Aalto University
School of Engineering

Power Electronics

ELEC-E8412 Power Electronics, 5 ECTS

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Fall 2022

Course Objectives

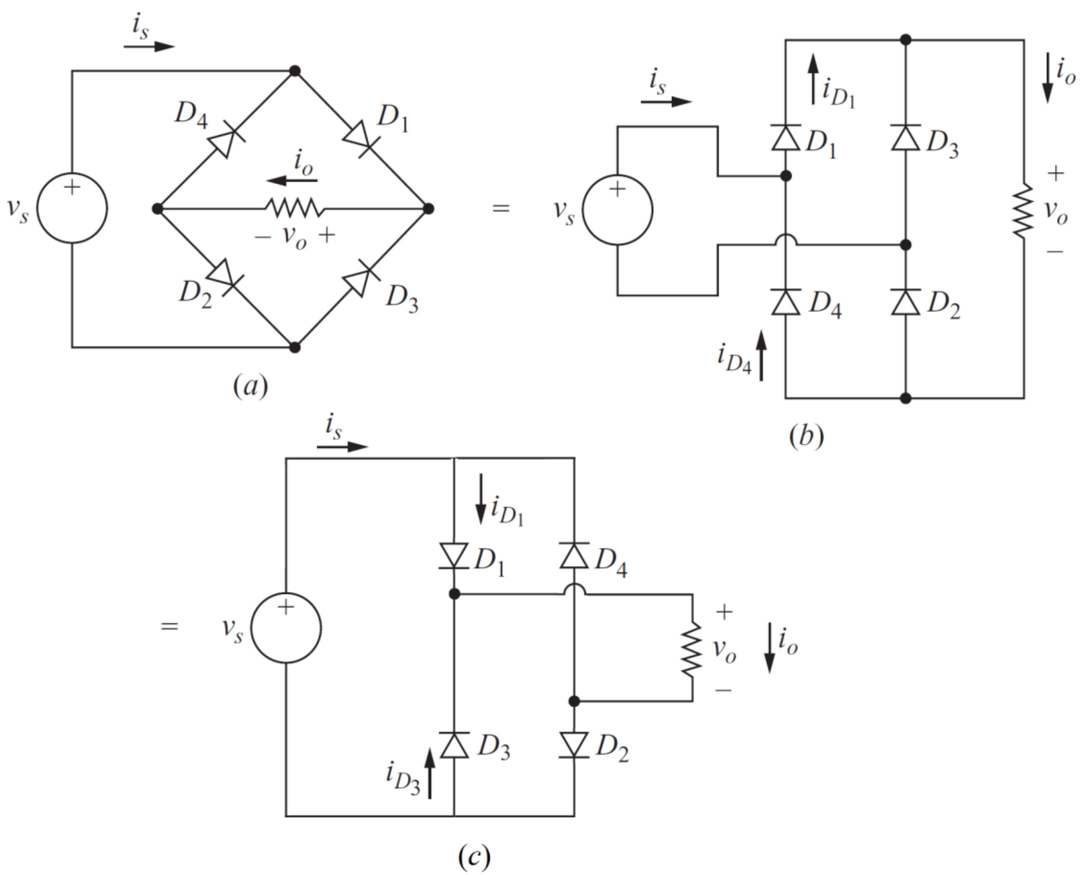
At the end of this course, you will be able to:

generally analyse the uncontrolled and controlled full-wave rectifiers with different loads and apply the power computation concepts from the previous chapter to these circuits.

Full-Wave Rectifiers

Single-phase Full-Wave Rectifier:

- The objective of a full-wave rectifier is to produce a voltage or current that is **purely dc** or has **some specified dc component**.
- The source is utilized in **entire cycle** (not half a cycle as used in half-wave rectifiers).



H-bridge rectifiers

Figure 4-1: Full-wave bridge rectifier. (*a*) Circuit diagram. (*b&c*) Alternative representation.

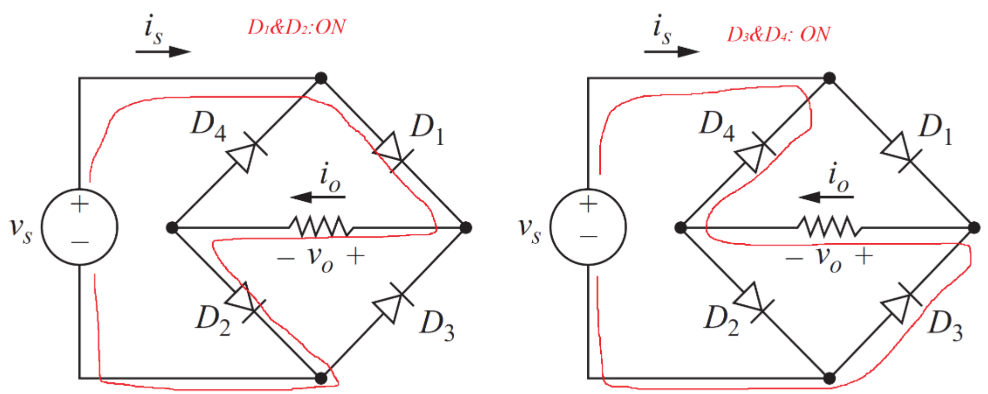


Figure 4-2: Voltages and currents of full-wave bridge rectifier.

1. Diodes D_1 and D_2 conduct together, and D_3 and D_4 conduct together. Kirchhoff's voltage shows that D_1 and D_3 cannot be ON at the same time (similarly, D_2 and D_4). The load current can be positive or zero but can never be negative.

