

# ECON-C5100 Digital Markets

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## Lecture 4: Auctions

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- Why auctions (price discovery, competition)
- Auction methods
- Private vs. common value auctions

# Example: Online auction for an apartment

Fredrikinkatu 58, Etu-Töölö, Helsinki

40 m<sup>2</sup> | KT, 1h, avok, kph, vh



Annettu	Asiakasnumero	Tyyppi	Summa
<b>01.01.2019</b>	<b>1493587</b>	<b>Kirjallinen tarjous</b>	<b>337 000,00 €</b>
30.12.2018	1500570	Kirjallinen tarjous	336 000,00 €
29.12.2018	1493587	Kirjallinen tarjous	335 000,00 €
27.12.2018	1500570	Kirjallinen tarjous	332 000,00 €
25.12.2018	1493587	Kirjallinen tarjous	331 000,00 €
25.12.2018	1500570	Kirjallinen tarjous	329 000,00 €
23.12.2018	1493587	Kirjallinen tarjous	328 000,00 €
22.12.2018	1500570	Kirjallinen tarjous	326 000,00 €
21.12.2018	1493587	Kirjallinen tarjous	324 000,00 €
20.12.2018	1499837	Kirjallinen tarjous	319 000,00 €
20.12.2018	1493587	Kirjallinen tarjous	318 000,00 €
19.12.2018	1500570	Kirjallinen tarjous	315 000,00 €
19.12.2018	1500082	Kirjallinen tarjous	307 000,00 €
19.12.2018	1493587	Kirjallinen tarjous	306 000,00 €

<https://www.kiinteistomaailma.fi/tarjouskauppa>  
(e.g. 50m2 case from Helsinki Sörnäinen on 26.5.2022)

Source: Kiinteistömaailma.

# Auction games

- You can participate with your team or individually.
- Your reservation price, i.e. the maximum price you are willing to pay for the item is determined as follows:
  - For your team, choose the person who would be first in alphabetical ordering
  - Take the last two digits of that persons' student number
  - As an example, I'd have 96 (IIRC)

## Ascending price auction: How should you bid?

We'll start with an English open-cry auction:

- Auctioneer announces the current going price
- You can bid to buy at the current price or exit
- Auction ends when just one bidder remains
- Final bidder wins, and pays her/his bid

## Sealed bid auction: How should you bid?

Let's try a different variant, now you place the bids in a sealed envelope\*.

The valuations change for this round, change the order of the digits in your reservation price, e.g. for me from 96 to 69.

- One round will determine the winner.
- The one with the highest bid wins, and pays her/his bid.

\* ) Use the poll in Presemo [presemo.aalto.fi/digimar](https://presemo.aalto.fi/digimar) to place bids.

- Online advertising is the key application
  - E.g. advertising made up 80% of Google's annual revenue in: 147 billion U.S. dollars in 2020.
  - Early enthusiasm of online auctions for other stuff (e.g. eBay) has quieted somewhat
- Data created through individual tracking and e.g. unique search term create thousands of ad sales opportunities *per second*.
- How to choose price for thousands of ads sold to billions of consumers?

