

# **Power Electronics**

## **ELEC-E8412 Power Electronics, 5 ECTS**

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# Course Objectives

At the end of this course, you will be able to:

generally analyse the uncontrolled and controlled half-wave rectifiers with different loads and apply the power computation concepts from the previous chapter to these circuits.

# Half-Wave Rectifiers

A **rectifier** converts **AC** to **DC**. The half-wave rectifier is used most often in **low-power** applications.

## 1. The Uncontrolled Half-Wave Rectifiers:

Assuming you have an AC source, and you are going to supply a DC load. For this process, you need an AC/DC rectifier.

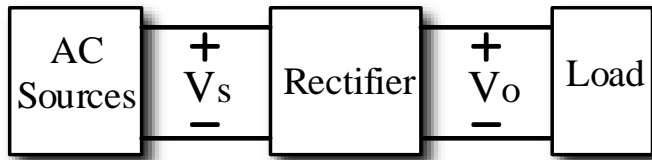
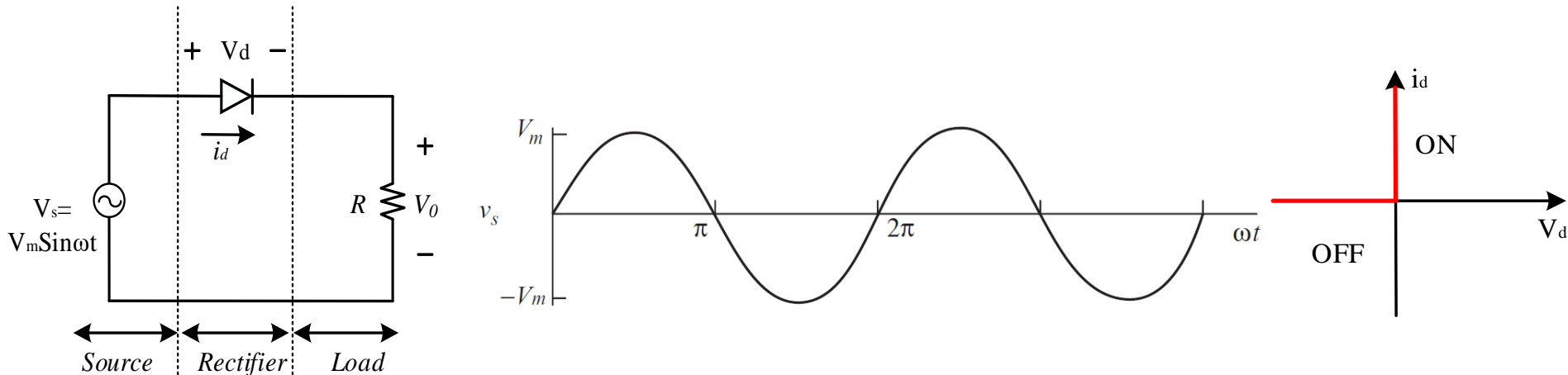


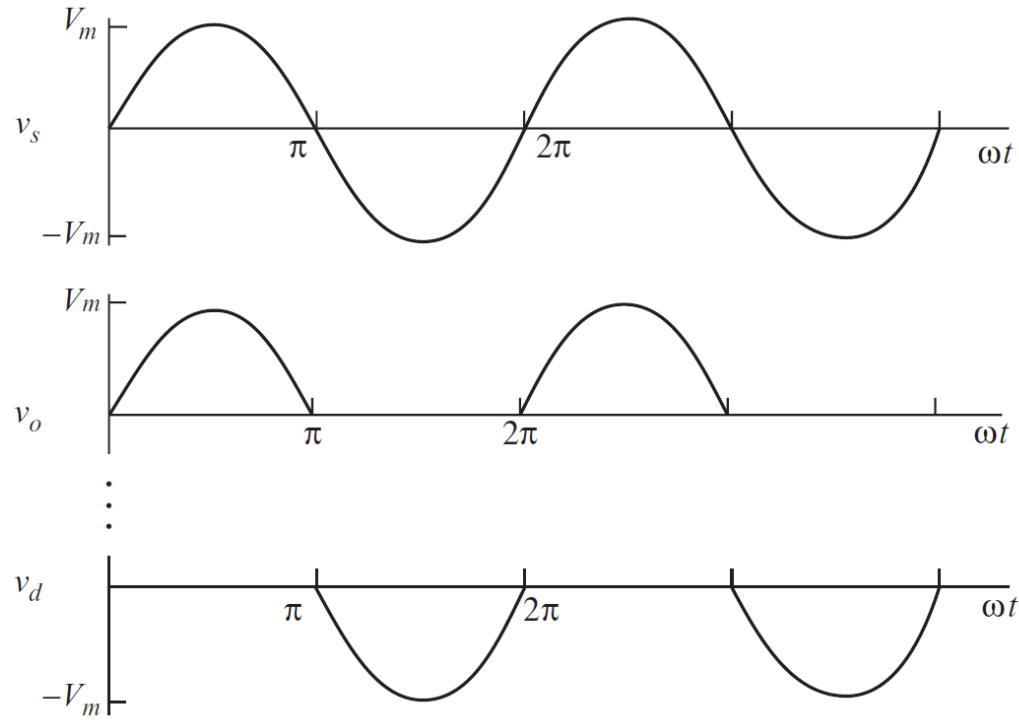
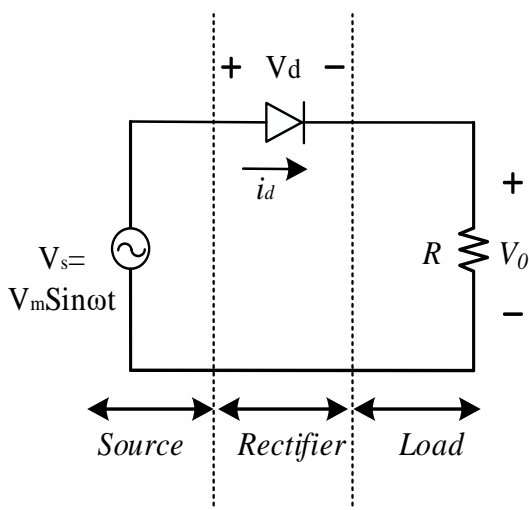
Figure 3-1