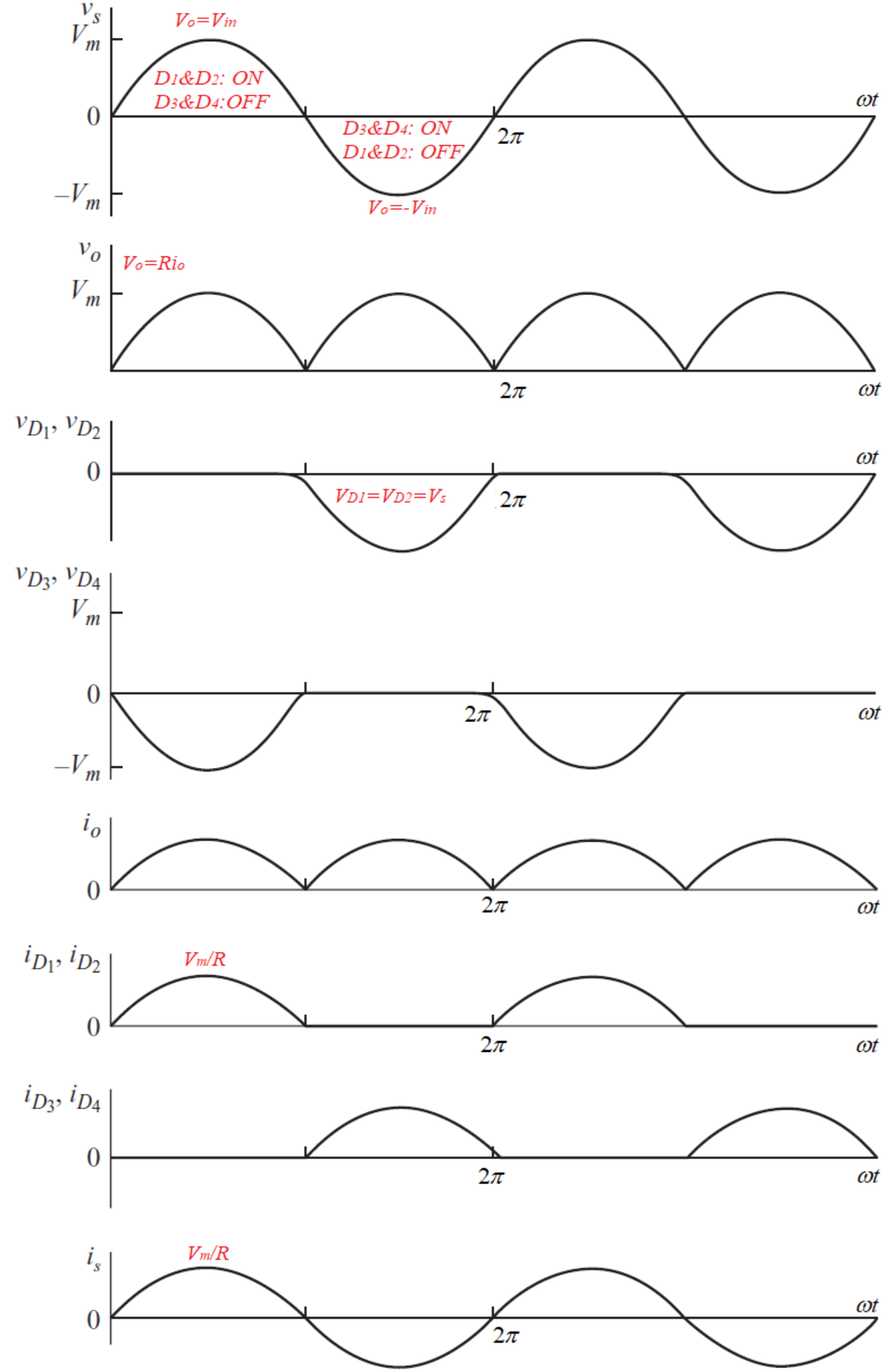
2. The voltage across the load is $+v_s$ when D_1 and D_2 are ON. The voltage across the load is $-v_s$ when D_3 and D_4 are ON.

3. The maximum voltage across a reverse-biased diode is the peak value of the source. This can be obtained by KVL around the loop containing the source, D_1, and D_3. With D_1 ON, the voltage across D_3 is $-v_s$.

4. The current entering the bridge from the source is i_{D1}-i_{D4}, which is symmetric about zero. Therefore, the average source current is zero.

5. The rms source current is the same as the rms load current. The source current is the same as the load current for one-half of the source period and is the negative of the load current for the other half. The squares of the load and source currents are the same, so the rms currents are equal.