# Reminder: Preferences online

Ads · Shop headphones

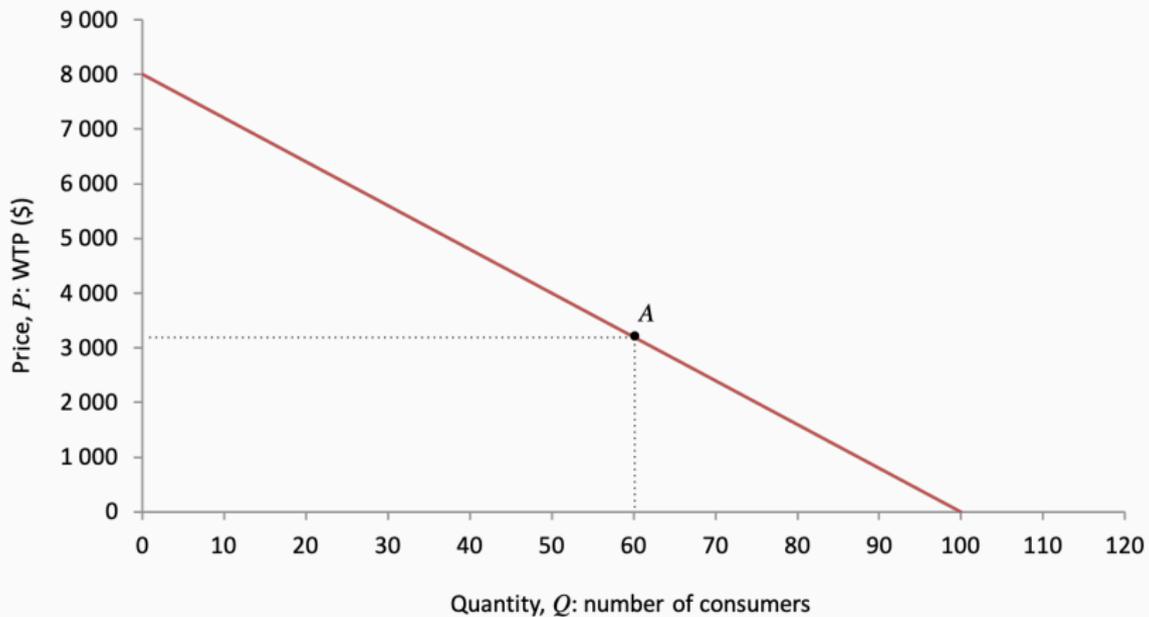
				
Apple AirPods mit LadeCase (...)	Essager - Kabellose TW...	Digitaler Stereo-Funk-Kopfhör...	Audeze LCD-5	Apple AirPods Pro
CHF 139.00	CHF 44.90	CHF 89.95	CHF 5'050.00	CHF 279.00
Apple	apfelkiste.ch	Pearl Schweiz	Thomann CH	Apple
Free shipping By Google	Free shipping By Smarketer	By Google	Free shipping By BiddingLab	Free shipping By Google

Ads · Shop headphones

				
In-ear Headphone...	Mifo O5 Plus Gen 2 Smar...	AirPods (3rd generation) ...	PX7 Carbon Edition...	AH-D1200 Headphone...
\$199.99	\$89.99	\$179.00	\$399.00	\$99.99
Grell Audio	Mifo US Store	Apple	Bowers & W...	Denon
Free shipping	Special offer	Special offer	Free shipping	★★★★★ (35)

Figure. Two identical Google searches, one done from Zurich, Switzerland (top) and another from Mexico City, Mexico (bottom)

# How to set prices?



Source: CORE, The Economy.

## Auctions as a tool for selling stuff

- Consider a situation where a seller has a single item for sale and there are a number of potential buyers.
- What is the right price to ask?
  - It will depend on how much buyers are willing to pay.
  - Buyers are not going to tell you their true value.
  - Auction is a mechanism for price discovery.
- Auctions also create a competition between buyers.

# Simple model

- Seller looking to sell one item.
- There are  $n$  buyers
  - Buyers have their own valuations for the item  $v_1, v_2, \dots, v_n$ .
  - These valuations are private information.
- We assume that the valuations are drawn from an uniform distribution  $[0, 100]$ .
  - In practice, the distribution of such valuations could be estimated from past decisions, or modeled based on some assumptions.
- Seller sets the rules for the auction.

## Ascending price auction

- Prices start at zero, and rises slowly.
- Buyers can bid at the current price or exit.
- Auction ends when just one bidder remains.
- Final bidder wins, and pays the price at which the second remaining bidder dropped out.

## Ascending price auction

- In this auction it is optimal for you to bid until the price is higher than your private value:
  - If you continue bidding above your value and win, you need to pay a price that is higher than your private value.
  - If you stop bidding below your value, you lose the potential gain from buying the item below your value.
- Bidder with the highest valuation will win and pay the second highest value.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - First exits at 25, second at 33 and the auction ends.

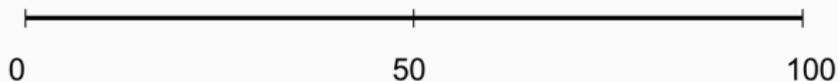
## Expected revenue from the ascending auction

- Assume two bidders with random values.
- Expected revenue from the auction is equal to the expected value of the second highest bid.
- Expected values for the bids
  - Highest value is  $66 \frac{2}{3}$ .
  - Second highest value is  $33 \frac{1}{3}$ .
- So the expected revenue for the seller is  $33 \frac{1}{3}$ .

