5.1 (L6.3) (Two correlated assets) The correlation ρ between assets A and B is 0.1, and other data are given in Table 1 (*Note* $\rho = \sigma_{AB}/(\sigma_A\sigma_B)$).

Table 1: Two Correlated Cases

Asset	$\bar{r}$	σ
A	10.0%	15%
B	18.0%	30%

a) Find the proportions α of A and $(1-\alpha)$ of B that define the portfolio of A and B which has the minimum standard deviation.
b) What is the value of this minimum standard deviation?
c) What is the expected return of this portfolio?

5.2 (L6.7) (Markowitz fun) There are just three assets with rates of return r_1, r_2 and r_3, respectively. The covariance matrix and the expected rates of return are

$$\mathbf{V} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \bar{\mathbf{r}} = \begin{bmatrix} 0.4 \\ 0.8 \\ 0.8 \end{bmatrix}$$

a) Find the minimum-variance portfolio.
b) If the risk-free rate is $r_f = 0.2$, find the efficient portfolio of risky assets.

5.3 (L6.1) (Shorting with margin) Suppose that to short a stock you are required to deposit an amount equal to $1.5X_0$, where X_0 is the initial price of the stock. At the end of first year the stock price is X_1 and you liquidate your position. If R is the total return of the stock, what is the total return on your short?

5.4 (L6.5) (Rain insurance) Kalle Virtanen's friend is planning to invest 1 M€ in a rock concert to be held 1 year from now. The friend figures that he will obtain 3 M€ revenue from his 1 M€ investment - unless, my goodness, it rains. If it rains, he will lose his entire investment. There is a 50% chance that it will rain the day of the concert. Kalle suggests that he buys rain insurance. He can buy one unit of insurance for 0.50 €, and this unit pays 1 € if it rains and nothing if it does not. He may purchase as many units as he wishes, up to 3 M€.
a) What is the expected rate of return on his investment if he buys u units of insurance? (The cost of insurance is in addition to his 1 M€ investment.)
b) What number of units will minimize the variance of his return? What is this minimum value? And what is the corresponding expected rate of return? (*Hint* Before calculating a general expression for variance, think about a simple answer.)

5.5 (L6.6) Suppose there are n assets which are uncorrelated. You may invest in any one, or in any combination of them. The mean rate of return $\bar{r}$ is the same for each asset, but the variances are different. The return of an asset i has a variance of σ_i^2 $(i = 1, 2, \ldots, n)$.
a) Show the situation on an $\bar{r} - \sigma$ diagram. Describe the efficient set.
b) Find the minimum-variance point. Express your result in terms of

$$\bar{\sigma}^2 = \left(\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \right)^{-1}.$$

5.6 (L6.8) (Tracking) Suppose that it is impractical to use all the assets that are incorporated into a specified portfolio (such as a given efficient portfolio). One alternative is to find the portfolio, made up of a given set of n stocks, that tracks the specified portfolio most closely - in the sense of minimizing the variance of the difference in returns.
Specifically, suppose that the target portfolio has (random) rate of return r_M. Suppose that there are n assets with (random) rates of return $r_1, r_2, \ldots, r_n$. We wish to find the portfolio rate of return

$$r = \alpha_1 r_1 + \alpha_2 r_2 + \ldots \alpha_n r_n$$

(with $\sum_{i=1}^{n} \alpha_i = 1$) minimizing $\text{Var}[r - r_M]$.
a) Find a set of equations for the α_i's.
b) Although this portfolio tracks the desired portfolio most closely in terms of variance, it may not have the desired the mean. Hence a logical approach is to minimize the variance of the tracking error subject to achieving a given mean return. As the mean is varied, this results in a family of portfolios that are efficient in a new sense - say, tracking efficient. Find the equation for the α_i's that are tracking efficient.