Aalto University
School of Engineering

Power Electronics
ELEC-E8412 Power Electronics, 5 ECTS

By: Edris Pouresmaeil

Department of Electrical Engineering and Automation (EEA)
Aalto University, 02150 Espoo, Finland

Office 3563, Maarintie 8

E-Mail : edris.pouresmaeil@aalto.fi

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Chapter 6:
dc/ac Inverter

The Main Objectives of this Session:

At the end of this session you will be able to:

- analyze operation of full-bridge inverter
- use pulse-width modulation (PWM) to synthesize the desired output voltage of the basic converter topologies
- analyze 3-phase inverter waveforms

Introduction:

Inverters are circuits that convert **dc** to **ac**. In this application, an **ac output** is synthesized from a **dc input** by closing and opening the switches in an appropriate sequence.

Applications:

- Inverters are used in applications such as **adjustable-speed ac motor drives**, **uninterruptible power supplies (UPS), running ac appliances from an automobile battery**, and integration of **renewable energy** sources into the ac grids/loads.

- Assuming you have solar panel installed in roof of your house and locally you use some of the power generated by this generation, but you are going send rest of the power back to the grid. Your source is **dc** and load (grid) is **ac**. In this situation, an **inverter** is need to convert power from **dc** to **ac**.

- Assuming you are going to inject power from wind power source to the grid. Wind speed is not constant, then the frequency is not constant. Therefore, we need to rectify it from ac to dc. Then, we need an inverter to provide a fixed frequency power for grid.

THE FULL-BRIDGE CONVERTER

The output voltage (v_o) can be $+V_{dc}$, $-V_{dc}$, or zero, depending on which switches are closed.

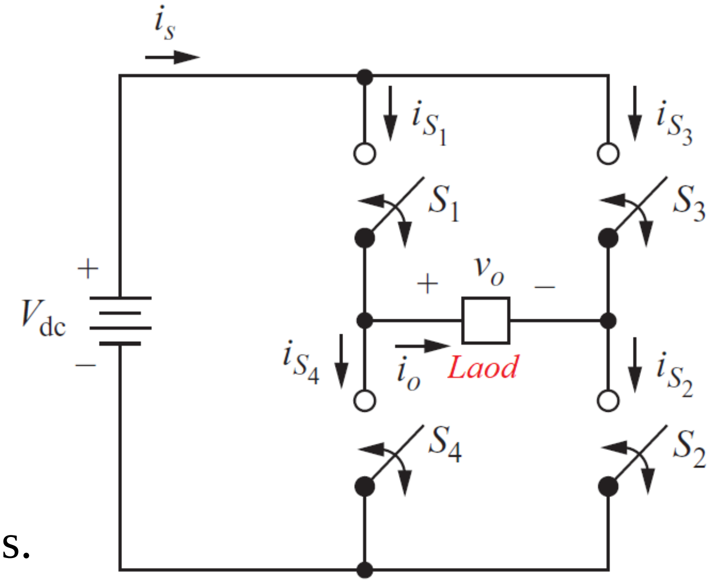


Figure 6-1: Full-bridge converter.

Loads can be:

- Resistor
- Winding of an electric motor (R-L load)
- Power Grid

Operation:

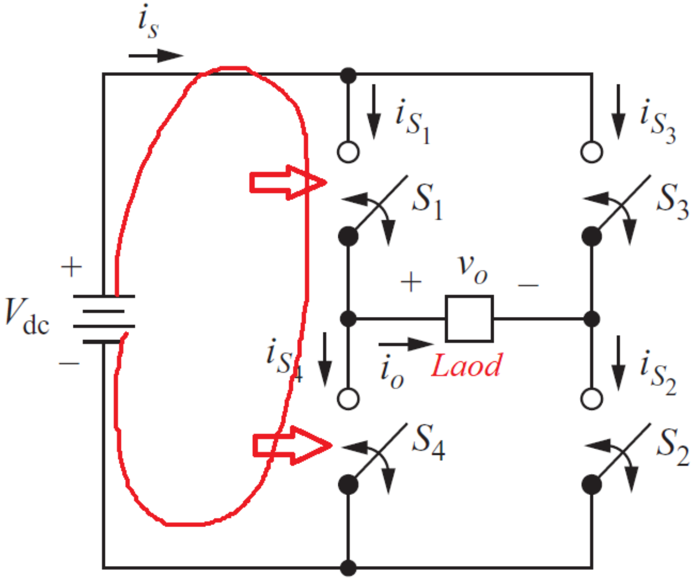
Each switch has two possible states (On or Off).

They are 4 switches, so they are 2×2×2×2=16 possibilities.

Some of these possibilities are not viable possibilities, for example it is not possible to turn on S_1 and S_4 at the same time. If so, a short circuit path will be provided for dc source.

They are only 4 states which make sense.

Switches Closed	Output Voltage v_o
S_1 and S_2	$+V_{dc}$
S_3 and S_4	$-V_{dc}$
S_1 and S_3	0
S_2 and S_4	0



S_1: ON S_2: ON → $v_o = V_{dc} = V_{in}$

S_3: ON S_4: ON → $v_o = -V_{dc} = -V_{in}$

S_1: ON S_3: ON → $v_o = 0$

S_2: ON S_4: ON → $v_o = 0$

- ❖ It is up to modulation scheme (switching scheme) to choose any of these modes.
- ❖ Note that S_1 and S_4 should not be closed at the same time, nor should S_2 and S_3. Otherwise, a short circuit would exist across the dc source.

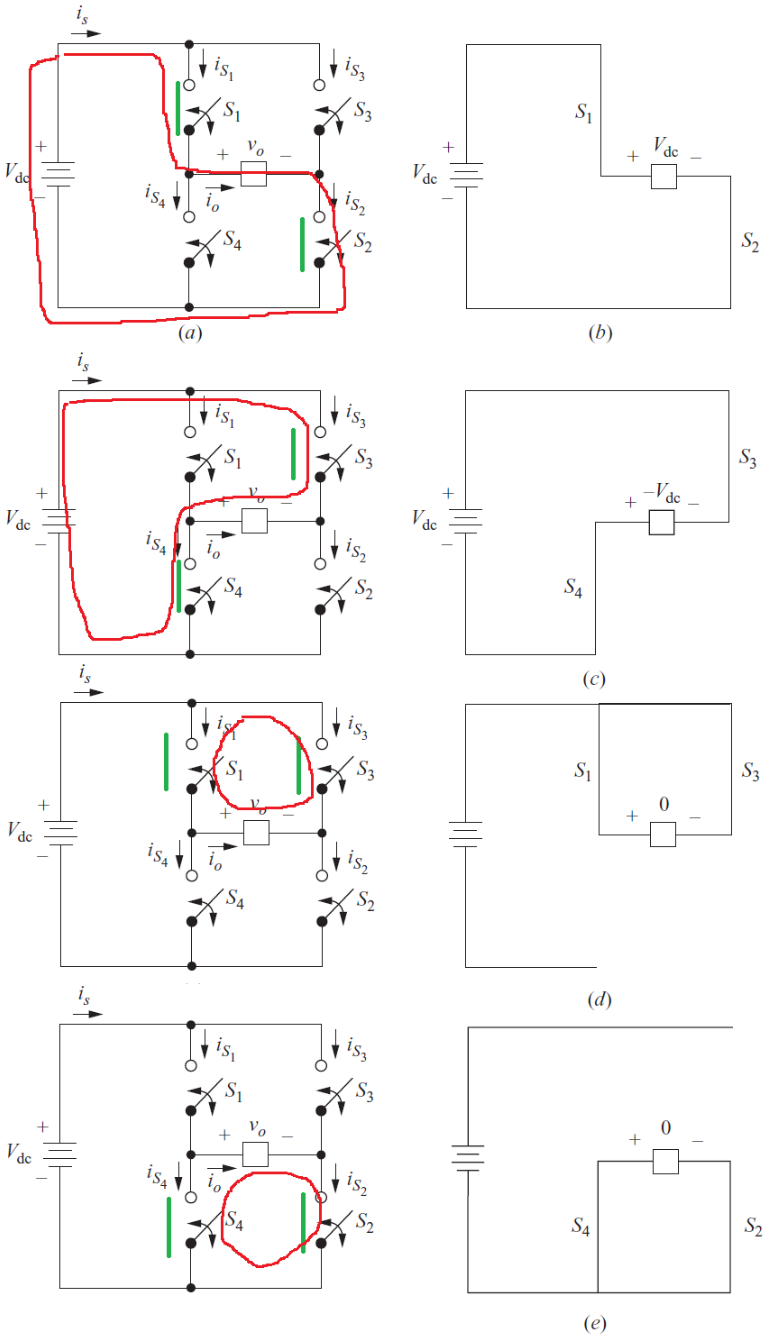


Figure 6-2 (a) Full-bridge converter; (b) S_1 and S_2 closed; (c) S_3 and S_4 closed; (d) S_1 and S_3 closed; (e) S_2 and S_4 closed.