- $V_o$  is purely DC or has a specified DC component
- AC source could be sinusoidal, could be a voltage source

## A. The Uncontrolled Half-Wave Rectifiers with resistive load:



$\left\{ \begin{array}{l} 0 < \omega t < \pi \\ 0 < \omega t < \pi \end{array} \right.$	assume	D is OFF	$\Rightarrow i_d = 0$	$\rightarrow V_o = 0$	$\rightarrow V_s = V_d > 0$	<span style="color: red;">✗ Wrong assumption</span>
	assume	D is ON	$\Rightarrow V_d = 0$	$\rightarrow V_o = V_s$	$\rightarrow i_d = \frac{V_s}{R} > 0$	<span style="color: green;">✓ Right assumption</span>



$\left\{ \begin{array}{l} \pi < \omega t < 2\pi \quad \text{assume } D \text{ is ON} \quad \times \text{Wrong assumption} \\ \Rightarrow V_d = 0 \rightarrow V_o = V_{in} \rightarrow i_d = \frac{V_{in}}{R} < 0 \end{array} \right.$

$\left\{ \begin{array}{l} \pi < \omega t < 2\pi \quad \text{assume } D \text{ is OFF} \quad \checkmark \text{Right assumption} \\ \Rightarrow i_d = 0 \rightarrow V_o = 0 \rightarrow V_d = V_{in} < 0 \end{array} \right.$

$$\langle V_o \rangle = V_{avr} = \frac{1}{2\pi} \int_0^{2\pi} V_o d(\omega t) = \frac{1}{2\pi} \int_0^{\pi} V_m \sin(\omega t) d(\omega t) + \frac{1}{2\pi} \int_{\pi}^{2\pi} 0 d(\omega t) = \frac{V_m}{\pi} \quad \leftarrow \text{The average value is not zero}$$

$$\langle i_d \rangle = \frac{V_m}{R\pi}$$

$$V_{orms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_o^2 d\omega t} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} [V_m \sin(\omega t)]^2 d(\omega t)} = \sqrt{\frac{1}{2\pi} \frac{V_m^2 \pi}{2}} = \frac{V_m}{2}$$

$$P_{out} = \frac{V_{o-rms}^2}{R}$$

$$I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_{orms}}{R} = \frac{V_m}{2R}$$

**Example:** If  $V_{in}(t)=170\sin(366t)$  and  $R=12\ \Omega$ , determine:

- The average value of the load current
- The rms value of the load current
- The apparent power supplied by the source
- Power factor of the circuit

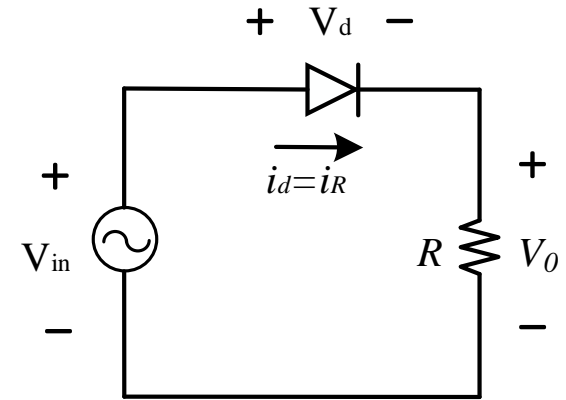
**Solution:**

$$(a) \quad \langle i_o \rangle = \langle i_R \rangle = \langle i_d \rangle = \frac{V_m}{R\pi} = \frac{170}{\pi \times 12} = 4.51 \quad (\text{A})$$

$$(b) \quad I_{in-rms} = I_{R-rms} = I_{d-rms} = I_{o-rms} = \frac{V_m}{2R} = \frac{170}{2 \times 12} = 7.08 \quad (\text{A})$$

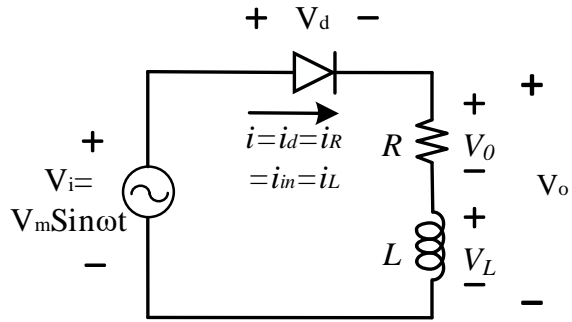
$$(c) \quad \text{apparent power} = V_{in-rms} \cdot I_{in-rms} = \frac{170}{\sqrt{2}} \times 7.08 = 851.47$$

$$(d) \quad \text{pf} = \frac{\text{average power}}{\text{apparent power}} = \frac{P_{out}}{V_{in-rms} \cdot I_{in-rms}} = \frac{R \times I_{o-rms}^2}{V_{in-rms} \cdot I_{in-rms}} = \frac{12 \times (7.08)^2}{\frac{170}{\sqrt{2}} \times 7.08} = \frac{\sqrt{2}}{2} = 0.707$$

