

- 11.1 (L13.3) (Sigma estimation) Traders in major financial institutions use the Black-Scholes formula in a backward fashion to infer other traders' estimates of  $\sigma$  from option prices. In fact, traders frequently quote sigmas to each other, rather than prices, to arrange trades. Suppose a call option on a stock that pays no dividend for 6 months has a strike price of 35 €, a premium of 2.15 €, and time to maturity of 7 weeks. The current risk-free rate is 7%, and the price of the underlying stock is 36.12 €. What is the implied volatility of the underlying security?
- 11.2 (L13.4) (Black-Scholes approximation) The first order Taylor approximation of  $N(d)$  is  $1/2 + d/\sqrt{2\pi}$ . Use this to derive the value of a call option when the stock price is at the present value of the strike price; that is,  $S = Ke^{-rT}$ . Specifically, show that  $C \approx 0.4S\sigma\sqrt{T}$ . Also show that  $\Delta \approx 1/2 + 0.2\sigma\sqrt{T}$ . Suppose the price of a stock is 62 € and the volatility is 20%. The risk-free rate is 10%. Use the above approximations to estimate the value of a 5-month European call option with a strike price  $K = 60$  for the mentioned stock.
- 11.3 (L13.6) (A special identity) Kalle Virtanen believes that for a derivative security with price  $P(S)$ , the values of  $\Delta$ ,  $\Gamma$  and  $\Theta$  are related. Show that in fact

$$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2\Gamma = rP.$$

- 11.4 (L13.11) (Pay-later options) Pay-later options are options for which the buyer is not required to pay the premium up front (i.e., at the time that the contract is entered into). At expiration, the holder of a pay-later option *must* exercise the option if it is in the money, in which case he pays the premium at that time. Otherwise the option is left unexercised and no premium is paid.
- The stock of the CCC Corporation is currently valued at 12 € and is assumed to possess all the properties of geometric Brownian motion. It has an expected annual return of 15%, an annual volatility of 20%, and the annual risk-free rate is 10%.
- Using a binomial lattice, determine the price of a call option of CCC stock maturing in 10 months' time with a strike price of 14 €. (Let the distance between nodes on your tree be 1 month in length.)
  - Using a similar methodology, determine the premium for a pay-later call with all the same parameters as the call in part a).
  - Compare your answers to part a) and b). Do the answers differ; if so why, if not why not? Under what conditions would you prefer to hold which option?