THE SQUARE-WAVE INVERTER

The switches connect the load to +Vdc when S1 and S2 are closed or to -Vdc when S3 and S4 are closed. The periodic switching of the load voltage between +Vdc and -Vdc produces a square wave voltage across the load.

Mode I: S_1 & S_2: ON $\quad 0 < t < \frac{T}{2} \Rightarrow v_o = +V_{in}$

Mode II: S_3 & S_4: ON $\quad \frac{T}{2} < t < T \Rightarrow v_o = -V_{in}$

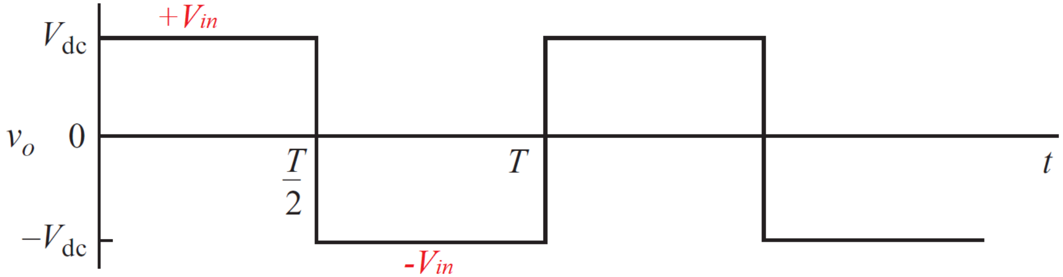
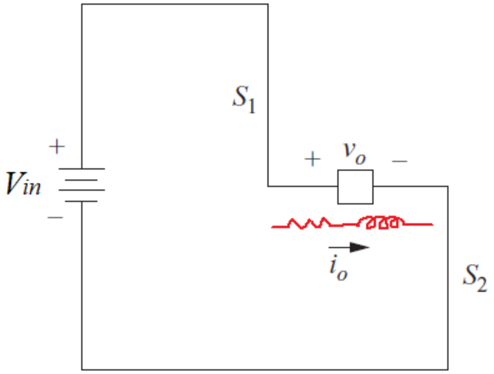


Figure 6-3 Square wave output voltage

Square-Wave Inverter with R-L load:

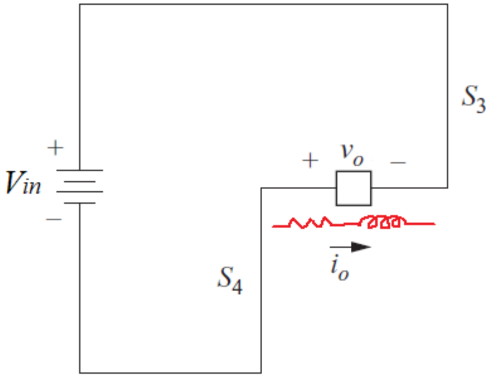
Mode I: S_1 & S_2: ON

$$Ri_o(t) + L\frac{di_o(t)}{dt} = V_{in} \Rightarrow i_o(t) = \frac{V_{in}}{R} + Ae^{-t/\tau} \quad (\tau = \frac{L}{R}) \qquad (1)$$



Mode II: S_3 & S_4: ON

$$Ri_o(t) + L\frac{di_o(t)}{dt} = -V_{in} \Rightarrow i_o(t) = -\frac{V_{in}}{R} + Be^{-(t-\frac{T}{2})/\tau} \quad (\tau = \frac{L}{R}) \qquad (2)$$



What are the values of **A** and **B**?

Assuming we are in steady state operation condition, when we start first mode, current starts from the minimum value and raises to the maximum value. Because this is periodic, it will be backed to the minimum value again in the next cycle.

$$i_o(0)=\mathrm{I}_{\min}\ ;\ i_o(\frac{T}{2})=\mathrm{I}_{\max};\ i_o(T)=\mathrm{I}_{\min}$$

$$\left.\begin{matrix} i_o(0)=\mathrm{I}_{\min} \\ (1) \end{matrix}\right\} \Rightarrow i_o(0)=\frac{V_{in}}{R}+Ae^{-0/\tau} \Rightarrow \mathrm{I}_{\min}=\frac{V_{in}}{R}+A \Rightarrow A=\mathrm{I}_{\min}-\frac{V_{in}}{R} \qquad A<0$$

$$\left.\begin{matrix} i_o(\frac{T}{2})=\mathrm{I}_{\max} \\ (2) \end{matrix}\right\} \Rightarrow i_o(\frac{T}{2})=-\frac{V_{in}}{R}+Be^{-(\frac{T}{2}-\frac{T}{2})/\tau} \Rightarrow \mathrm{I}_{\max}=-\frac{V_{in}}{R}+B \Rightarrow B=\frac{V_{in}}{R}+\mathrm{I}_{\max} \qquad B>0$$

$$i_o(t)=\begin{cases} \dfrac{V_{in}}{R}+(\mathrm{I}_{\min}-\dfrac{V_{in}}{R})e^{-t/\tau} & 0<t<\dfrac{T}{2} \\ \dfrac{-V_{in}}{R}+(\mathrm{I}_{\max}+\dfrac{V_{in}}{R})e^{-(t-\frac{T}{2})/\tau} & \dfrac{T}{2}<t<T \end{cases} \qquad (3)$$

Eq. (1) describes the operation in First Mode and Eq. (2) describes the operation in Second Mode. There is an overlap between these two operating modes which is at t=T/2.

$$i_o(\frac{T}{2})=\underbrace{\frac{V_{in}}{R}+Ae^{-\frac{T}{2}/\tau}}_{(1)}=\underbrace{-\frac{V_{in}}{R}+Be^{-(\frac{T}{2}-\frac{T}{2})/\tau}=-\frac{V_{in}}{R}+Be^{0}}_{(2)} \qquad (4)$$

Under steady state operation condition, the currents in beginning of cycle is equal to ending of cycle.

$$i_o(0)=i_o(T)=\underbrace{\frac{V_{in}}{R}+Ae^{0}}_{(1)}=\underbrace{-\frac{V_{in}}{R}+Be^{-(T-\frac{T}{2})/\tau}}_{(2)} \qquad (5)$$

$$\left.\begin{array}{l}\underbrace{\frac{2V_{in}}{R}+Ae^{-\frac{T}{2}/\tau}=B}_{(4)}\\ \underbrace{-\frac{2V_{in}}{R}+Be^{-\frac{T}{2}/\tau}=A}_{(5)}\end{array}\right\}(4)+(5)\Rightarrow Ae^{-\frac{T}{2}/\tau}+Be^{-\frac{T}{2}/\tau}=A+B\Rightarrow(A+B)e^{-\frac{T}{2}/\tau}=A+B \qquad (6)$$

Eq. (6) is possible if A=-B.