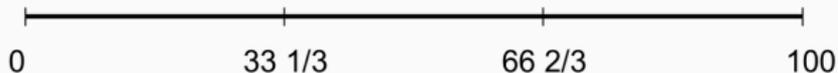
# Role of competition

More competition increases the bids and the expected value for the seller:

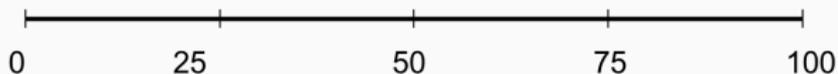
One bidder



Two bidders



Three bidders



*Figure.* Expected bid values drawn from uniform  $[0, 100]$  distribution as the number of bidders increases.

## Why auctions have been popular online

- Arranging an auction is a trade-off between the benefit of price discovery vs. the cost of arranging an auction.
- An auction can be costly for both the seller and the buyers.
  - Transaction costs, think of a traditional auction selling arts.
  - But also indirect costs for both sides (delay, hassle).
- Online both the advantage and disadvantage change:
  - Many more participants possible online vs. physically.
  - Lower costs of arranging.
  - Emergence of popular sites has made it easier for people to know where to find the auction action.

# Illustration: Trade-off between auctions and posting prices

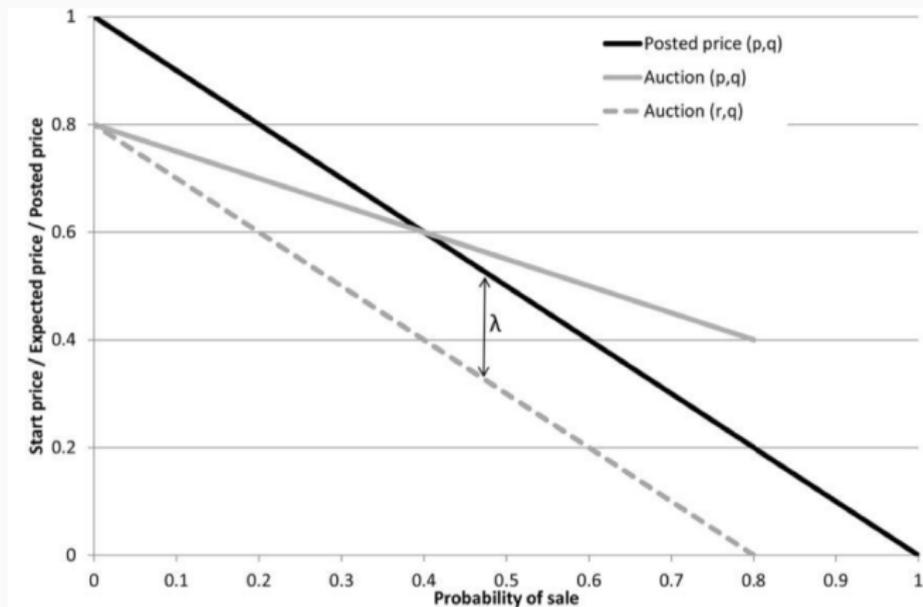


Figure. Trade-off between auctions and posting prices.

Figure: Einav et al. 2018.

- Seller asks for bids from potential buyers.
- There is only one round of bidding.
- Buyers deliver their bids so that the other buyers do not observe their bids.
- Highest bidder wins and pays their bid.
- Or a variation: Winning bidder pays the second highest bid.
- Common applications
  - Procurement contracts for commodities and services.
  - Private M&A ownership transactions between firms.

## Sealed bidding – Second price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays the second highest bid.

*How should you bid?*

## Second price auction – Optimal bidding

- Again, it is optimal for you to bid your private value.
- In equilibrium, everyone bids their value.
- Bidder with the highest valuation will win and pay the second highest value, exactly as in the ascending auction.
- Example with three bidders
  - Assume that the valuations are 25, 33, 75.
  - Winner has value 75, pays 33, as in ascending auction.

## Sealed bidding – First price auction

- Buyers submit their bids.
- Seller opens the bids.
- Bidder who submitted the highest bid wins.
- Winner pays her/his bid.

*How does the optimal bidding change?*

## First price auction – Optimal bidding

- Optimal bid is less than your true value.
- A higher bid (but still below your value)
  - Increases your chances to win.
  - Decreases your profit if you do win.
- Optimal bid depends on what you think the others will bid.
- Variation: Descending price auction
  - Price starts high, is lowered until someone buys.
  - Highest bidder wins and pays her bid.
- We need to consider an equilibrium analysis.