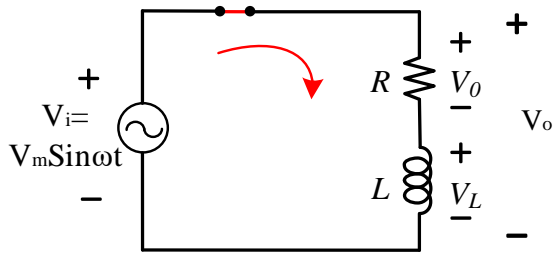


## B. The Uncontrolled Half-Wave Rectifiers with R-L load:

**Example:** Providing power to the field winding of a DC machine or armature winding of a DC machine (winding has internal resistance  $R$  and self-inductance  $L$ ).



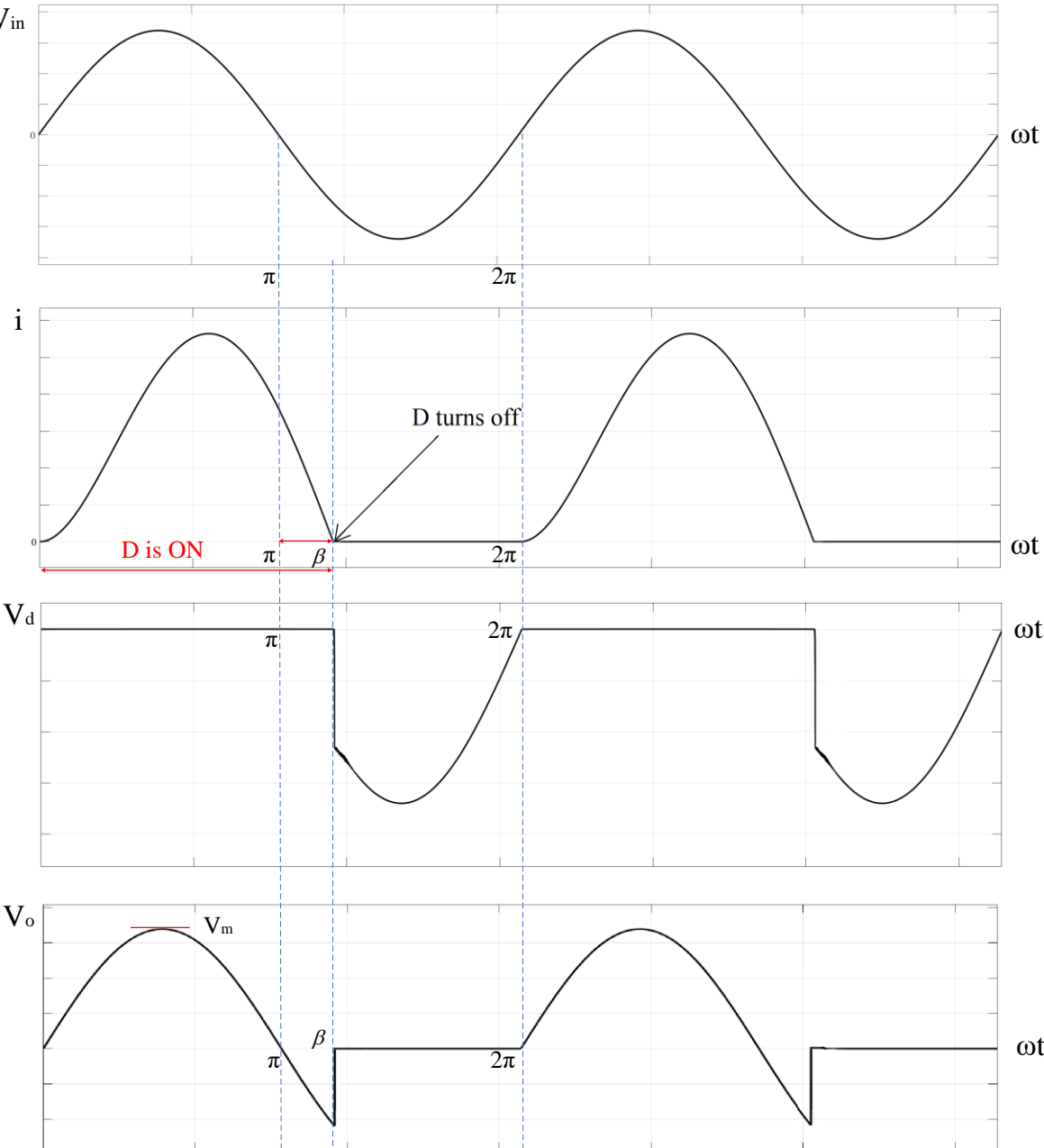
$\begin{cases} 0 < \omega t < \pi & \text{assume } D \text{ is OFF} \Rightarrow V_{in} = V_d > 0 \times \\ 0 < \omega t < \pi & \text{assume } D \text{ is ON} \Rightarrow V_d = 0 \rightarrow V_o = V_{in} \checkmark \end{cases}$