From Eq.s (1) and (2):

$$\left.\begin{array}{l}\mathrm{I}_{\min}=A+\frac{V_{in}}{R}=\\ \mathrm{I}_{\max}=B-\frac{V_{in}}{R}\end{array}\right\}\xrightarrow{A=-B}\left.\begin{array}{l}\mathrm{I}_{\min}=A+\frac{V_{in}}{R}=\\ \mathrm{I}_{\max}=-A-\frac{V_{in}}{R}\end{array}\right\}\Rightarrow\mathrm{I}_{\min}=-\mathrm{I}_{\max} \qquad (7)$$

Substituting $I_{\max}$ for $I_{\min}$ in Eq. (3) and solving for $I_{\max}$,

$$\mathrm{I}_{\max}=-\mathrm{I}_{\min}=\frac{V_{in}}{R}\left(\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right) \qquad (8)$$

- In the first mode the current of switches and the current of output are the same, otherwise they are zero. In fact, the current in second mode is equal to zero for switches 1 and 2.
- In the second mode the current of switches and the current of output are opposite, otherwise they are zero. In fact, the current in first mode is equal to zero for switches 3 and 4.
- Source current is sum of switches currents.

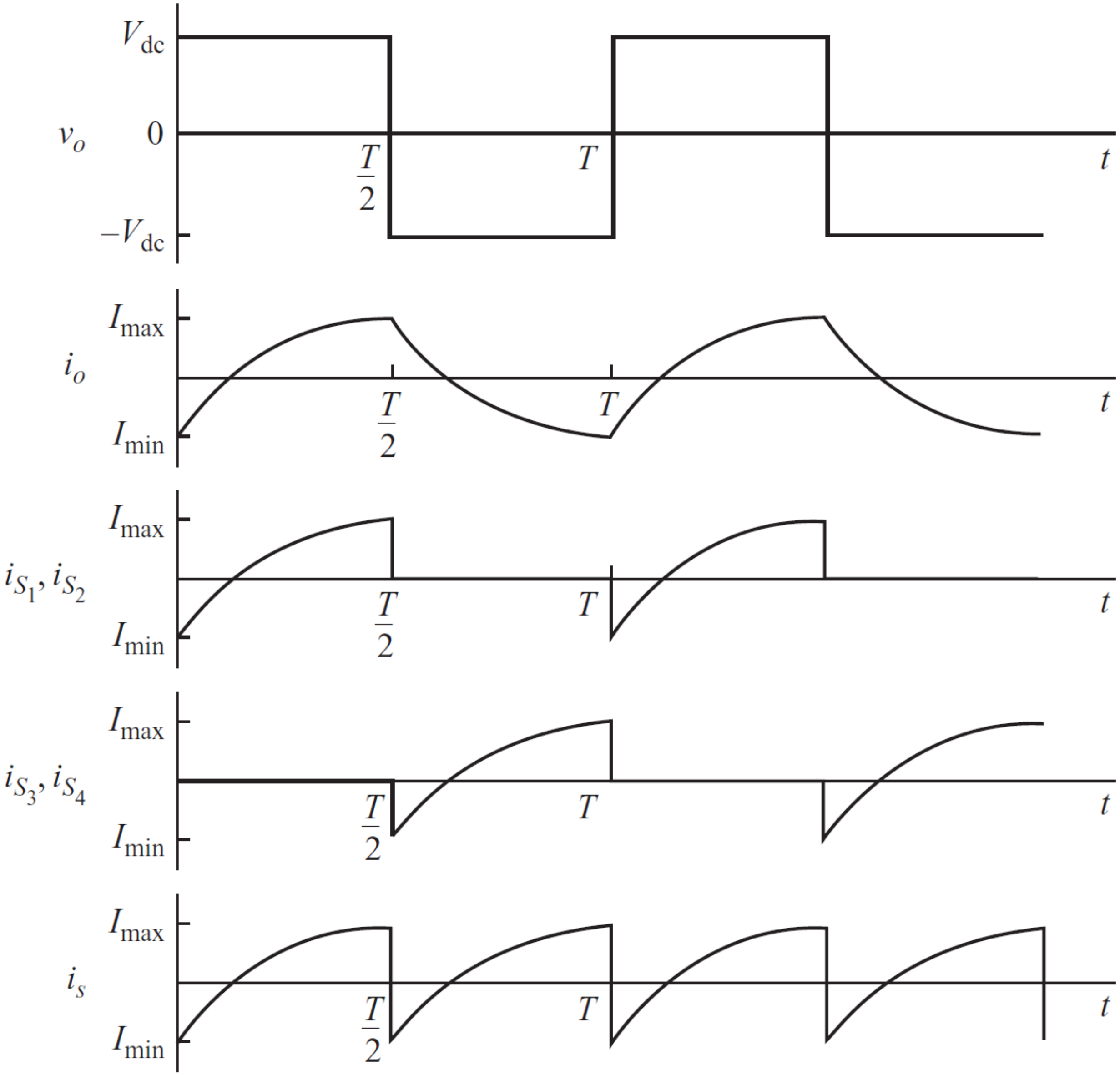


Figure 6-4 Square wave output voltage and steady-state current waveform for an *RL* load.

Power absorbed by the load can be determined from RI_{rms}^2.

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,d(t)} = \sqrt{\frac{2}{T}\int_0^{T/2} \left[\frac{V_{\text{dc}}}{R} + \left(I_{\text{min}} - \frac{V_{\text{dc}}}{R}\right)e^{-t/\tau}\right]^2 dt} \tag{9}$$

If the switches are ideal, the power supplied by the source must be the same as absorbed by the load. Power from a dc source is determined from:

$$P_{dc} = V_{dc} \times I_s \tag{10}$$

Example: The full-bridge inverter of Fig. 6-1 has a switching sequence that produces a square wave voltage across a series *RL* load. The switching frequency is 60 Hz, V_{dc}=100 V, R=10 Ω, and L=25 mH. Determine (*a*) an expression for load current, (*b*) the power absorbed by the load, and (*c*) the average current in the dc source.

Solution

(*a*) From the parameters given,

$$T = 1/f = 1/60 = 0.0167s$$

$$\tau = L/R = 0.025/10 = 0.0025s$$

$$T/2\tau = 3.33$$

Equation (8) is used to determine the maximum and minimum current.

$$\mathrm{I}_{\max} = -\mathrm{I}_{\min} = \frac{V_{in}}{R}\left(\frac{1-e^{-T/2\tau}}{1+e^{-T/2\tau}}\right) = \frac{100}{10}\left(\frac{1-e^{-3.33}}{1+e^{-3.33}}\right) = 9.31\ \mathrm{A}$$

Equation (3) is then evaluated to give load current.

$$i_o(t) = \frac{V_{in}}{R} + (\mathrm{I}_{\min} - \frac{V_{in}}{R})e^{-t/\tau} = \frac{100}{10} + (-9.31 - \frac{100}{10})e^{-t/0.0025} = 10 - 19.31e^{-t/0.0025} \qquad 0 \le \mathrm{t} \le \frac{1}{120}$$

$$i_o(t) = \frac{-V_{in}}{R} + (\mathrm{I}_{\max} + \frac{V_{in}}{R})e^{-(t-\frac{T}{2})/\tau} = -\frac{100}{10} + (9.31 + \frac{100}{10})e^{-(t-0.0167/2)/0.0025} = -10 + 19.31e^{-(t-0.00835)/0.0025} \qquad \frac{1}{120} \le \mathrm{t} \le \frac{1}{60}$$

(b) Power absorbed by the load can be determined from RI_{rms}^2, where I_{rms} is computed from Eq. (9).

$$\mathrm{I}_{rms} = \sqrt{120\int_0^{1/120}\left[10 - 19.31e^{-t/0.0025}\right]^2 dt} = 6.64\ \mathrm{A}$$

