

7.1 (L8.1) (A simple portfolio) Someone who believes that the collection of all stocks satisfies a single-factor model with the market portfolio serving as the factor gives you information on three stocks which make up a portfolio. (See Table 1.) In addition, you know that the market portfolio has an expected rate of return of 12% and a standard deviation of 18%. The risk-free rate is 5%.

- a) What is the portfolio's expected rate of return?
- b) Assuming the factor model is accurate, what is the standard deviation of this rate of return?

Table 1: Simple Portfolio.

Stock	Beta	Standard deviation of random error term	Weight in portfolio
A	1.10	7.0%	20%
B	0.80	2.3%	50%
C	1.00	1.0%	30%

7.2 (L8.2) (APT factors) Two stock are believed to satisfy the two-factor model

$$r_1 = a_1 + 2f_1 + f_2$$

$$r_2 = a_2 + 3f_1 + 4f_2.$$

In addition, there is a risk-free asset with a rate of return  $r_f = 10\%$ . It is known that  $\bar{r}_1 = 15\%$  and  $\bar{r}_2 = 20\%$ . What are the values of  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  for this model?

7.3 (L8.4) (Variance estimate) Let  $r_i$ , for  $i = 1, 2, \dots, n$ , be independent samples of a return  $r$  of mean  $\bar{r}$  and variance  $\sigma^2$ . Define the estimates

$$\hat{r} = \frac{1}{n} \sum_{i=1}^n r_i$$

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (r_i - \hat{r})^2.$$

Show that  $\mathbb{E}[s^2] = \sigma^2$ .

7.4 (L8.7) (Clever, but no cigar) Kalle Virtanen figured out a clever way to get 24 samples of monthly returns in just over one year instead of only 12 samples; he takes overlapping samples; that is, the first sample covers Jan 1 to Feb 1, and the second sample covers Jan 15 to Feb 15, and so forth. He figures that the error in his estimate of  $\bar{r}$ , the mean monthly return, will be reduced by this method. Analyse Kalle's idea. How does the variance of his estimate compare with that of the usual method of using 12 non-overlapping monthly returns?

7.5 (L9.1) (Certainty equivalent) An investor has utility function  $U(x) = x^{1/4}$  for salary. He has a new job offer which pays 80000€ with a bonus. The bonus will be 0€, 10000€, 20000€, 30000€, 40000€, 50000€, or 60000€, each with equal probability. What is the certainty equivalent of this job offer?