## First price auction – Equilibrium

- Recall the concept of Nash equilibrium: the strategic interaction now takes place through bidding strategies.
- A set of bidding strategies is a Nash equilibrium if each bidder's strategy maximizes his expected payoff given the strategies of the others.
- Bidders do not know their opponent's values, i.e. we are in incomplete information setting.
- In equilibrium, all bidding strategies maximize the expected payoff of the bidder taking in to account the uncertainty about opponent values.

## First price auction – Equilibrium

- In the above first price sealed bid auction example, there is an equilibrium in which both bidders bid half their value.
- Bidder with the highest value wins.
- In expectation, highest value is  $66 \frac{2}{3}$ , so the expected revenue is  $33 \frac{1}{3}$ . Which is the same as in the second price auction.
- Turns out this is a quite general result.

- Auction outcome is efficient if the high value bidder wins.
- The Nash equilibrium outcome is the same in several auction mechanisms:
  - The high value bidder wins.
  - The expected auction price equals the expected value of the second highest bidder.
- First discovered by Vickrey (1961), extended by Myerson (1981) and Riley and Samuelson (1981).

# Revenue Equivalence Theorem

The Revenue Equivalence Theorem:

- Take any auction that allocates the goods efficiently and offers no profit to a zero value bidder.
- Each such auction has the same expected profits for every bidder valuation and the same expected revenue for the seller.

# Revenue Equivalence Theorem

- Intuition: Auctions are a method to extract the valuations of bidders.
  - For each bidder it is 1) the probability of winning and 2) the expected price in the case of winning that matter.
  - In an equilibrium of the bidding game, a bidder correctly perceives how her bids map to the other bidders' chances of winning and prices.
- It's assumed that bidders are subjective expected utility maximizers and can quantify the uncertainty over other bidders' private information.

- Auctions are an efficient way to organize markets.
  - They offer a method for price discovery and to induce competition.
  - Cost of arranging an auction low online.
- Several auction methods, but in theory often same expected revenues when an efficient method is employed.
- Details of the auction design matter in practice.

# Materials for this week

Online resources (for Lecture 3):

- **Perfect competition.** [www.core-econ.org](http://www.core-econ.org) 8.5 and 8.8.
- **Monopolies.** Here MRU section on Monopolies [mru.org](http://mru.org): [Monopoly](#) is better suited for the course than [www.core-econ.org](http://www.core-econ.org) 7.5.1.
- **Oligopolies.** MRU on Cournot [mru.org](http://mru.org): [Cournot](#).

Reading assignment 2 (for Lecture 4):

- Einav, Lina, Chiara Farronato, Jonathan Levin and Neel Sundaresan (2018) “Auctions versus Posted Prices in Online Markets”. Very selective reading expected: Introduction, with the exception of the literature review in the end, and Section II.
- Easley, David and Jon Kleinberg (2010), Chapter 9 from “Networks, Crowds, and Markets: Reasoning about a Highly Connected World”. Advanced material (9.7) not obligatory.

## Market design

- Auction design
- Ad auctions
- Market design

# Appendix

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# Auctions are loved by economists!

- Three Nobel prizes
- Including the 2020 prize to Paul Milgrom and Robert Wilson:
  - Common value auctions
  - Implementation of auctions
- Others main contributions
  - William Vickrey (1996): formal auction theory
  - Roger Myerson (2007): revelation principle

## Additional topics in auctions: Common values

- In this course, we mostly assume that the buyers have some *private* valuation for the item.
- Reverse is also possible, it may be that the value of the item once acquired is the same for all buyers, but the value during the auction is uncertain.
- These are named *common* value auctions.
- Here optimal bidding strategies change: you want to bid cautiously to avoid *Winner's curse* (see Appendix).

## Additional topics in auctions: Winner's curse

- In a common value auction the bidder with the highest valuation on the value of the item, i.e. the most optimistic bidder, wins.
- A bidder who fails to take this into account pays, on average, more than the item is worth.

See Paul Milgrom's treatment on the topic in "Auctions and Bidding: A Primer." Journal of Economic Perspectives.

<https://www.aeaweb.org/articles?id=10.1257/jep.3.3.3>

- Auctions can be generalized to auctions when many identical items are sold simultaneously.
  - Uniform price auctions where all successful bidders get the same price.
  - Pay-as-bid auctions where each winning bidder pays her own price.
- All-pay auctions where bidders submit bids, highest bidder wins, and everyone has to pay their own bid.
  - Winner pays less in expectation, because everyone pays.
  - Not often seen in monetary auctions, but can be used to model e.g. R&D competition or lobbying