6. The fundamental frequency of the output voltage is 2ω where ω is the frequency of the ac input since two periods of the output occur for every period of the input.



$$V_o(\omega t) = \left|V_{in}(\omega t)\right|$$

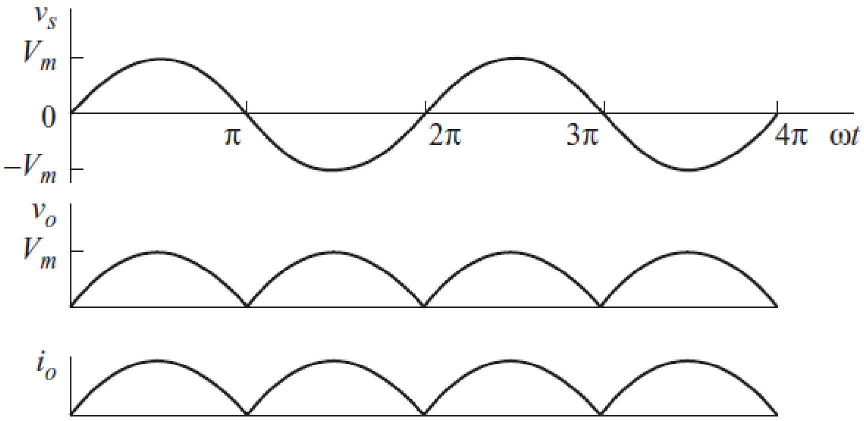
$$\langle V_o(\omega t)\rangle = \frac{1}{2\pi}\int_0^{2\pi}\left|V_m \operatorname{Sin}(\omega t)\right| d(\omega t) = \frac{2}{2\pi}\int_0^{\pi} V_m \operatorname{Sin}(\omega t) d(\omega t) = \frac{2V_m}{\pi}$$

$$\langle i_o(\omega t)\rangle = \frac{\langle V_o(\omega t)\rangle}{R} = \frac{2V_m}{\pi R}$$

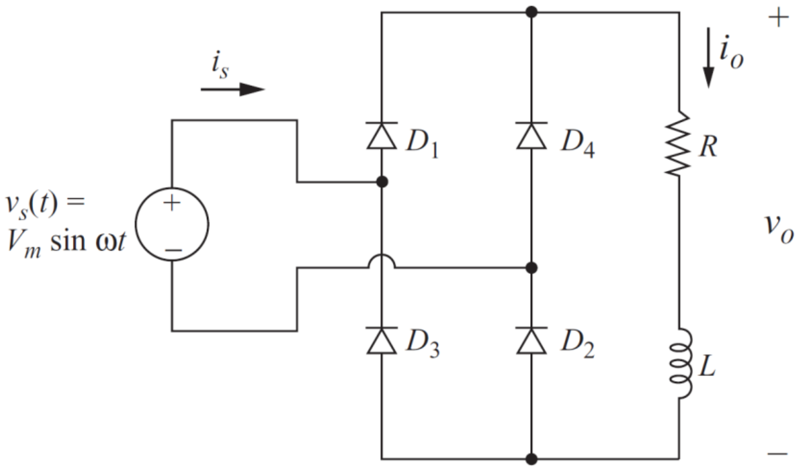
$$V_{o,rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left|V_m \operatorname{Sin}(\omega t)\right|^2 d(\omega t)} = \sqrt{\frac{1}{2\pi}\times\frac{V_m^2}{2}\times 2\pi} = \frac{V_m}{\sqrt{2}}$$

$$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{V_m}{\sqrt{2}R} = \frac{I_m}{\sqrt{2}}$$

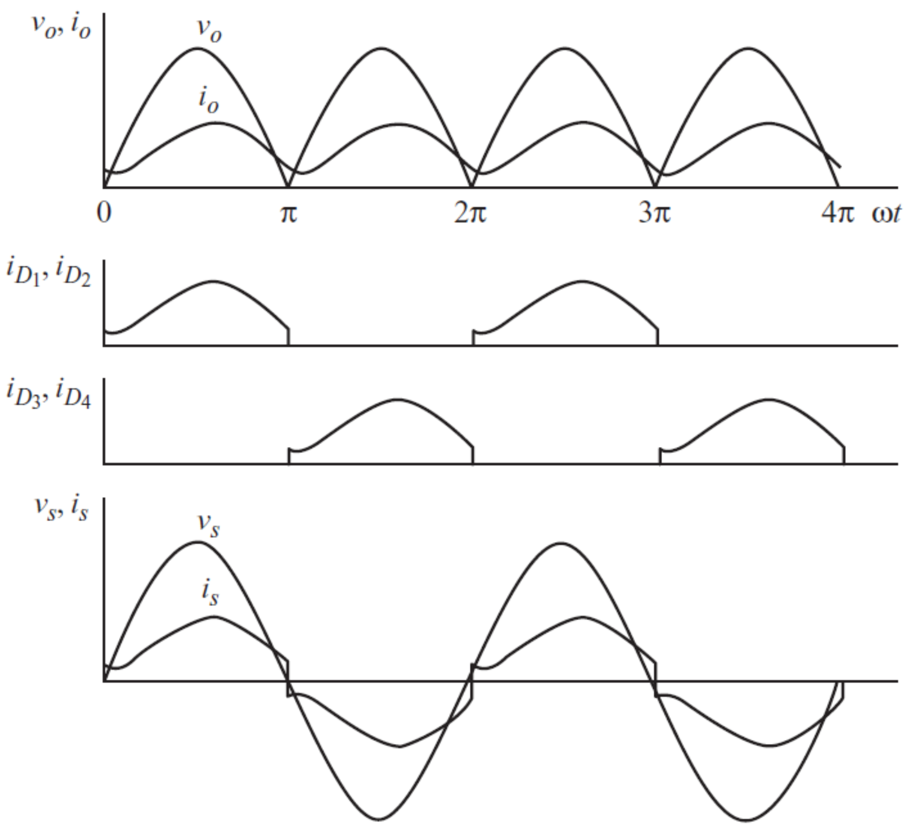
$$pf = \frac{\text{average output power}}{\text{apparent input power}} = \frac{R\times I_{o,rms}{}^2}{V_{in,rms}\times I_{in,rms}} = \frac{R\times(\frac{V_m}{\sqrt{2}R})^2}{\frac{V_m}{\sqrt{2}}\times\frac{V_m}{\sqrt{2}R}} = 1$$