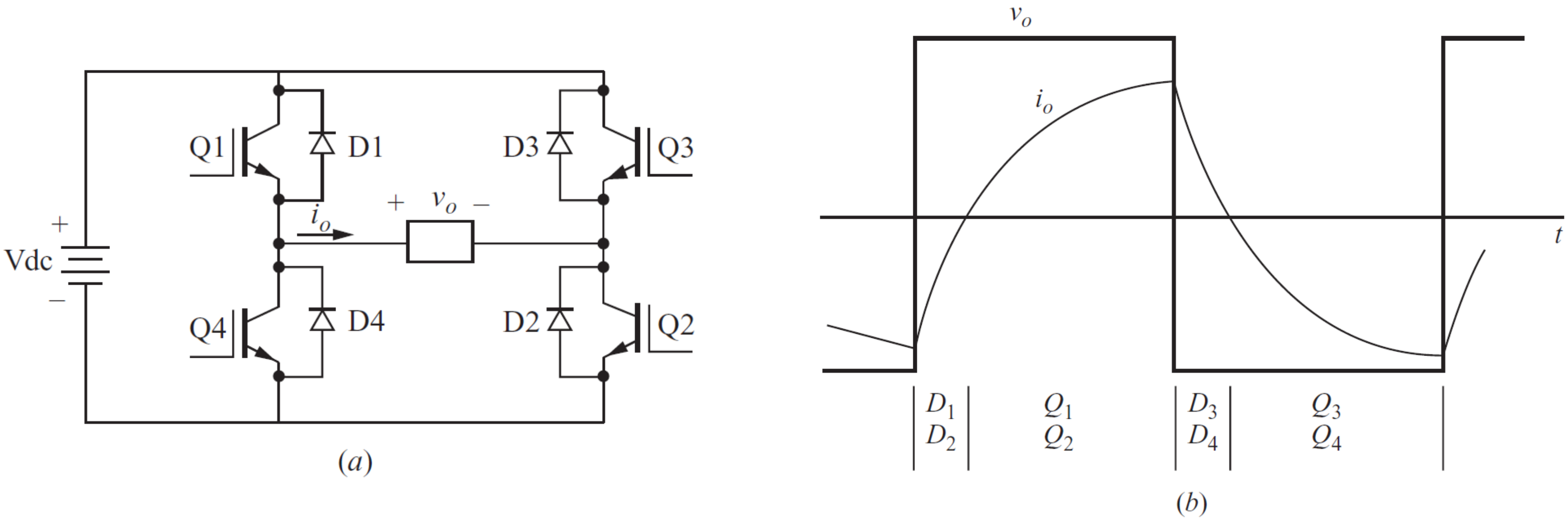
Power absorbed by the load is:

$$P = RI_{rms}^2 = 10 \times (6.64)^2 = 441\ \mathrm{W}$$

(c) Average source current can also be computed by equating source and load power, assuming a lossless converter. Using Eq. (10):

$$I_s = \frac{P_{dc}}{V_{dc}} = \frac{441}{100} = 4.41\ \mathrm{A}$$

The switch currents in Fig. 6-4 show that the switches in the full-bridge circuit must be capable of carrying both positive and negative currents for *RL* loads. However, real electronic devices may conduct current in one direction only. This problem is solved by placing feedback diodes in parallel (antiparallel) with each switch. When the current in the switch must be negative, the feedback diode carries the current.



(*a*) (*b*)

Figure 6-5 (*a*) Full-bridge inverter using IGBTs; (*b*) Steady-state current for an *RL* load.

In mode I, the output current is negative, therefore D1 and D2 are ON. When the output current becomes positive but still conduction is needed, so Q1 and Q2 turn ON.

In mode II, the output current is positive, therefore D3 and D4 are ON. When the output current becomes negative but still conduction is needed, so Q3 and Q4 turn ON.

Q: Draw the current waveform for the first diode (D_1) and the first transistor (Q_1).

PULSE-WIDTH-MODULATED (PWM)

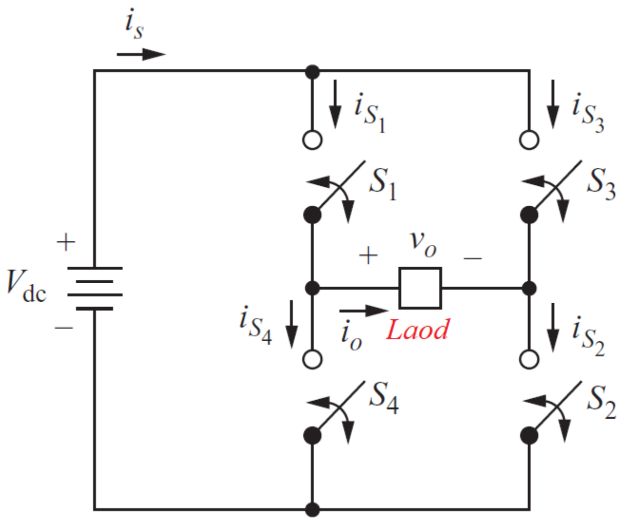
Pulse-width modulation (PWM) provides a way to decrease the total harmonic distortion of load current.

Advantage: The fundamental harmonic is very far from the undesired harmonics; therefore, it will be easy to design a filter and the filter size will be small.

A PWM inverter output, with some filtering, can generally meet THD requirements more easily than the square wave switching scheme.

Disadvantage: The switches should be turned ON and OFF more frequently to generate desired frequency for output signal. This switching frequency is much higher than the output signal frequency. Therefore, the switching losses are higher. In addition, the control circuits are more complex.

- The power stage is not changing, but the switching control change.



Bipolar Switching

They are **two** signals (triangular waveform and a symmetric fixed frequency signal), and the switching decisions are made based the intersections between these two signals.

a) If the triangular waveform is larger than the Sine waveform, S_3 and S_4 turn ON. In fact the negative voltage is applied to the load during this time interval.

b) If the Sine waveform is larger than the triangular waveform, S_1 and S_2 turn ON. In fact the positive voltage is applied to the load during this time interval.

c) The output voltage can be either positive or negative, therefore it is called **Bipolar.**

$$\textit{if}\ \ V_{sine} > V_{tri}\ \Rightarrow S_1\ \&\ S_2\ :\ \mathrm{ON} \Rightarrow V_o = V_{in}$$

$$\textit{if}\ \ V_{sine} < V_{tri}\ \Rightarrow S_3\ \&\ S_4\ :\ \mathrm{ON} \Rightarrow V_o = -V_{in}$$

- The output voltage does not look like sinusoidal and it has a very large harmonic. In fact, it is consisted of fundamental frequency and harmonic frequencies.
- The undesired harmonics are very high frequencies.

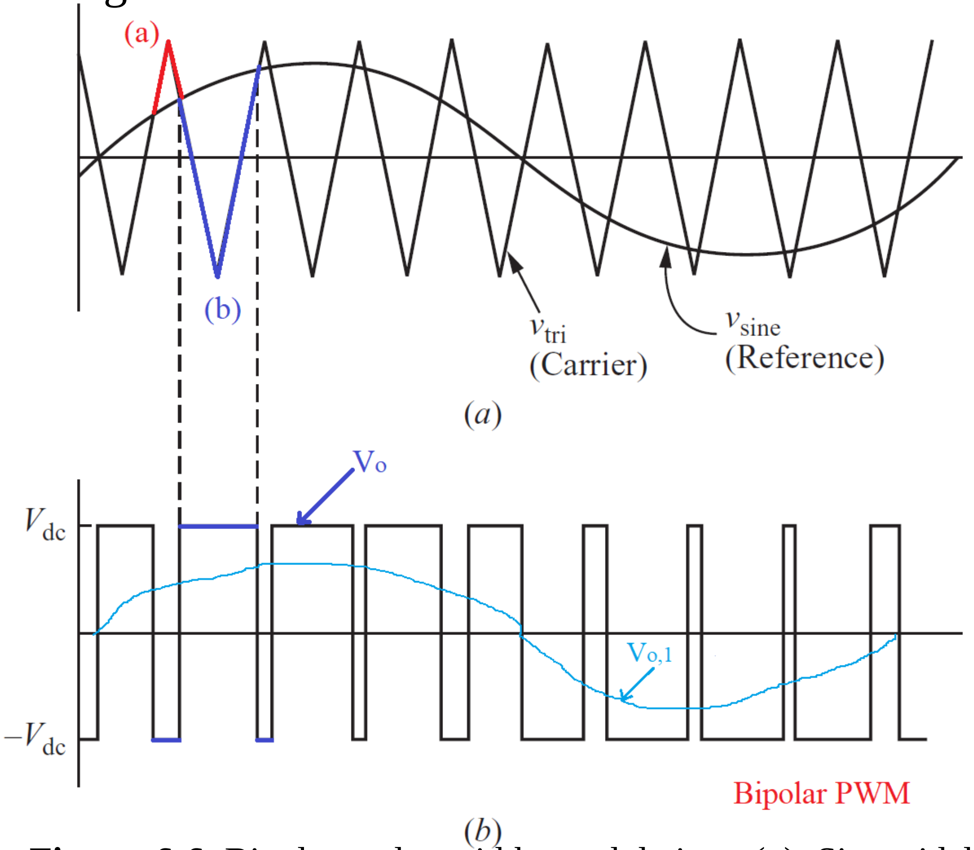


Figure 6-6 Bipolar pulse-width modulation. (*a*) Sinusoidal reference and triangular carrier; (*b*) Output voltage.