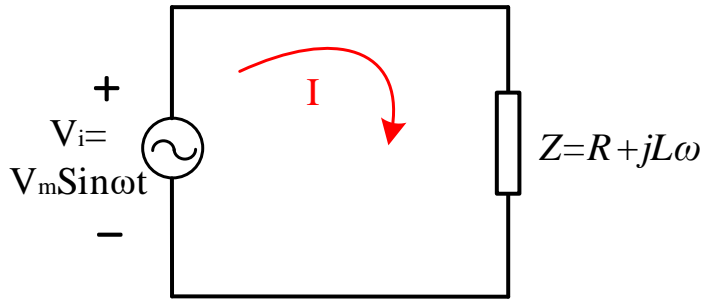


$$V_{in} = V_R + V_L \Rightarrow V_m \sin \omega t = Ri + L \frac{di}{dt}$$

$$\Rightarrow i(t) = \underbrace{\frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t - \theta)}_{\text{steady-state response}} + \underbrace{Ae^{-t/L/R}}_{\text{natural response}}$$

$$\theta = \tan^{-1}\left(\frac{L\omega}{R}\right) > 0$$





$$0 = Ri + L \frac{di}{dt} \longrightarrow i(t) = Ae^{-t/L/R}$$

$$I = \frac{V_m}{|Z|} \text{Sin}(\omega t - \angle Z)$$

$$A = ? \text{ initial condition } i(0) = 0 \Rightarrow 0 = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \text{Sin}(-\theta) + A \Rightarrow A = \frac{V_m \text{Sin } \theta}{\sqrt{R^2 + (L\omega)^2}}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \left[ \text{Sin}(\omega t - \theta) + \text{Sin } \theta e^{-t/L/R} \right]$$

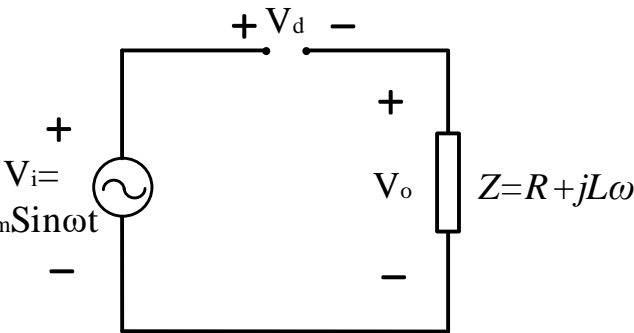
$\begin{cases} \text{D is ON} & \text{for } 0 < \omega t < \beta & \beta > \pi \\ \text{D is OFF} & \text{for } \beta < \omega t < 2\pi & \text{because } i(t) \text{ tends to become negative} \end{cases}$

$$\text{D is OFF} \Rightarrow i_d = 0 \Rightarrow V_R = V_L = V_o = 0 \Rightarrow V_{in} = V_d$$

$$\text{when } \omega t = \beta \Rightarrow i(\omega t = \beta) = 0$$

$$i(\beta) = \frac{V_m}{\sqrt{R^2 + (L\omega)^2}} \left[ \text{Sin}(\beta - \theta) + \text{Sin } \theta e^{-\beta/L/R} \right] = 0$$

\* There is no closed-form sloution to find the  $\beta$  and it must be calculated by software programing like MATLAB but the value of  $\beta$  is always as  $\pi \leq \beta \leq 2\pi$



D is OFF

\*  $\beta$  is called as conduction angle or turn off angle of diode.

### C. The Uncontrolled Half-Wave Rectifiers with R-C load:

$$D \text{ is ON} \Rightarrow i_d > 0 \text{ or } i_d = i_C + i_R > 0$$

$$\text{if } D \text{ is ON} \Rightarrow V_{in} = V_C = V_o = V_m \sin(\omega t)$$

$$i_C + i_R > 0$$

$$C \frac{dV_C}{dt} + \frac{V_o}{R} > 0$$

$$C \frac{d(V_m \sin(\omega t))}{dt} + \frac{V_m \sin(\omega t)}{R} > 0 \rightarrow C\omega V_m \cos(\omega t) + \frac{V_m}{R} \sin(\omega t) > 0$$

$\theta$  is the time at which the diode is going to turn off because its current turns to get negative.

Until  $\omega t = \pi/2$  both terms are positive. After  $\omega t > \pi/2$  the first term get negative and D turns off at  $\omega t = \theta$ . So, the capacitor is getting discharged through the resistor.

$$C\omega V_m \cos \theta + \frac{V_m}{R} \sin \theta = 0$$

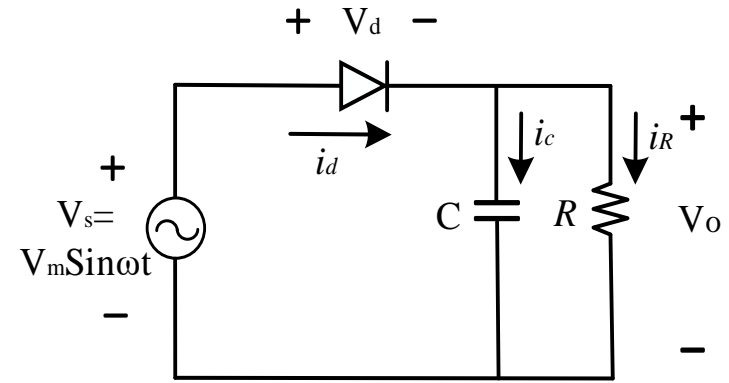
$$\tan \theta = -RC\omega \rightarrow \theta = \tan^{-1}(-RC\omega) = \pi - \tan^{-1}(RC\omega)$$

$$RC\omega > 0 \rightarrow 0 < \tan^{-1}(RC\omega) < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < \theta < \pi$$

