4.1 (L5.1) (Capital budgeting) A firm is considering funding several proposed projects that have the financial properties shown in Table 1. The available budget is 600 000 €. What set of projects would be recommended by the approximate method based on benefit-cost ratios? What is the optimal set of projects (using net present value)?

Table 1: Financial properties of the proposed projects.

Project	Outlay (1 000 €)	Present value of benefits (1 000 €)
1	100	200
2	300	500
3	200	300
4	150	200
5	150	250

4.2 (L5.3) (Two-period budget) A company has identified a number of promising projects, as indicated in Table 3. The cash flows for the first 2 years are shown (they are all negative).

Table 2: A list of projects.

	Cash flow (1 000 €)		
Project	year 1	year 2	NPV (1 000 €)
1	-90	-58	150
2	-80	-80	200
3	-50	-100	100
4	-20	-64	100
5	-40	-50	120
6	-80	-20	150
7	-80	-100	240

The cash flows in later years are positive, and net present value of each project is shown. The company managers have decided that they can allocate up to 250 000 € in each the first 2 years to fund these projects. If less than 250 000 € is used the first year, the balance can be invested at 10% and used to augment the next year's budget. Which projects should be funded? Formulate the problem as an optimization problem.

4.3 (L5.4) (Bond matrix) Suppose that we face a known sequence of future monetary obligations. In cash flow matching problem, we design a portfolio that will provide the necessary cash as required for the obligations. We formulate this optimization problem in matrix form as follows. Let the number of bonds be m and the time horizon be n. The cash flow streams of bond j can be denoted as a $\mathbf{c}_j \in \mathbb{R}^{n\times 1}$ and the yearly obligations as $\mathbf{b} \in \mathbb{R}^{n\times 1}$. We denote the bond matrix that has columns of the cash flows $\mathbf{c}_j$ as $\mathbf{C} \in \mathbb{R}^{n\times m}$. Furthermore, the prices of the bonds can be denoted as $\mathbf{p} \in \mathbb{R}^{m\times 1}$ and the numbers of the bonds in the portfolio as $\mathbf{x} \in \mathbb{R}^{m\times 1}$. These notations give the cash flow matching problem as

$$\begin{aligned} \min \quad & \mathbf{p}^T\mathbf{x} \\ \text{s.t.} \quad & \mathbf{C}\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

a) The cash flow structure of a cash flow matching problem is presented in Table 3. Define $\mathbf{C}$, $\mathbf{b}$, $\mathbf{p}$ and $\mathbf{x}$.
b) Suppose the bonds are priced according to a conventional spot rate curve. The price vector $\mathbf{p}$ can be then written as

$$\mathbf{C}^T\mathbf{v} = \mathbf{p},$$

where $\mathbf{v} \in \mathbb{R}^{n\times 1}$ is a vector of the discount rates. Moreover, if the portfolio $\mathbf{x}^*$ matches the obligations exactly, we have

$$\mathbf{C}\mathbf{x}^* = \mathbf{b}.$$

Show that the price $\mathbf{p}^T\mathbf{x}^*$ of the portfolio is $\mathbf{v}^T\mathbf{b}$ and interpret this.
c) The optimization problem presented above seeks a solution that matches the obligations each year exactly. If the cash flows cannot be matched exactly, the present value of the portfolio is greater than the present value of the obligations. How does this model differ from immunization of a portfolio? What factor of portfolio immunization is neglected in this approach? Which approach is better?

Table 3: Bonds of exercise 3.

	Bonds											
Year	1	2	3	4	5	6	7	8	9	10	Required	Actual
1	10	7	8	6	7	5	10	8	7	100	100	171.74
2	10	7	8	6	7	5	10	8	107		200	200.00
3	10	7	8	6	7	5	110	108			800	800.00
4	10	7	8	6	7	105					100	119.34
5	10	7	8	106	107						800	800.00
6	110	107	108								1200	1200.00
p	109	94.8	99.5	93.1	97.2	92.9	110	104	102	95.2	2381.14	
x	0	11.215	0	6.807	0	0	0	6.302	0.283	0	Cost	

4.4 (L5.12) (Two-stage growth) When pricing financial instruments, the dividend discount model can be expanded by taking more growth phases into account. Consider Nokia Corp. that shared 1439M€ of dividends in year 2003. Suppose that the dividends grow at a constant rate $G = 1.3$ in the first five years (that is, during years 2004-2008), and the dividends grow at rate $g = 1.05$ from year 2009 onwards.

a) Formulate the general formula for two-stage dividend discount model for valuing a publicly traded company. The growth rate is constant G for k years and then g from year $k + 1$ onwards. The dividend of the first year D_0 is paid immediately.

b) What is the market value of Nokia Oyj, if it is valued solely based on the shared dividends? Assume a constant interest rate $r = 0.1$ and that first dividend is paid immediately.