Unipolar Switching

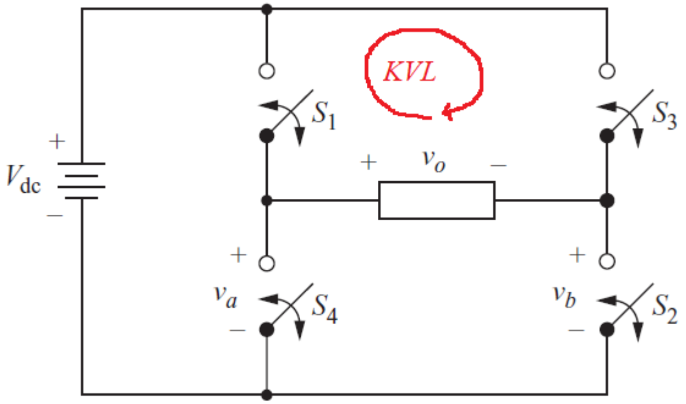
In a unipolar switching scheme for pulse-width modulation, the output is switched either from high to zero or from low to zero, rather than between high and low as in bipolar switching. One unipolar switching scheme has switch controls in Fig. 6-7 as follows:

$$\textit{if}\ \ V_{sine} > V_{tri} \ \Rightarrow S_1 \ : \ \text{ON}$$

$$\textit{if}\ \ V_{sine} < V_{tri} \ \Rightarrow S_4 \ : \ \text{ON}$$

$$\textit{if}\ \ -V_{sine} < V_{tri} \ \Rightarrow S_2 \ : \ \text{ON}$$

$$\textit{if}\ \ -V_{sine} > V_{tri} \ \Rightarrow S_3 \ : \ \text{ON}$$



- This is call **Unipolar** because when the reference is positive, switching is between zero and +Vdc, and when the reference is negative, switching is between zero and -Vdc.

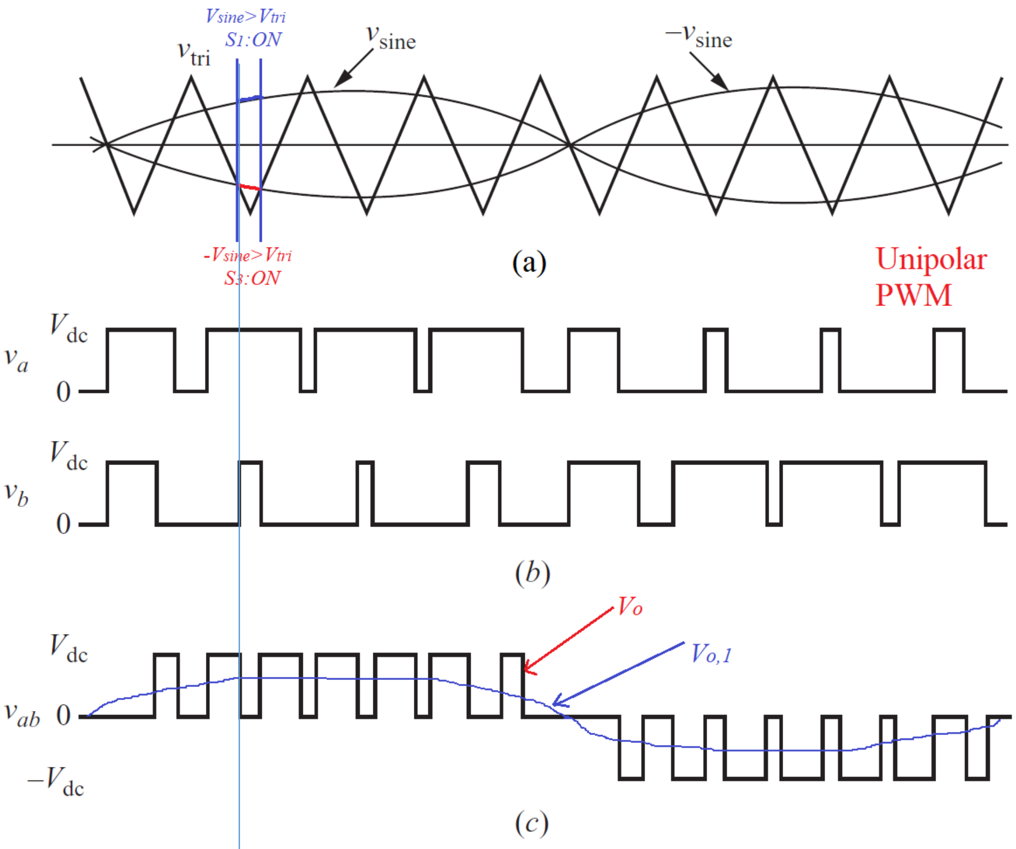
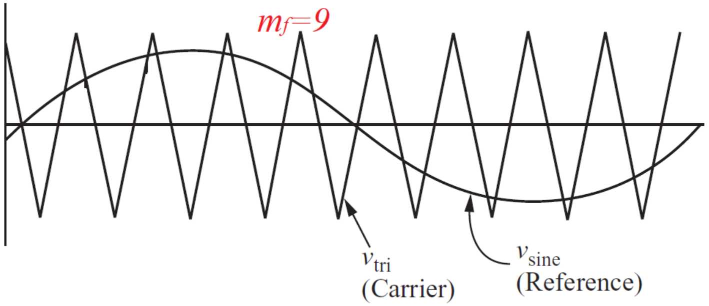


Figure 6-7 Unipolar PWM; (*a*) Reference and carrier signals; (*b*) Bridge voltages *va* and *vb*; (*c*) Output voltage.

Definitions:

1. Frequency Modulation Ratio (m_f):

$$m_f = \frac{f_{carrier}}{f_{reference}} = \frac{f_{tri}}{f_{\sin e}}$$

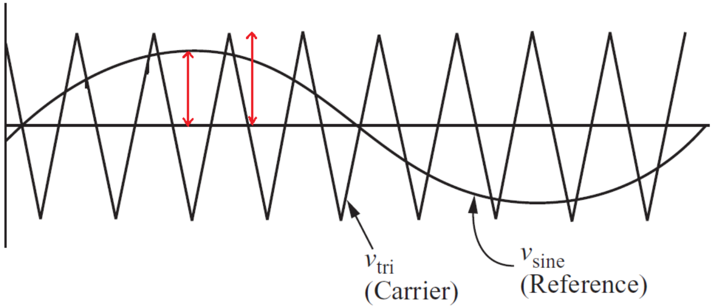


2. Amplitude Modulation Ratio (m_a):

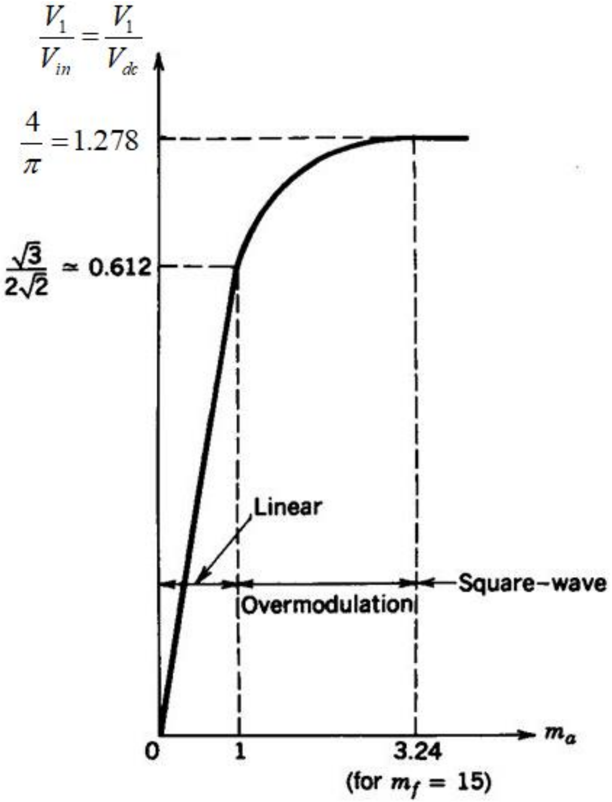
Variable

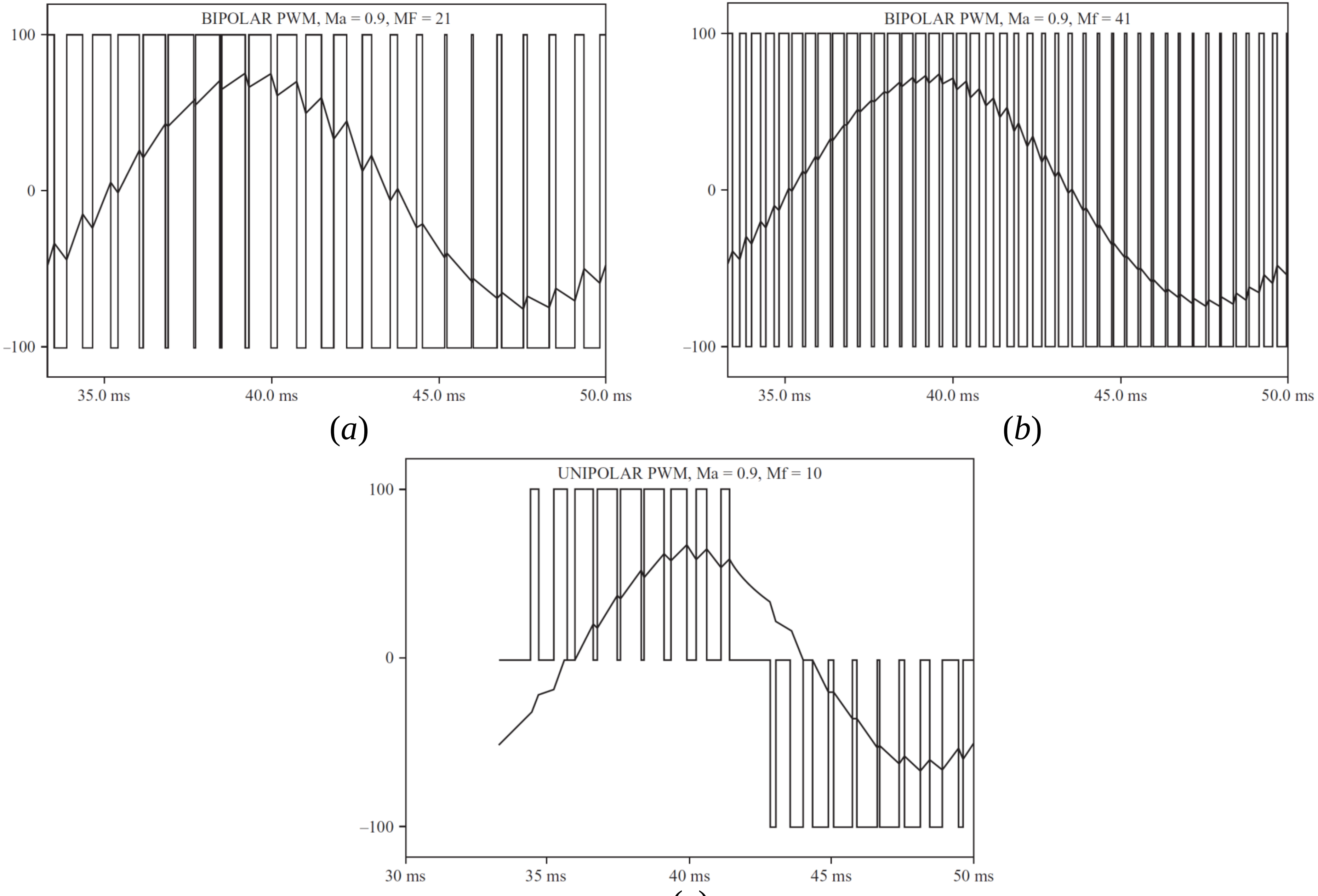
$$m_a = \frac{V_{reference,peak}}{V_{carrier,peak}} = \frac{V_{\sin e,peak}}{V_{tri,peak}}$$

Constant



- If $m_a \leq 1$, the amplitude of the fundamental frequency of the output voltage V_1 is linearly proportional to m_a. That is, $V_1 = m_a V_{dc}$.
- The amplitude of the fundamental frequency of the PWM output is controlled by *ma*. This is significant in the case of an unregulated dc supply voltage because the value of *ma* can be adjusted to compensate for variations in the dc supply voltage, producing a constant-amplitude output.
- *ma* can be varied to change the amplitude of the output. If *ma* is greater than 1, the amplitude of the output increases with *ma*, but not linearly.





(*a*) (*b*)

(*c*)

Figure 6-8 Voltage and current for (*a*) bipolar PWM with m_f=21, (*b*) bipolar PWM with m_f=41, (*c*) Unipolar PWM with m_f=10.

THREE-PHASE INVERTERS

The Six-Step Inverter

Figure 6-9 shows a circuit that produces a **three-phase ac** output from a **dc** input.

t	0-T/6	T/6-2T/6	2T/6-3T/6	3T/6-4T/6	4T/6-5T/6	5T/6-T
S1	ON	ON	ON	OFF	OFF	OFF
S2	OFF	ON	ON	ON	OFF	OFF
S3	OFF	OFF	ON	ON	ON	OFF
S4	OFF	OFF	OFF	ON	ON	ON
S5	ON	OFF	OFF	OFF	ON	ON
S6	ON	ON	OFF	OFF	OFF	ON