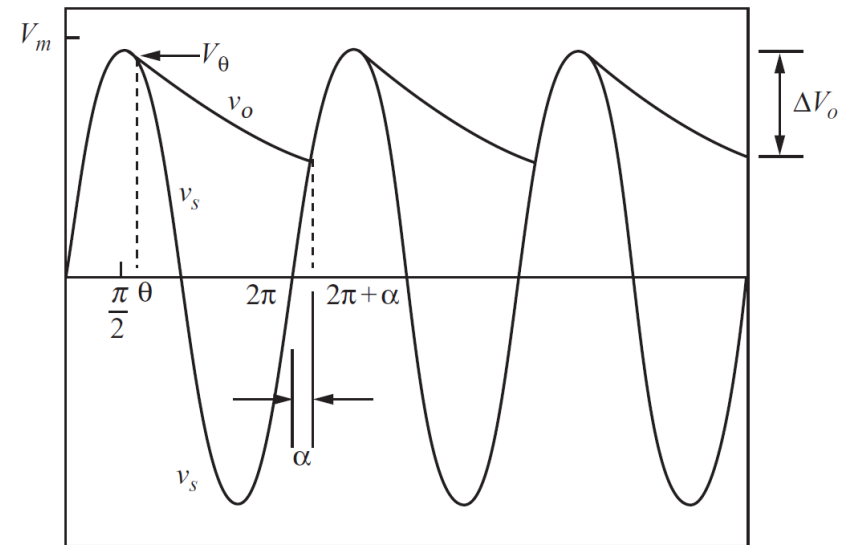
$$\text{if } RC\omega \gg 0 \Rightarrow \tan^{-1}(RC\omega) \rightarrow \frac{\pi^-}{2} \Rightarrow \theta \rightarrow \frac{\pi^+}{2}$$

In practical circuits where the time constant is large,

$$\theta \approx \frac{\pi}{2} \text{ and } V_m \sin \theta \approx V_m$$



**Half-wave rectifier with RC load**



**Input and output voltages**



D turns OFF:

In this mode capacitor C discharges through resistor R.

$$\text{if D is OFF: } i_C + i_R = 0 \rightarrow C \frac{dV_C}{dt} + \frac{V_C}{R} = 0$$

$$\frac{dV_C}{V_C} + \frac{dt}{RC} = 0 \rightarrow \frac{dV_C}{V_C} = -\frac{dt}{RC}$$

$$\int_{V_\theta}^{V_C} \frac{dV_C}{V_C} = -\int_{\theta}^{\omega t} \frac{dt}{RC} \rightarrow \int_{V_m \sin \theta}^{V_C} \frac{dV_C}{V_C} = -\frac{1}{\omega RC} \int_{\theta}^{\omega t} d\omega t \rightarrow \ln \frac{V_C}{V_\theta} = -\left(\frac{\omega t - \theta}{\omega RC}\right)$$

$$V_C = v_o = V_m \sin \theta e^{-\frac{(\omega t - \theta)}{\omega RC}}$$

$$\text{for } \omega t = 2\pi + \alpha \rightarrow V_C = V_m \sin \theta e^{-\frac{(2\pi + \alpha - \theta)}{\omega RC}}$$

$$\text{if D turns ON at } \omega t = 2\pi + \alpha \Rightarrow V_m \sin(2\pi + \alpha) - V_m \sin \theta e^{-\frac{(2\pi + \alpha - \theta)}{\omega RC}} = 0$$

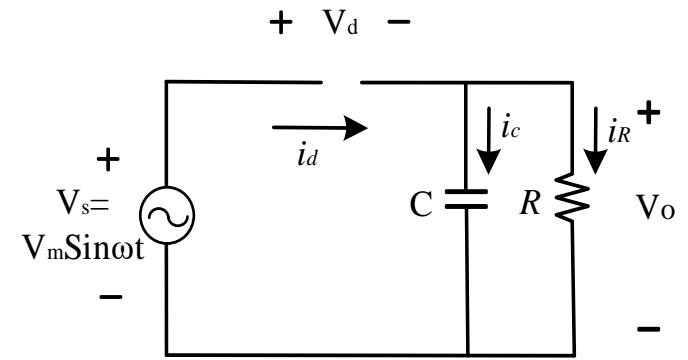
$$*\sin \alpha - \sin \theta e^{-\frac{(2\pi + \alpha - \theta)}{\omega RC}} = 0$$

