$

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Move q terms to the left side

$$q^2 y(k) - 1.3qy(k) - qu(k) = -0.4y(k) - 0.4u(k)$$

Develop the left side as factors according to q and choose the state variables as follows

$$q\underbrace{\left[q\underbrace{y(k)}_{x_1(k)} - 1.3y(k) - u(k)\right]}_{x_2(k)} = -0.4y(k) - 0.4u(k)$$

$$x_1(k) = y(k);\ x_2(k) = x_1(k+1) - 1.3x_1(k) - u(k)$$

$$x_2(k+1) = -0.4x_1(k) - 0.4u(k)$$

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The following state representation is obtained

$$x_1(k+1) = 1.3x_1(k) + x_2(k) + u(k)$$
$$x_2(k+1) = -0.4x_1(k) - 0.4u(k)$$
$$y(k) = x_1(k)$$

and in matrix form

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \overbrace{\begin{bmatrix} 1.3 & 1 \\ -0.4 & 0 \end{bmatrix}}^{F} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \overbrace{\begin{bmatrix} 1 \\ -0.4 \end{bmatrix}}^{G} u(k)$$

$$y(k) = \overbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}^{H} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

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Pulse transfer function

$$\Xi(z) = \frac{Y(z)}{U(z)} = H(zI - F)^{-1}G$$

$$= \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z & 1 \\ -0.4 & z-1.3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.4 \end{bmatrix}}{z(z-1.3)+0.4} = \frac{z-0.4}{(z-0.8)(z-0.5)}$$

Zeros: $z_1 = 0.4$

Poles: $p_1 = 0.8$

$p_2 = 0.5$

The poles are inside the unit circle, and the system is therefore stable.

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Is there a need for discrete-time control theory?

- •Analog systems can be imitated
- •There exists discrete-time systems without an analog correspondence
- •Sampling can cause problems to discrete-time systems
- •Digital controller can even beat the analog controller (fastness)
- •Discrete-time control theory is needed !

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Grounds and basic building blocks

•Sampling theorem (Shannon -49)
•Difference equations (-48)
•Integral transformations
•Z-transformation (Hurewicz -47)

•State-space-representation (Kalman -55)

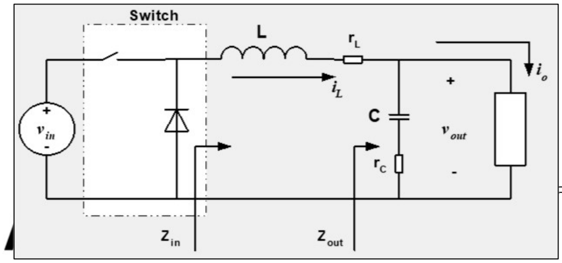
Grounds and basic building blocks …

- Optimal control (Bellman -57)
- Stochastic control, LQG (Kalman -65)
- Algebral system theory (Rosenbrock -70)
- Identification, Adaptive control
- Robust control (-80)
- Soft computing, -80, fuzzy control, neural networks, cloud computing, computational sciences (-2000)...

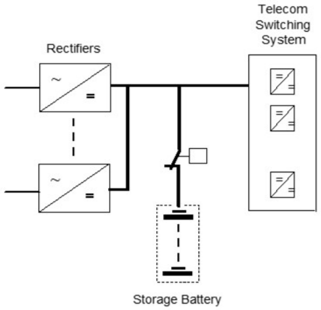
An example of controller design (only an example:no need to learn very carefully)

Controller design of a switched-mode power supply (Buck-type; output voltage smaller than input)

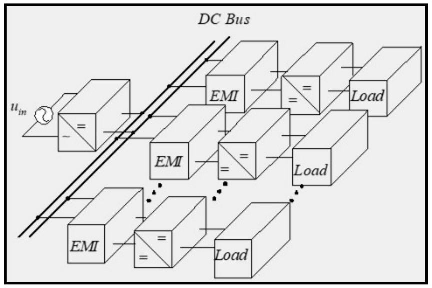
The switch operates at high frequency, e.g. 100 kHz. By changing the on/off time ratio of the switch (duty cycle) power is transferred into load and the output voltage level is controlled.



By connecting AC/DC power supplies in parallel, and providing a battery for back-up a suitable power system for a Telecom load (for example) is obtained.