Full-Bridge Rectifier with R-L load:



(a)



(b)

Figure 4-3 (a) Bridge rectifier with an RL load; and (b) Voltages and currents.

$$if\ \ v_s > 0 \rightarrow \begin{cases} D_1 \,\&\, D_2 \text{ are } ON \\ D_3 \,\&\, D_4 \text{ are } OFF \end{cases}$$

$$if\ \ v_s < 0 \rightarrow \begin{cases} D_1 \,\&\, D_2 \text{ are } OFF \\ D_3 \,\&\, D_4 \text{ are } ON \end{cases}$$

The v_o will be absolute value of v_s ($v_o = |v_s|$)

$$v_o = L\frac{di_o}{dt} + Ri_o \rightarrow |V_m \,\text{Sin}\, \omega t| = L\frac{di_o}{dt} + Ri_o$$

Full-Bridge Rectifier with R-C load:

$$if\ v_s > 0 \rightarrow \begin{cases} D_1 \,\&\, D_2 \text{ are } ON \\ D_3 \,\&\, D_4 \text{ are } OFF \end{cases}$$

$$if\ v_s < 0 \rightarrow \begin{cases} D_1 \,\&\, D_2 \text{ are } OFF \\ D_3 \,\&\, D_4 \text{ are } ON \end{cases}$$

The analysis proceeds exactly as for the half-wave rectifier.

$$v_o(\omega t) = \begin{cases} |V_m \sin \omega t| & \text{one diode pair on} \\ (V_m \sin \theta) e^{-(\omega t - \theta)/\omega RC} & \text{diodes off} \end{cases}$$

Θ is the angle where the diodes become reverse biased

$$\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi$$

At that boundary point:

$$(V_m \sin \theta) e^{-(\pi + \alpha - \theta)/\omega RC} = -V_m \sin(\pi + \alpha)$$

or

$$(\sin \theta) e^{-(\pi + \alpha - \theta)/\omega RC} - \sin \alpha = 0$$

The peak-to-peak voltage variation, or ripple is:

$$\Delta V_o = V_m - |V_m \sin(\pi + \alpha)| = V_m(1 - \sin \alpha)$$

In practical circuits where RCW≫ π:

$$\Delta V_o \approx \frac{V_m \pi}{\omega RC} = \frac{V_m}{2fRC}$$

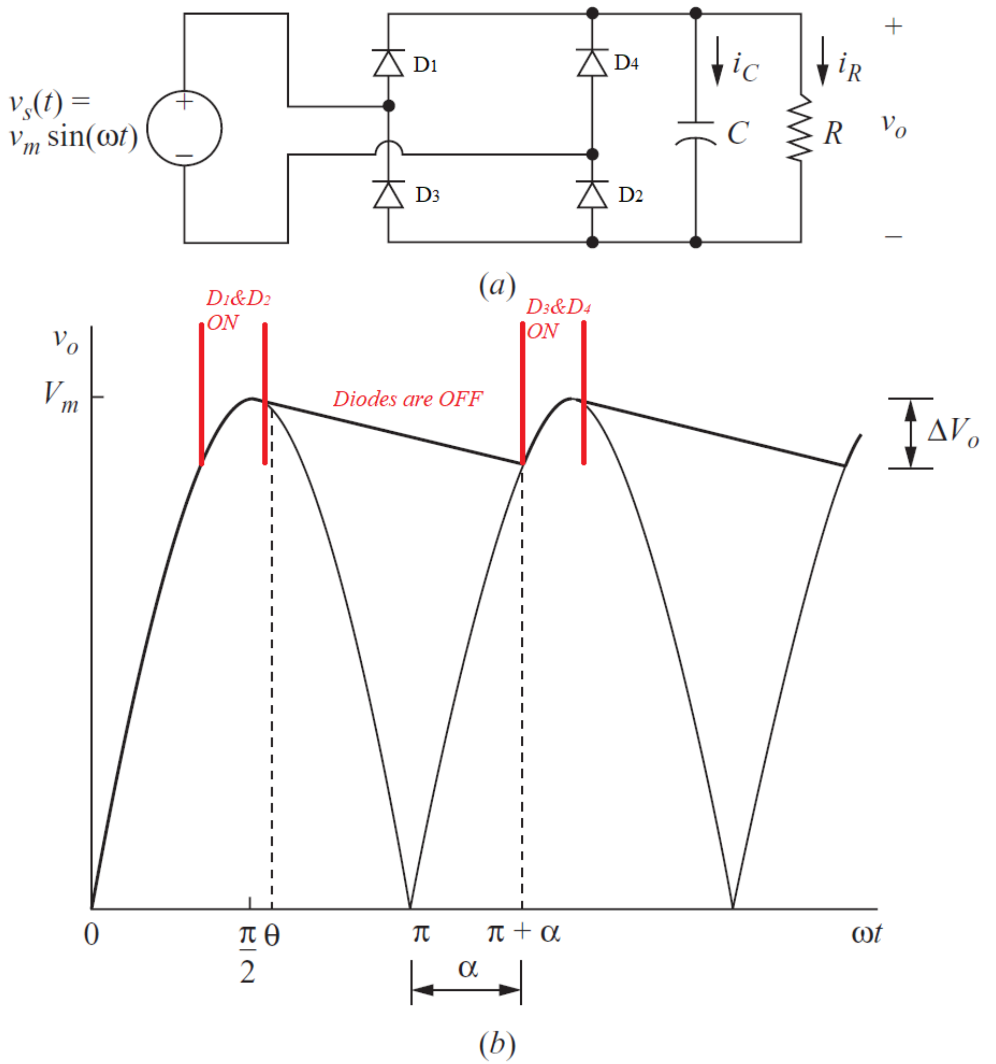


Figure 4-4 (*a*) Full-wave rectifier with capacitance filter; (*b*) Source and output voltage.