No closed form solution for  $\alpha$   
must be solved numerically

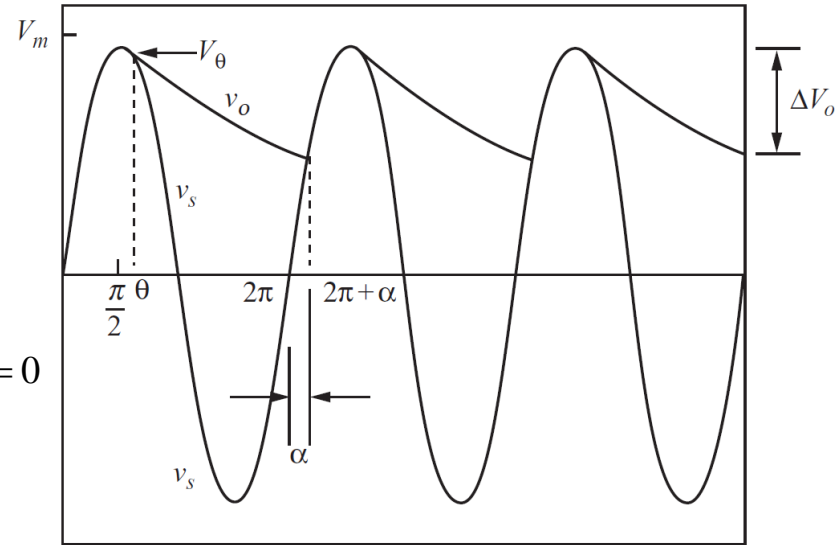
$$V_o(\omega t) = V_C(\omega t) = \begin{cases} V_m \sin \theta e^{-\frac{(\omega t - \theta)}{\omega RC}} & \theta \leq \omega t \leq 2\pi + \alpha \text{ (diode is OFF)} \\ V_m \sin \omega t & 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \text{ (diode is ON)} \end{cases}$$

$$i_C = C \frac{dV_C}{dt} = \begin{cases} -\frac{V_m \sin \theta e^{-\frac{(\omega t - \theta)}{\omega RC}}}{R} & \text{diode is OFF} \\ C\omega V_m \cos \omega t & \text{diode is ON} \end{cases}$$

where  $\theta = \pi - \tan^{-1}(RC\omega)$  and  $\alpha$  is the numerical solution of \*.



Half-wave rectifier with RC load



Input and output voltages

# What is the peak to peak ripple of output voltage ( $\Delta V_o$ )?

The maximum output voltage is  $V_m$ . The minimum output voltage occurs at  $\omega t = 2\pi + \alpha$ , which can be computed from  $V_m \sin \alpha$ . Then, the peak-to-peak ripple of output voltage can be expressed as:

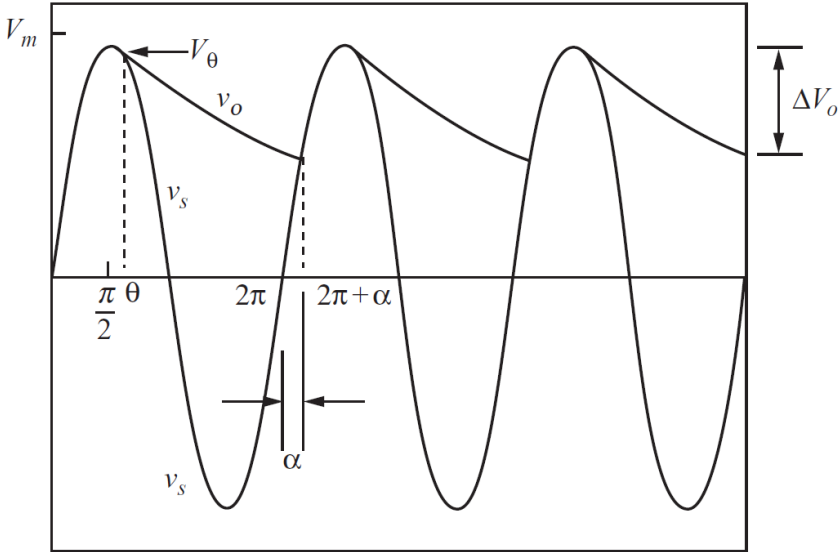
$$\Delta V_o = V_m - V_m \sin \alpha = V_m (1 - \sin \alpha)$$

It is not easy to use this equation if it is not existed. For this reason we need to consider some approximations.

The peak-to-peak ripple is approximately:

$$\Delta V_o \approx V_m \left( \frac{2\pi}{\omega RC} \right) = \left( \frac{V_m}{fRC} \right)$$

\*This approximation is valid if:  $\left\{ \begin{array}{l} \omega RC \gg 1 \\ or \\ RC \gg T \end{array} \right.$



**Example:** The half-wave rectifier with RC load has a 120-V rms source at 60 Hz,  $R= 500 (\Omega)$ , and  $C=100 (\mu\text{F})$ . Determine:

- an expression for output voltage
- the peak-to-peak voltage variation on the output
- an expression for capacitor current
- the peak diode current
- the value of  $C$  such that  $\Delta V_o$  is 1 percent of  $V_m$ .

**Solution:**

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi \times 60)(500)(10^{-6}) = 18.85 \text{ rad}$$

The angle  $\theta$  is determined as  $\rightarrow \theta = \pi - \tan^{-1}(RC\omega) = \pi - \tan^{-1}(18.85) = 1.62 \text{ rad} = 93^\circ$

The angle  $\alpha$  is determined as  $\rightarrow \sin \alpha - \sin \theta e^{-(2\pi+\alpha-\theta)/\omega RC} = 0 \rightarrow \sin \alpha - \sin(1.62)e^{-(2\pi+\alpha-1.62)/18.85} = 0$

yielding :  $\alpha = 0.843 \text{ rad} = 48^\circ$

- a. Output voltage is expressed as:

$$V_o(\omega t) = \begin{cases} V_m \sin \theta e^{-(\omega t - \theta)/\omega RC} & \theta \leq \omega t \leq 2\pi + \alpha \\ V_m \sin \omega t & 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \end{cases} \Rightarrow \begin{cases} 169.7 e^{-(\omega t - 1.62)/18.85} & \theta \leq \omega t \leq 2\pi + \alpha \\ 169.7 \sin \omega t & 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \end{cases}$$