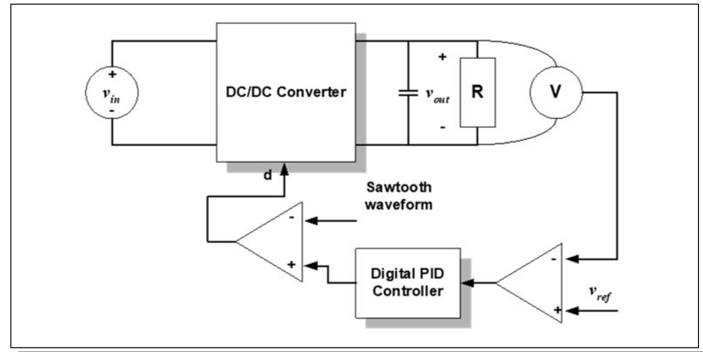
A difficult multivariable system, MIMO = multiple input, multiple output. C.f. SISO = single input, single output



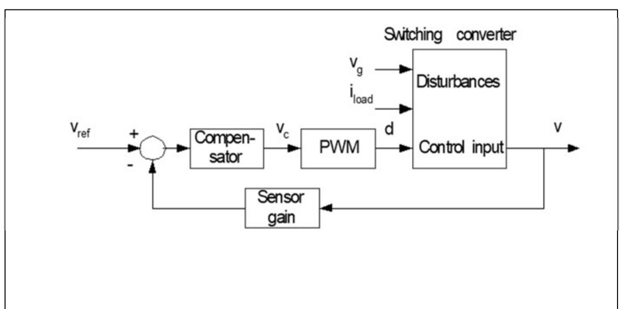
Modern industrial power systems are complex and difficult;
System blocks are connected in series and in parallel. They are provided with input filters (EMI), loads are changing and complex.

Systematic methods are needed = theory is needed!
Today's magic buzzword is **digitalization!**

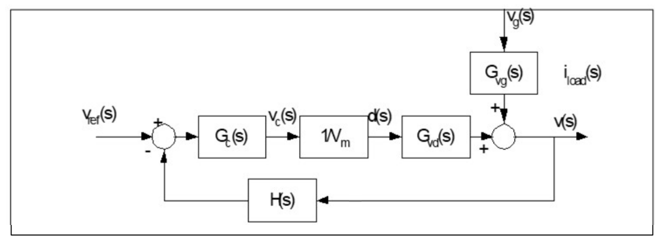
Negative feedback = feedback control, *RC*-circuit is the load. The output voltage is compared to reference, and a discrete-time PID-controller is used to control the switch.



Systems can be described by block diagrams PWM (pulse width modulation) transforms the controller output signal to the duty cycle of the switch; basically that belongs to the operation of the actuator Input voltage. V_g and load current i_{load} are disturbances from control viewpoint. *Robustness* means the ability of the controller to tolerate disturbances and modeling errors.



By writing the dynamic equations of the system blocks the performance can be studied.



$$G_{vd}(s) = \frac{V_{in}R(1+sCr_c)}{LC(R+r_c)s^2+(RCr_c+RCr_l+Cr_lr_c+L)s+R+r_l}$$

$$G_{vg}(s) = \frac{DR(1+sCr_c)}{LC(R+r_c)s^2+(RCr_c+RCr_l+Cr_lr_c+L)s+R+r_l}$$

A lead/lag compensator has been designed for the process by using the Bode diagram as a design tool. The insufficient phase margin (2 degrees) of the open loop has been improved to (50 degrees). Stability must always be guaranteed.

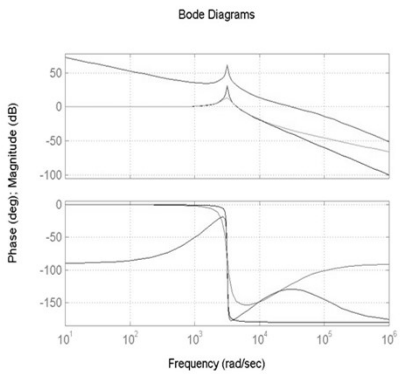
$$G_c(s)=G_{cm}\frac{\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{\omega_l}{s}\right)}{1+\frac{s}{\omega_p}}$$

$G_{cm}=33.8, \omega_z=10681\ (rad/s)$

$\omega_l=1257 (rad/s), \omega_p=91106 (rad/s)$

$\omega_c^* \approx 4500\ (rad/s),\ \phi_m^* \approx 2^\circ$

$\omega_c \approx 30000\ (rad/s),\ \phi_m \approx 50^\circ$



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But the compensator can be approximated by a PID-controller:

$$G_c(s)=G_{cm}\frac{\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{\omega_l}{s}\right)}{1+\frac{s}{\omega_p}}=K\left(1+\frac{1}{sT_i}\right)\left(1+\frac{sT_d}{1+\frac{T_d}{N}s}\right)\approx K\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+\frac{T_d}{N}s}\right)=G_{PID}(s)$$