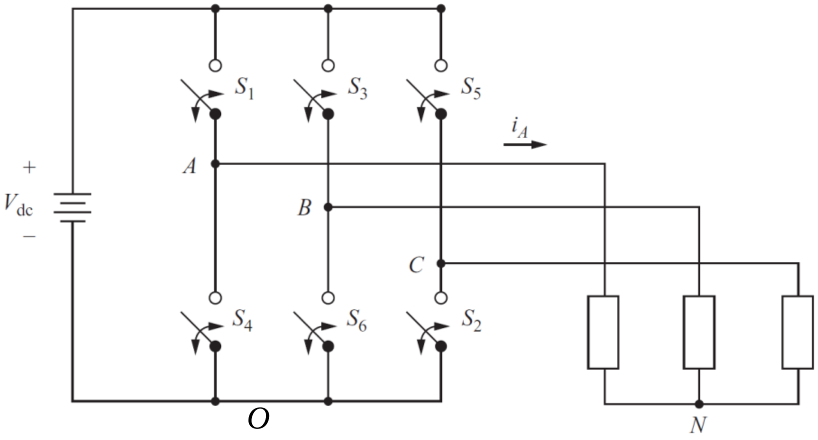
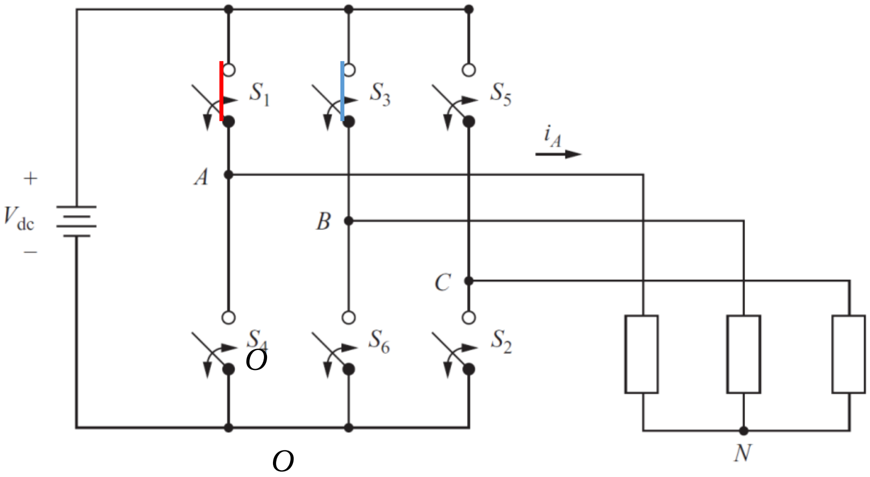
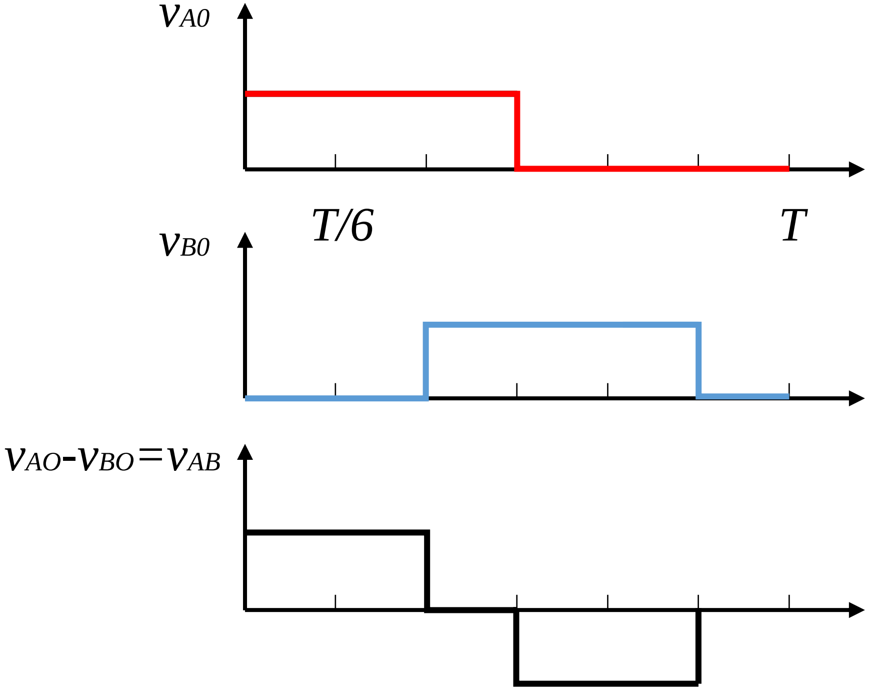


Figure 6-9 Three-phase inverter.

- Each switch has a duty ratio of 50 percent.
- Switching action takes place every T/6 time interval or 60 degree angle interval.
- Switches pairs (S1, S4), (S2, S5), and (S3, S6) close and open opposite of each other.
- Switch pairs are not closed at the same time, to prevent short circuit across the source.
- The instantaneous voltages v_{A0}, v_{B0}, and v_{C0} are $+V_{dc}$ or zero, and line-to-line output voltages v_{AB}, v_{BC}, and v_{CA} are $+V_{dc}$, 0, or $-V_{dc}$.





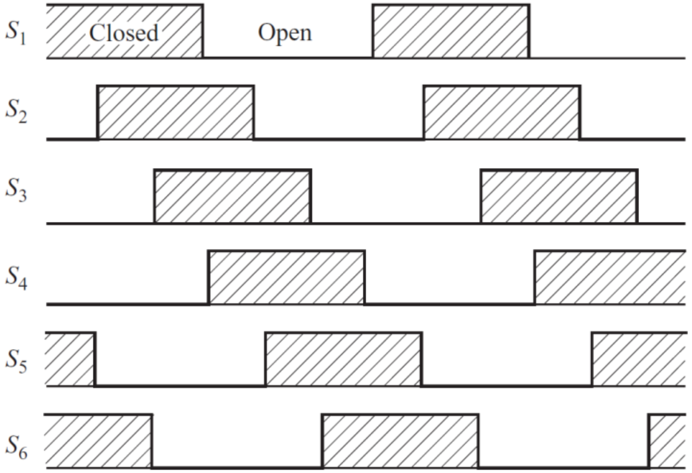


Figure 6-10 Switching sequence for six-step output.

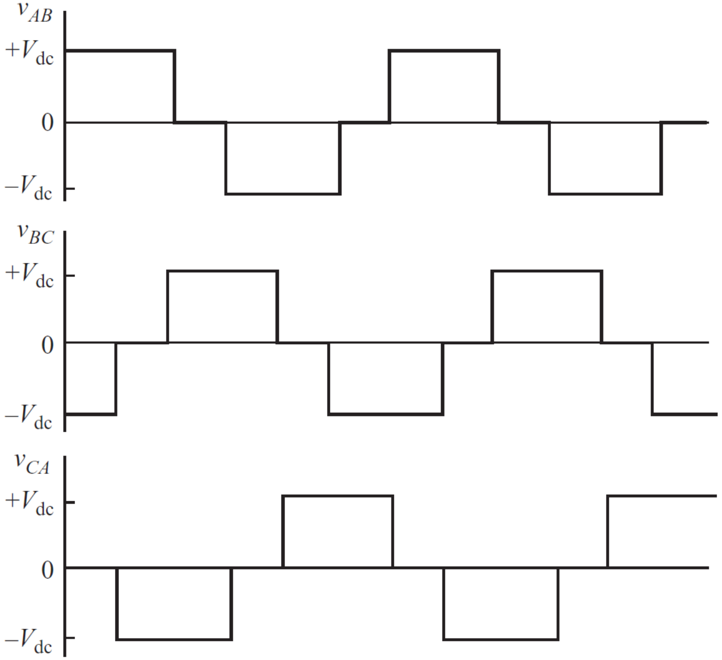


Figure 6-11 Line-to-line output voltages.

Voltage of null respect to ground (VNO):

By applying KCL in N:

$$i_A + i_B + i_C = 0$$

Assuming this is a symmetric load:

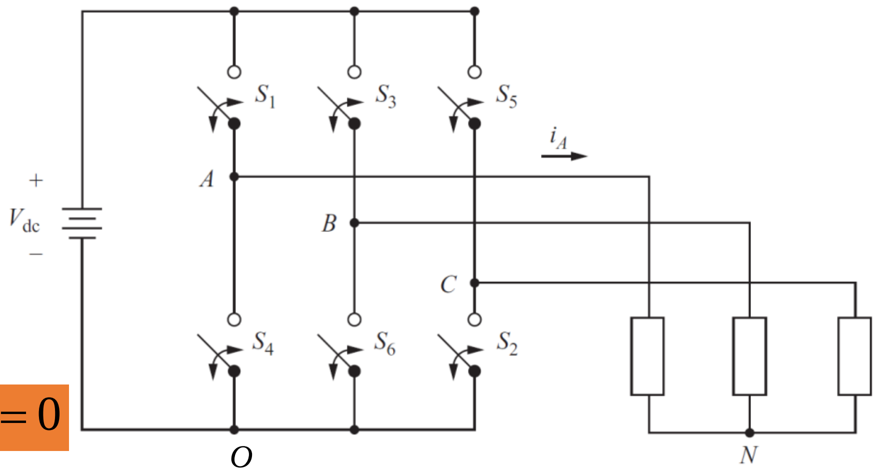
$$i_A + i_B + i_C = \frac{V_{AN}}{Z} + \frac{V_{BN}}{Z} + \frac{V_{CN}}{Z} = 0 \rightarrow V_{AN} + V_{BN} + V_{CN} = 0$$