Note that the approximate peak-to-peak ripple voltage for the full-wave rectifier is one-half that of the half-wave rectifier.

Example: The full-wave rectifier of Fig. 4-4(*a*) has a 120 V source at 60 Hz, R=500 Ω, and C=100 μF. (*a*) Determine the peak-to-peak voltage variation of the output. (*b*) Determine the value of capacitance that would reduce the output voltage ripple to 1percent of the dc value.

Solution

$$V_m = 120\sqrt{2} = 169.7\ V$$

$$\omega RC = (2\pi \times 60)(500)(10)^{-6} = 18.85$$

The angle θ is determined as: $\theta = \text{-}\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93^o$

$$V_m \text{ Sin}\,\theta = 169.7 \times \text{Sin}(93^o) = 169.5 \text{ V}$$

The angle α is determined by the numerical solution of following equation:

$$(\text{Sin}\,\theta)\text{e}^{-(\pi+\alpha-\theta)/\omega RC} - \text{Sin}\,\alpha = 0 \ \Rightarrow\ \text{Sin}(1.62)\text{e}^{-(\pi+\alpha-1.62)/18.85} - \text{Sin}\,\alpha = 0$$

$$\alpha = 1.06 \text{ rad=}60.6^o$$

(*a*) Peak-to-peak output voltage is described as:

$$\Delta\text{Vo=}V_m(1-\text{Sin}\,\alpha) = 169.7\left[1-\text{Sin}(1.06)\right] = 22\ V$$

Note that this ripple value is half of ripple value in a half-wave rectifier.

(b) With the ripple limited to 1 percent, the output voltage will be held close to V_m.

$$\Delta V_o \approx \frac{V_m}{2fRC} \ \Rightarrow\ \frac{\Delta V_o}{V_m} = 0.01 \approx \frac{1}{2fRC} \ \Rightarrow\ C \approx \frac{1}{2fR(\frac{\Delta V_o}{V_m})} = \frac{1}{2(60)(500)(0.01)} = 1670\mu F$$

Controlled Full-Bridge Rectifier:

Controlling is possible by substituting controlled switches such as thyristors (SCRs) for the diodes.

I_{G1} and I_{G2} are sent to S_1 and S_2 and turn them ON at the same time. With 180 degree phase shift triggering happening for I_{G3} and I_{G4}.

Average output voltage is:

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1 + \cos\alpha)$$

Average output current is then:

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)} = \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The rms current in the source is the same as the rms current in the load.

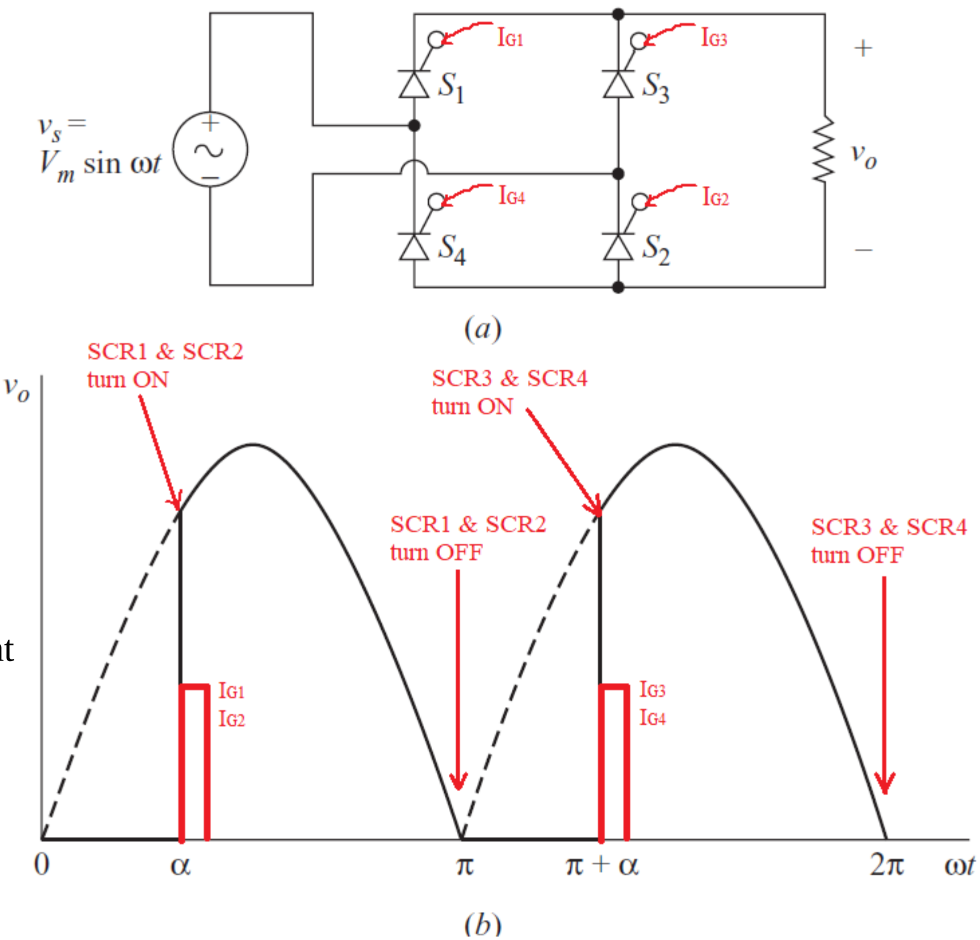


Figure 4-5 (*a*) Controlled full-wave bridge rectifier; (*b*) Output for a resistive load.

