10.1 (L11.1) (Stock lattice) A stock with current value $S(0) = 100$ has an expected growth rate of its logarithm of $\nu = 12\%$ and a volatility of that growth rate of $\sigma = 20\%$. Find suitable parameters of a binomial lattice representing this stock with a basic elementary period of 3 months. Draw the lattice and enter the node values of 1 year. What are the probabilities of attaining the various final nodes?

10.2 (American put option) Consider the stock of Exercise 10.1. An American put option has been written on the stock with a strike price of 90 € and an expiration date after 1 year. Using a binomial lattice with a basic elementary period of 3 months, find the probability that it will be beneficial to exercise this option. The yearly risk-free rate is 10%.

10.3 (L12.1) (Bull spread) An investor who is bullish about a stock (believing that it will rise) may wish to construct a *bull spread* for that stock. One way to construct such a spread is to buy a call with strike price K_1 and sell a call with the same expiration date but with a strike price of $K_2 > K_1$. Draw the payoff curve for such a spread. Is the initial cost of the spread positive or negative?

10.4 (L12.5) (Fixed dividend) Suppose that a stock will pay a dividend of amount D at time τ. We wish to determine the price of a European call option on this stock using the lattice method. Accordingly, the time interval $[0, T]$ covering the life of the option is divided into N intervals, and hence $N + 1$ time periods are assigned. Assume that the dividend date τ occurs somewhere between period k and $k + 1$. One approach to the problem would be to establish a lattice of stock prices in the usual way, but subtract D from the nodes at period k. This produces a tree with nodes that do not recombine, as shown in Figure 1.

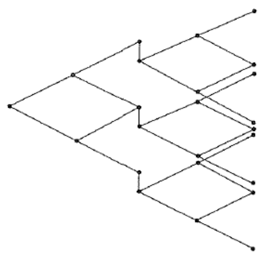

Figure 1: Non-recombining dividend tree.

The problem can be solved this way, but there is another representation that does recombine. Since the dividend amount is known, we regard it as a non-random component of the stock price. At any time before the dividend we regard the price as having two components: a random component S^* and a deterministic component equal to the present value of the future dividend. The random component S^* is described by a lattice with initial value $S(0) - De^{-r\tau}$ and with u and d determined by the volatility σ of the stock. The option is evaluated on this lattice. The only modification that must be made in the computation is that when valuing the option at a node, the stock price used in the valuation formula is not just S^* at that node, but rather $S = S^* + De^{-r(\tau-t)}$ for $t < \tau$. Use this technique to find the value of a 6-month call option with $S(0) = 50, K = 50, \sigma = 20\%, r = 10\%$, and $D = 3$ to be paid in $3\frac{1}{2}$ months.

10.5 (L12.9) (My coin) There are two propositions:

a) I flip a coin. If it is heads, you are paid 3 €; if it is tail, you are paid 0 €. It cost you 1 € to participate in this proposition. You may do so at any level, or repeatedly, and the payoffs scale accordingly.

b) You may keep your money in your pocket (earning no interest).

Here is a third proposition:

c) I flip the coin three times. If at least two of the flips are heads, you are paid 27 €; otherwise zero. How much is this proposition worth?