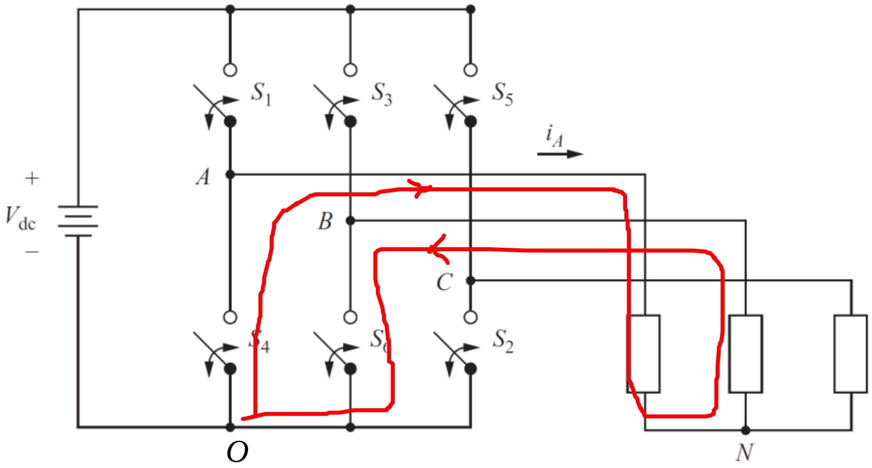
By applying KVL:

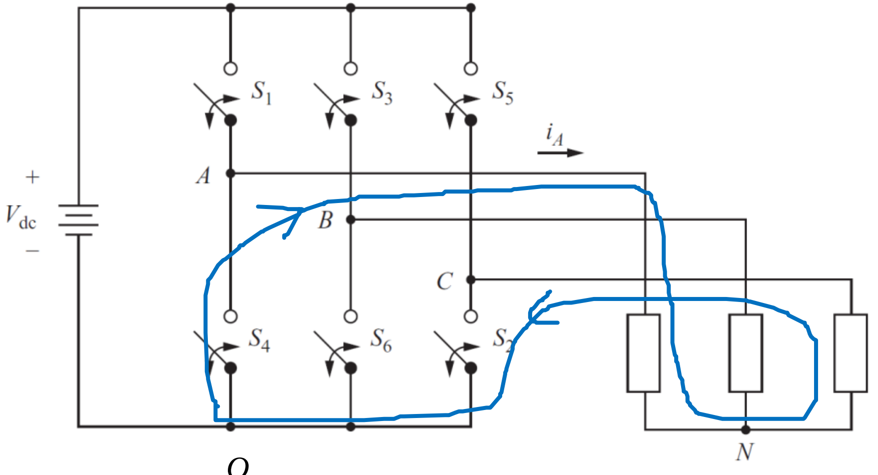
$$-V_{AO} + V_{AN} - V_{BN} + V_{BO} = 0$$

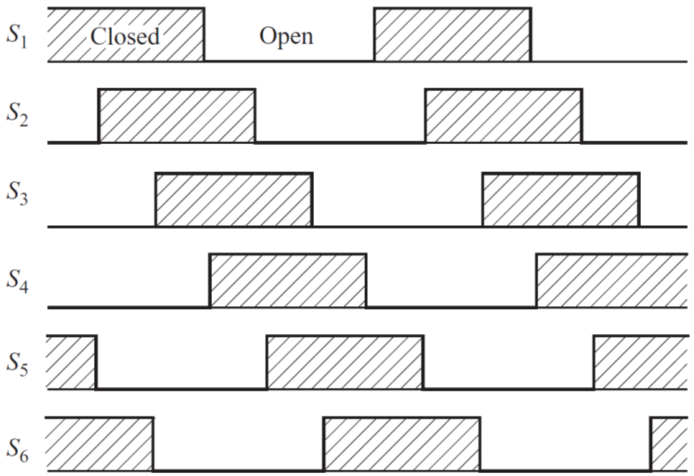
$$-V_{AO} + V_{AN} - V_{CN} + V_{CO} = 0$$

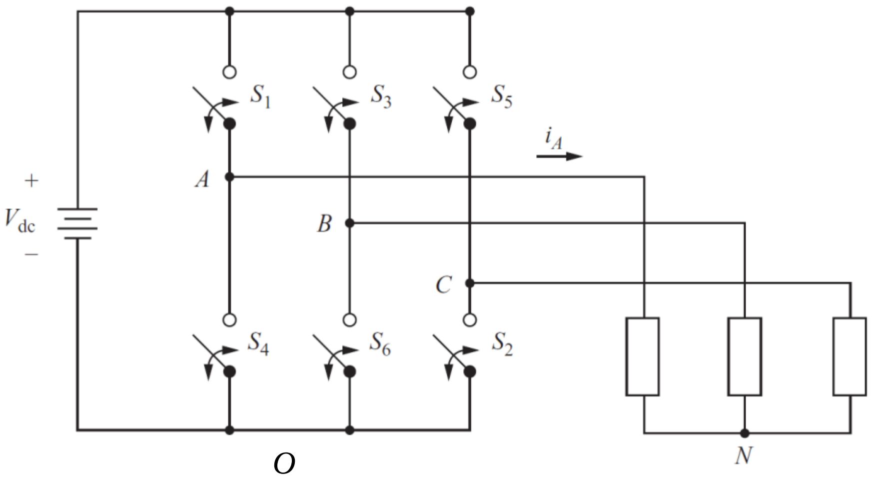
$$V_{AN} = \frac{2V_A - V_B - V_C}{3}$$



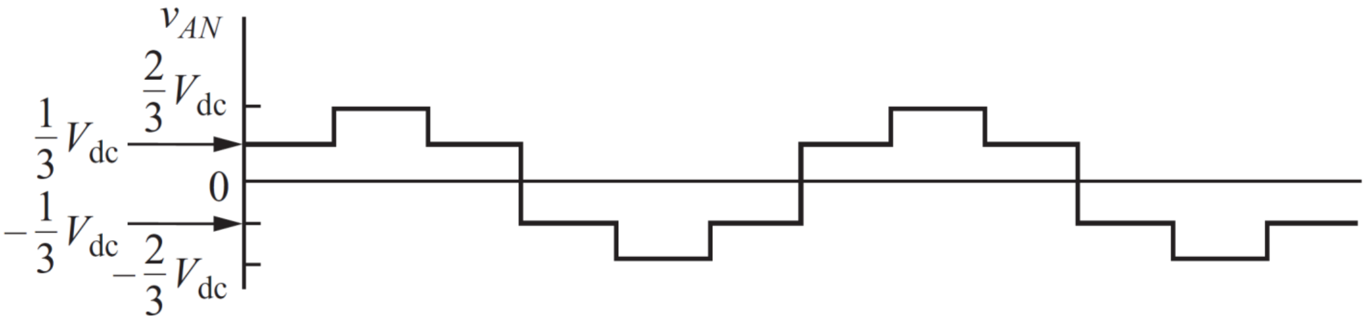








$$V_{AN}=\frac{2V_A-V_B-V_C}{3}\Rightarrow\begin{cases}0<t<\frac{T}{6} & \rightarrow V_{AN}=\frac{2V_{dc}-0-V_{dc}}{3}=\frac{V_{dc}}{3}\\ \frac{T}{6}<t<\frac{2T}{6} & \rightarrow V_{AN}=\frac{2V_{dc}-0-0}{3}=\frac{2V_{dc}}{3}\end{cases}$$



$$V_{NO}=V_{AO}-V_{AN}$$

Questions and comments are most welcome!