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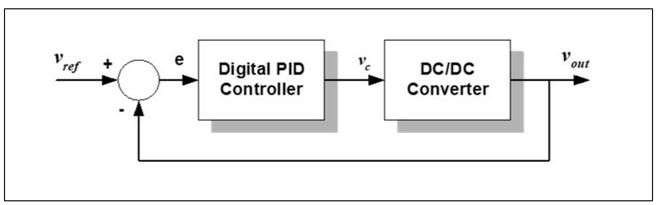
The control algorithm can be *discretized* directly for computer based control (details are presented later in the course)



$$v_c(z)=K\left(bv_{ref}(z)-v_{out}(z)\right)+K\frac{T}{T_i}\frac{z}{z-1}\left(v_{ref}(z)-v_{out}(z)\right)-K\frac{T_d(z-1)}{\left(T+\frac{T_d}{N}\right)z-\frac{T_d}{N}}v_{out}(z)$$

The operating frequency of the measurement and control are now ”new” concepts that must be considered. Here the sampling frequency is 20 kHz meaning that the sampling time is 1/20000 s.

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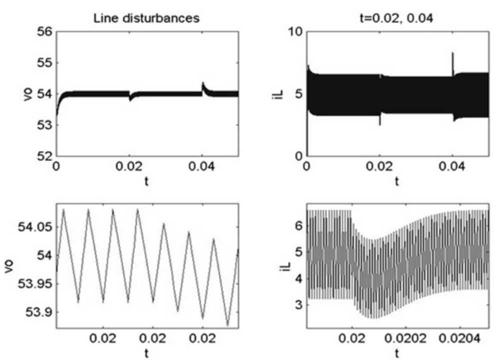
The performance of the controller can be simulated. Below the output voltage and inductor current are shown. The zoomed figures show clearly the operation of the switch.



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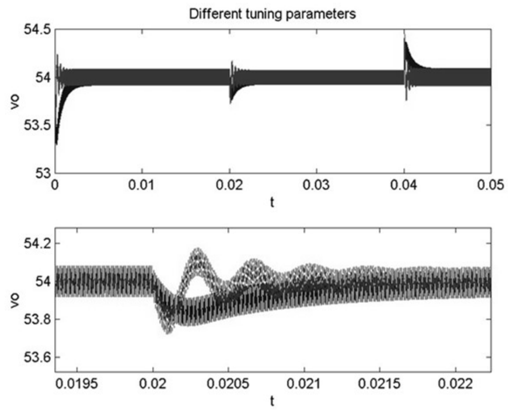
The performance can be made faster by changing the tuning of the PID controller. That leads to increasing oscillations however.



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Controller design procedure

- Construction of the process model from physical equations or by *identification*.
- The model *analysis* and *linearisation* when needed; construction of the transfer functions
- Formulation of the control problem
- Controller design in time or frequency domain
- *Discretization* and *implementation* of the controller e.g. by a digital signal processor (DSP)
- Alternatively: discretization of the process model and controller design directly in discrete time directly

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Simulation

- To be used: Matlab/Simulink with different toolboxes....
- Control System Toolbox
- Identification Toolbox
- Optimization Toolbox
- Model Predictive Toolbox
- Fuzzy Toolbox
- Neural Networks Toolbox
- Symbolic Toolbox
- Statistical Toolbox
- Comsol Multiphysics

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Challenges

- Process knowledge
- Measurement technology
- Computer technology
- Control Theory

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History of discrete-time systems

- 2. world war, radars, computers, numerical computation
- Pioneering period ~1955
  - 1 - 40 (1961) - 160 (1962) applications (supervisory-mode, where setpoint values etc. are calculated to help the operator)
- DDC ~ 1962 (Direct Digital Control = computer controls the process directly)
- in 1975 the first digital automation system (Honeywell TDC 2000)
- Programmable logic controllers (PLC) from the 70's.
- Efficient signal procesors (DSP) from 90's, embedded systems, mechatronics
- Wireless automation, networks, energy technology, digitalization = current trends
- New *era*: today and tomorrow belong to the digital age!

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