Example: The full-wave controlled bridge rectifier of Fig. 4-5 (*a)* has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40 degree. Determine the average current in the load, the power absorbed by the load, and the source voltamperes

Solution:

The average output voltage is determined from

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2\,(20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

Controlled Full-Bridge Rectifier R-L load:

Case 1: Inductor is small or firing angle α is large, so the output current is discontinuous. It means that it starts from zero, goes somewhere up and comes back to zero and remains there for the next half of the cycle to repeat.

Case 2: Inductor is large or firing angle α is small, so the output current is continuous and current continuously flow though in load (before the current reaches zero, the next triggering starts).

How do we understand whether we are in continuous mode or in discontinuous mode?

$$\text{if } \alpha < \tan^{-1}\left(\frac{L\omega}{R}\right) \Rightarrow \text{ Continuous Current Happens}$$

otherwise: Discontinuous Current Happens

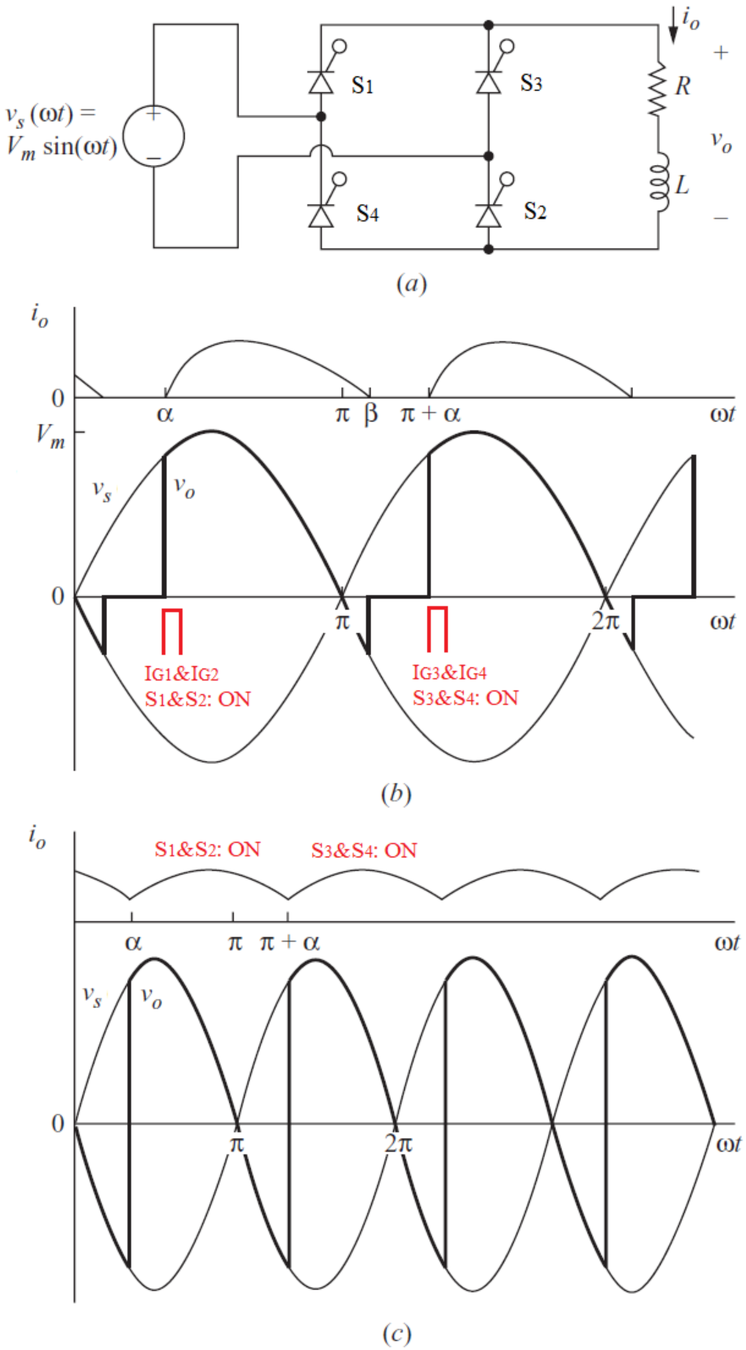


Figure 4-6 (*a*) Controlled rectifier with *RL* load; (*b*) Discontinuous current; (*c*) Continuous current.

Three-Phase Rectifiers:

Three-phase rectifiers are commonly used in industry to produce a dc voltage and current for large loads.