- b. Peak-to-peak output voltage can be expressed as:

$$\Delta V_o = V_m(1 - \sin \alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

c. The capacitor current is determined as:

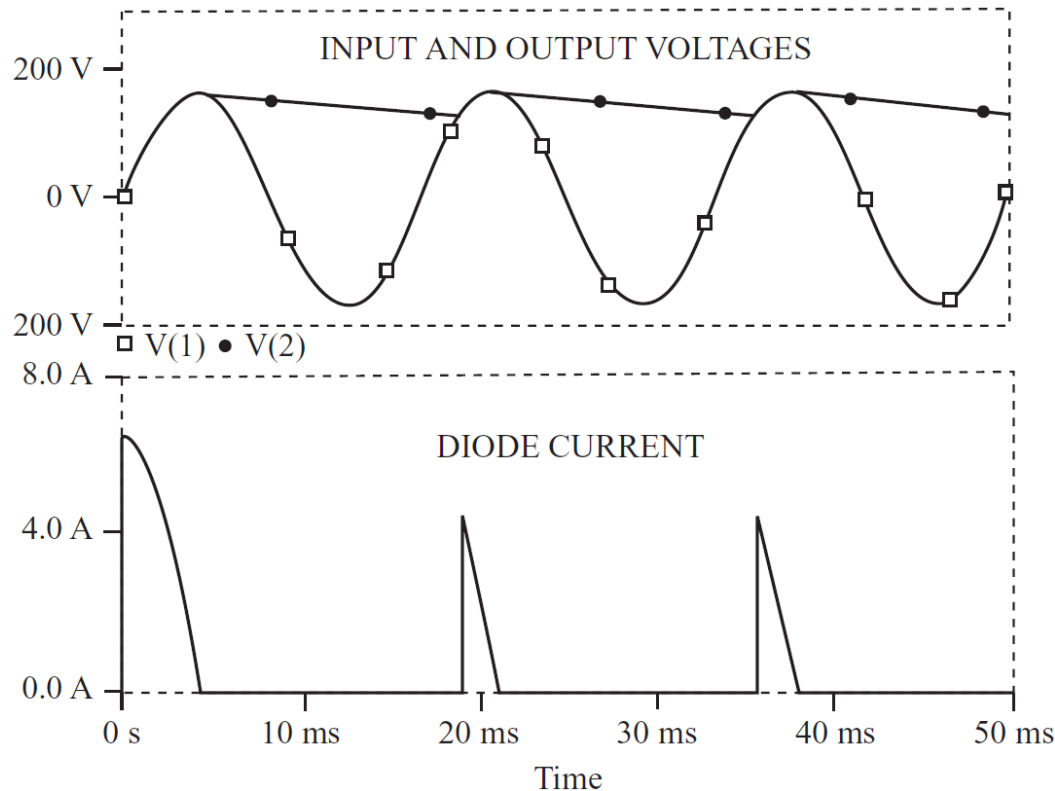
$$i_c(\omega t) = \begin{cases} -\frac{V_m \sin \theta e^{-(\omega t - \theta)/\omega RC}}{R} & \theta \leq \omega t \leq 2\pi + \alpha \\ C\omega V_m \cos \omega t & 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \end{cases} \Rightarrow \begin{cases} -0.339 e^{-(\omega t - 1.62)/18.85} \text{ A} & \theta \leq \omega t \leq 2\pi + \alpha \\ 6.4 \cos(\omega t) \text{ A} & 2\pi + \alpha \leq \omega t \leq 2\pi + \theta \end{cases}$$

d. Peak diode current is determined as:

$$i_{d,\text{peak}} = V_m \left( C\omega \cos \alpha + \frac{\sin \alpha}{R} \right) = \sqrt{2}(120) \left[ 377(10)^{-4} \cos 0.843 + \frac{\sin 0.843}{500} \right] = 4.26 + 0.34 = 4.50 \text{ A}$$

e. For  $\Delta V_o = 0.01 V_m$ , C can be calculated as:

$$C \approx \left( \frac{V_m}{fR(\Delta V_o)} \right) = \left( \frac{V_m}{60 \times 500 \times 0.01 V_m} \right) = \frac{1}{300} \text{ F} = 3333 \mu\text{F}$$



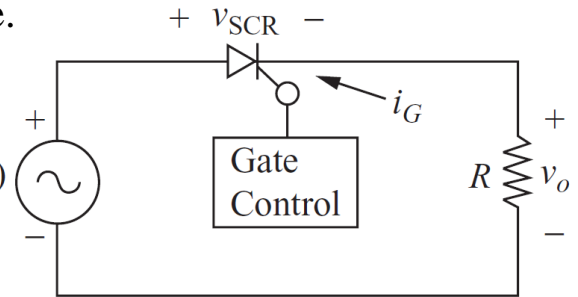
## 2. The Controlled Half-Wave Rectifier:

The half-wave rectifiers analyzed previously are classified as uncontrolled rectifiers. A way to control the output of a half-wave rectifier is to use an SCR instead of a diode.

