2.1 (L3.8) (Variable-rate mortgage) The Lehtinen family just took out a variable-rate mortgage on their new home. The mortgage value is 100 000 €, the term is 30 years, and initially the interest rate is 4%. The interest rate is guaranteed for 5 years, after which time the rate will be adjusted according to prevailing rates. The new rate can be applied to their loan either by changing the payment amount or by changing the length of the mortgage.
a) What is the original yearly mortgage payment? (Assume payments are yearly.)
b) What will be the mortgage balance after 5 years?
c) If the interest rate on the mortgage changes to 5% after 5 years, what will be the new yearly payment that keeps the termination time the same?
d) Under the interest change in c., what will be the new term if the payments remain the same?

In exercises 2.2 and 2.3 the coupon payments are made every 6 months ($m = 2$). The face values F of the bonds are 100 scores and the annual coupon payment is C is the nominal value×coupon rate. In formula (3.3) of the course book notations $c = C/(mF)$ and $y = \lambda/m$ are used.

2.2 (L3.10) (Bond price and duration) Find the price and duration of a 10-year, 8% bond that is trading at a yield of 10%.

2.3 (L3.14) (Duration limit) Show that the limiting value of duration as maturity is increased to infinity is

$$D \to \frac{1 + \frac{\lambda}{m}}{\lambda}.$$

For the bonds in Table (3.6) of the course book (where $\lambda = 0.05$ ja $m = 2$) we obtain $D \to 20.5$. Note that for large λ this limiting value approaches $1/m$, and hence the duration for large yields tends to be relatively short.

2.4 (L3.6) (The biweekly mortgage) Here is a proposal that has been advanced as a way for homeowners to save thousands of dollars on mortgage payments: pay biweekly instead of monthly. Specifically, if monthly payments are x, it is suggested that one instead pays x/2 every two weeks (for a total of 26 payments per year). This will pay down the mortgage faster, saving interest. The savings are surprisingly dramatic for this seemingly minor modification-often cutting the total interest payment by over one-third Assume a loan amount of 150 000 € for 20 years at 4% interest, compounded monthly.
a) Under a monthly payment program, what are the monthly payments and the total interest paid over the course of the 20 years?
b) Using the biweekly program, when will the loan be completely repaid, and what are the savings in total interest paid over the monthly program? (You may assume biweekly compounding for this part.)
c) Which factors do the savings consist of? Is this a newly invented money-making-machine? Compare the annual payments.

2.5 A young couple is currently living on rent in an apartment with a monthly rent of 600 €. They have saved 10 000 € for buying an own apartment, and have consulted a real estate broker about the new apartment. The broker is offering them a two-room apartment with a price of 100 000 €, and with monthly costs of 120 € (consisting of maintenance and water payments). The apartment is in good condition and it requires no renovation. However, after three years the condominium will be renovating the windows of the apartment, causing a 2000 € payment at the start of the third year.

The couple has decided to take a 20-year annuity debt of 90 000 €, with an annual interest of 5% and with an opening payment of 100 €. However, because only 70% of the value of the apartment (70 000 €) can be used as collateral for the loan, the couple buys a partial collateral by the government for 15% of the value of the apartment (15 000 €) with a price of 375 € (2.5% of the size of the collateral), and the remaining 5000 € will be covered with a free collateral from the parents of the husband.

a) Considering only the payments during the time of the loan, should the couple move into the apartment? Note that effectively only 70% of the interest have to be paid, because 30% of the interests of the first apartment can be reduced from taxes.

b) What if only the payments to bank, government and the condominium are considered, and no amortizations are included in the calculations (amortizations increase the wealth of the couple)? Use the 5% interest rate for discounting.

c) What if the prevalent interest rate is 2.3% (12-month Euribor + 0.7 marginal)?