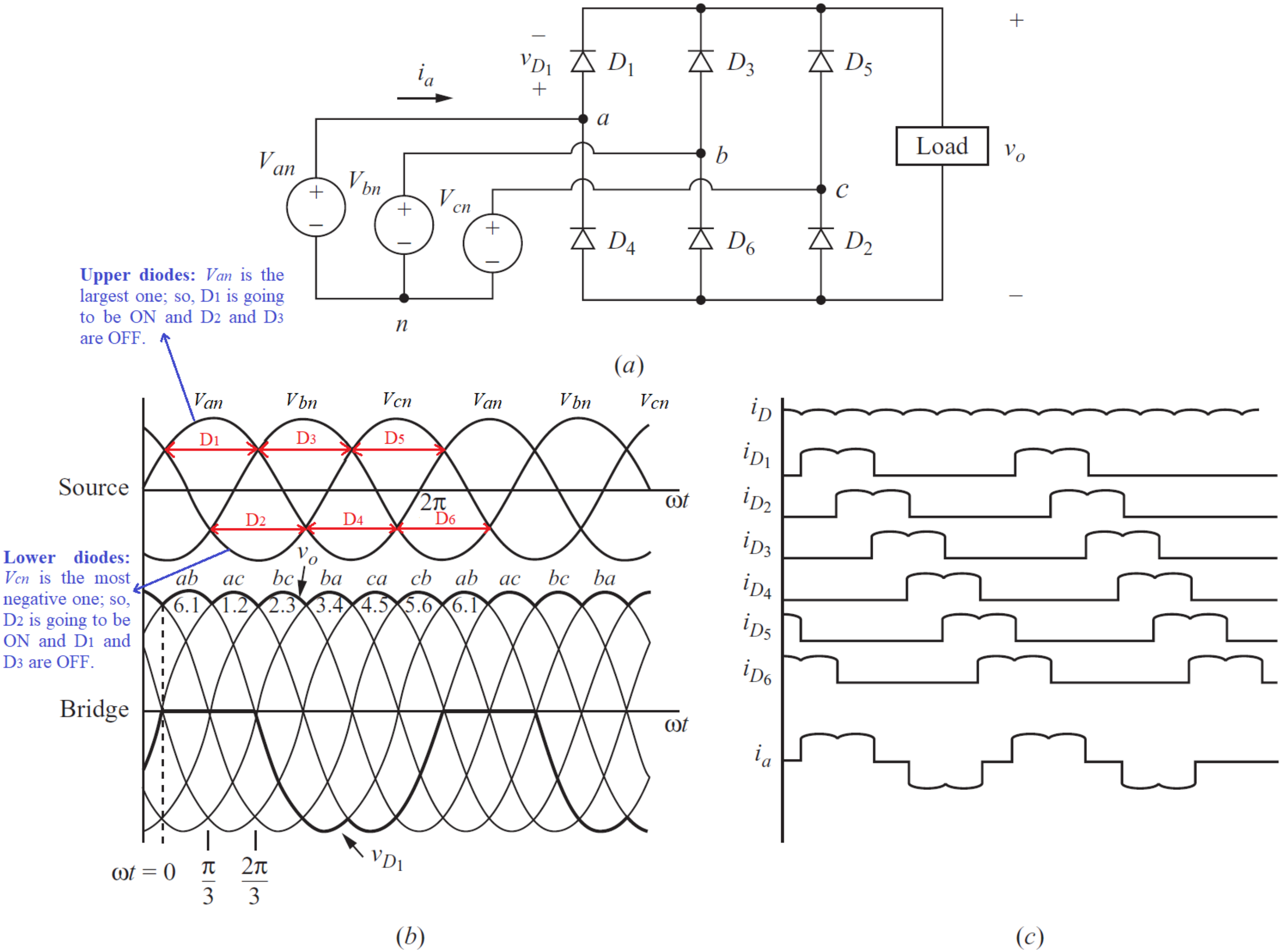


Figure 4-7 (*a*) Three-phase full-bridge rectifier; (*b*) Source and output voltages; (*c*) Currents for a resistive load.

Three-Phase Rectifiers:

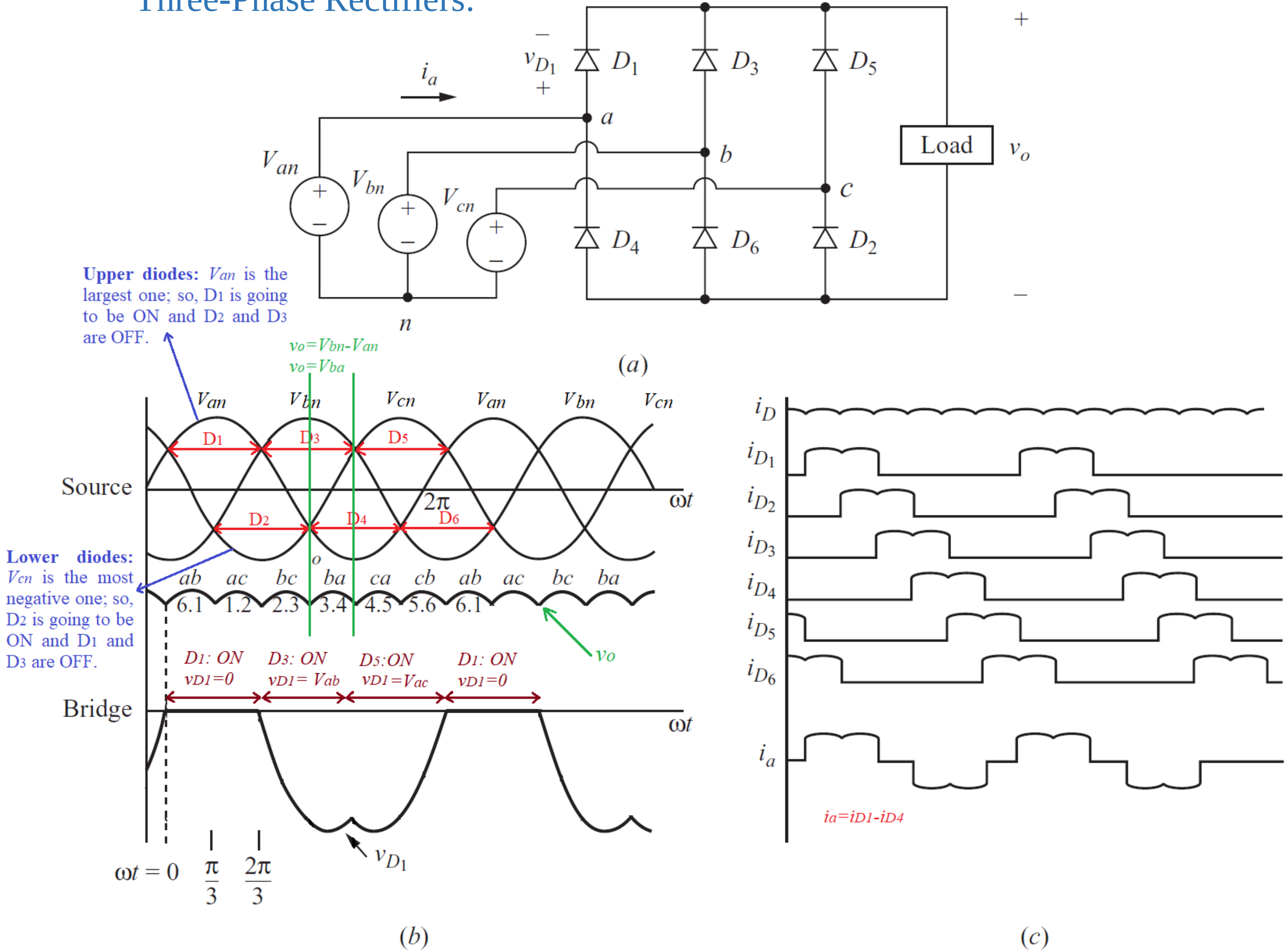
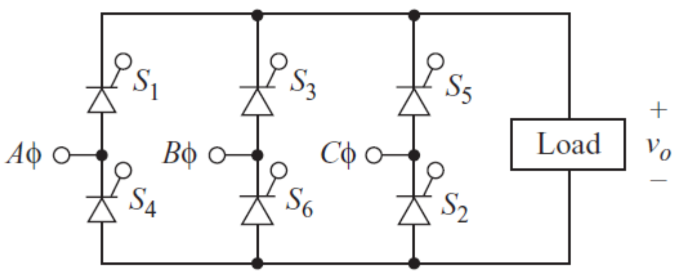


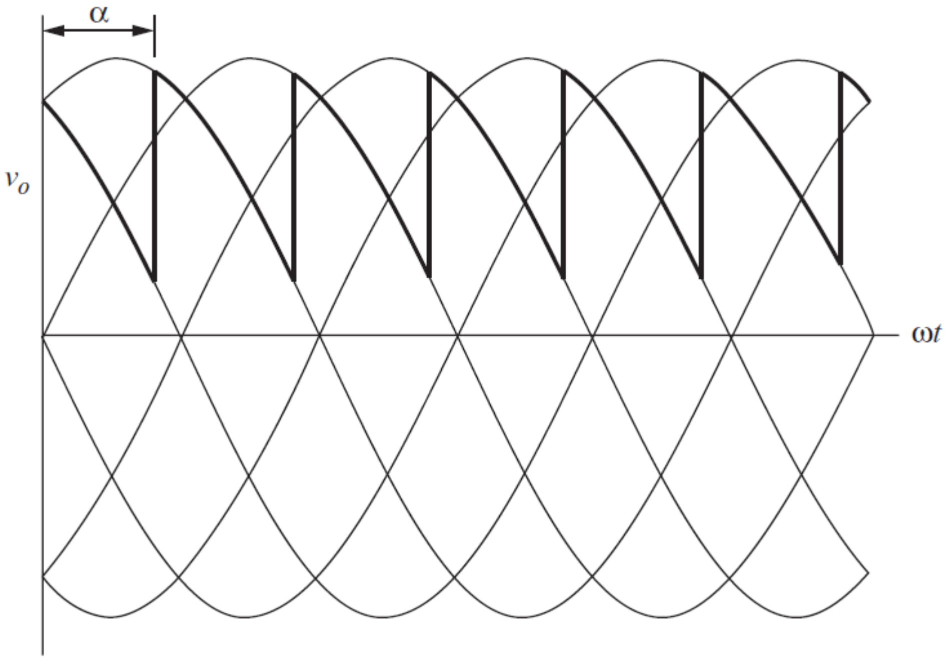
Figure 4-8 (*a*) Three-phase full-bridge rectifier; (*b*) Source and output voltages; (*c*) Currents for a resistive load.

Controlled Three-Phase Rectifier:

The output of the three-phase rectifier can be controlled by substituting SCRs for diodes.



(a)



(b)

Figure 4-9 (*a*) A controlled three-phase rectifier; (*b*) Output voltage for α= 45.

Questions and comments are most welcome!