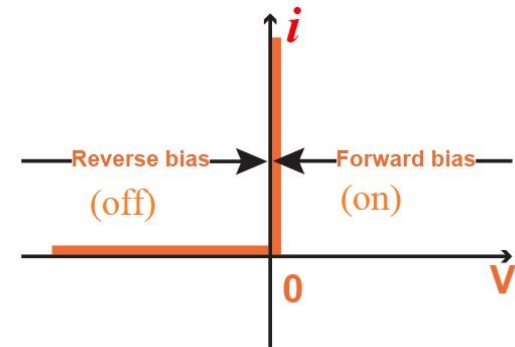
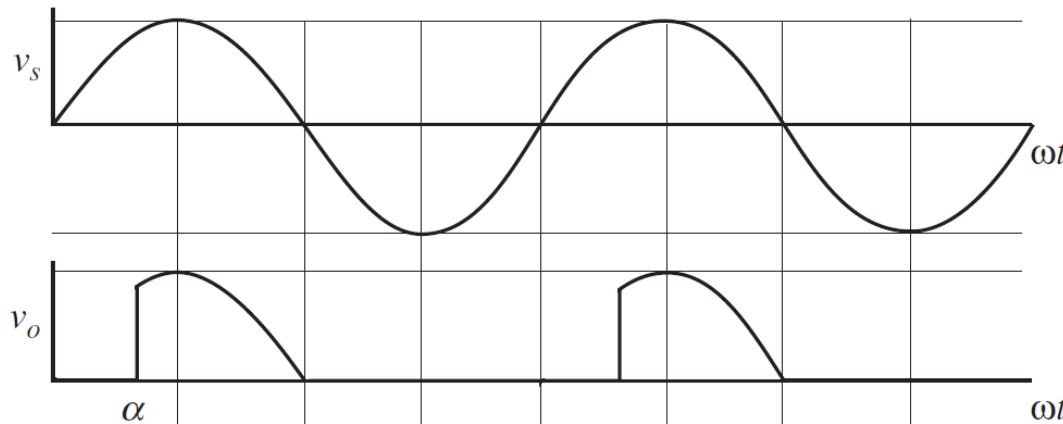
Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ( $v_{SCR} > 0$ ).
2. A current must be applied to the gate of the SCR.

$$v_s = V_m \sin(\omega t)$$



A basic controlled rectifier



$0 < \omega t < \alpha$ , SCR is forward blocking (OFF)

$\alpha < \omega t < \pi$ , SCR is ON

$\pi < \omega t < 2\pi$ , SCR is reversed biased

@  $\omega t = \alpha$ , the SCR is triggered and starts conducting

@  $\omega t = \pi$ , the SCR current reduces to zero and it stops conducting

**$\alpha$  is called the delay angle**

Voltage waveforms

The average (DC) voltage across the load resistor can be calculated as:

$$V_o = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{2\pi} (1 + \cos \alpha)$$

The power absorbed by the resistor is  $V_{rms}^2 / R$ , where the rms voltage across the resistor is computed from

$$\begin{aligned} V_{o,rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} [V_o(\omega t)]^2 d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t) d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} V_m^2 \times \left(\frac{1 - \cos 2\omega t}{2}\right) d(\omega t)} \\ &= \frac{V_m}{2} \sqrt{\left(\frac{\pi - \alpha}{\pi}\right) + \left(\frac{\sin 2\alpha - \sin 2\pi}{2\pi}\right)} = \frac{V_m}{2} \sqrt{\left(1 - \frac{\alpha}{\pi}\right) + \left(\frac{\sin 2\alpha}{2\pi}\right)} \quad \alpha \text{ in rad} \end{aligned}$$

$$\text{Power Factor (pf)} = \frac{\text{average power}}{\text{apparent input power}} = \frac{P_{in} = P_{out}}{V_{in,rms} \times I_{rms}} = \frac{V_{rms}^2 / R}{V_{in,rms} \times \frac{V_{o,rms}}{R}} = \frac{V_{o,rms}}{V_{in,rms}}$$

$$= \frac{\frac{V_m}{2} \sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}}}{\frac{V_m}{\sqrt{2}}} = \sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}} \quad \alpha \text{ in rad}$$

**Example:** Design a circuit to produce an average voltage of 40 V across a 100- $\Omega$  load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

**Solution:** In an uncontrolled half-wave rectifier, the average voltage will be  $V_m/\pi=(120\sqrt{2})/\pi=54$  V.

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right] = \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2^\circ = 1.07 \text{ rad}$$

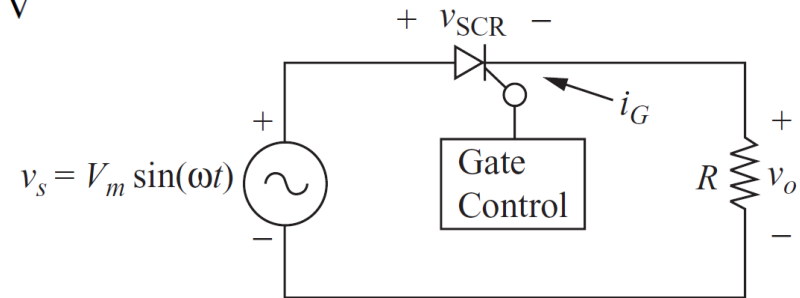
$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2} \sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is:

$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

The power factor of the circuit is:

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S, \text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$



**Questions and comments are  
most welcome!**

