Power system stability

- **Ability to maintain operation during and after situations such as:**
  - large load changes
  - Large generation changes
  - Network switching operations
  - Network faults

- **Equilibrium: production = demand**
  - In transmission system X >> R

  -> real power balance ~ frequency
  reactive power balance ~ voltage

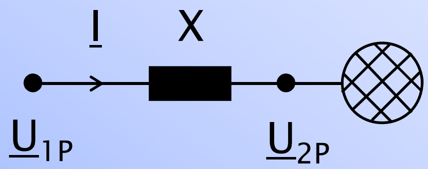
- **After disturbance an oscillation of energy follows:**
  - is the oscillation attenuated ?

- **decisive factor is the synchronization of generators:**
  Steady-state stability & Transient stability

Steady state stability

- **The ability to maintain the synchronization in normal state**
  - when frequency decreases the generators **increase** their production
  - when frequency increases the generators **decrease** their production
- **When load is increased the generators must be able to control U and f**
- **Synchronizing power:**
  - defines the change of power when frequency is increased
  - in stable case:
    - positive when frequency is decreased
    - negative when frequency is increased
  - in unstable case:
    - negative when frequency is decreased
      - frequency will be further decreased
    - positive when frequency is increased
      - frequency will be further increased
- **Steady-state maximum power:**
  - theoretical maximum limit for a stable power transmission
  - synchronizing power is zero at steady-state maximum power
- **Steady-state stability can be lost with no oscillation of power**

The power-angle equation



$$\underline{U}_{1P} = U_{1P}\angle 0°$$
$$\underline{U}_{2P} = U_{2P}\angle -\delta$$

$$\underline{S} = 3\,\underline{U}_{1P}\,\underline{I}^*$$

$$\underline{I} = \frac{\underline{U}_{1P} - \underline{U}_{2P}}{jX} = \frac{U_{1P}\angle 0° - U_{2P}\angle -\delta}{jX}$$

$$= \frac{U_{1P}}{jX} - \frac{U_{2P}\cos(-\delta) + j\,U_{2P}\sin(-\delta)}{jX}$$

$$= \frac{U_{1P}}{jX} - \frac{U_{2P}\cos\delta - j\,U_{2P}\sin\delta}{jX}$$

$$= \frac{U_{2P}\sin\delta}{X} + j\left(\frac{-U_{1P}}{X} + \frac{U_{2P}\cos\delta}{X}\right)$$

$$\Rightarrow \underline{S} = 3\,\underline{U}_{1P}\,\underline{I}^* \quad ; \quad \underline{U}_{1P} = U_{1P}\angle 0°$$

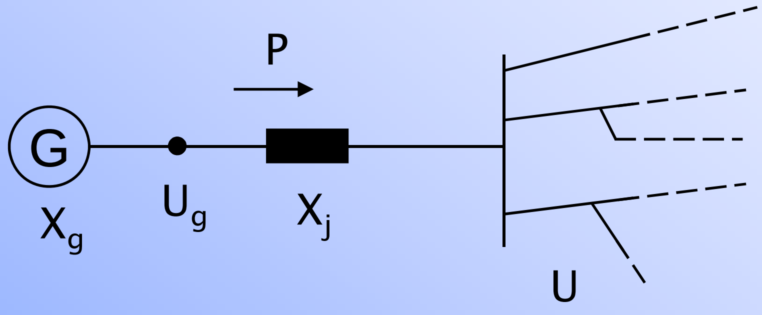
$$= \frac{3\,U_{1P}U_{2P}}{X}\sin\delta + j\left(\frac{3U_{1P}^2}{X} - \frac{3U_{1P}U_{2P}}{X}\cos\delta\right)$$

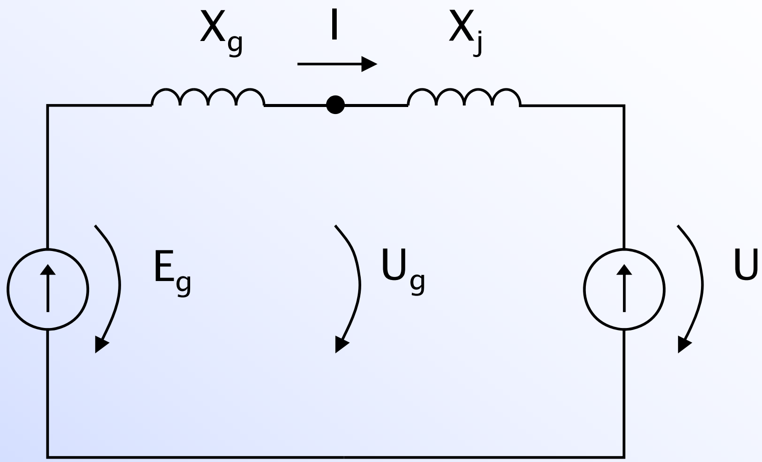
By line voltages

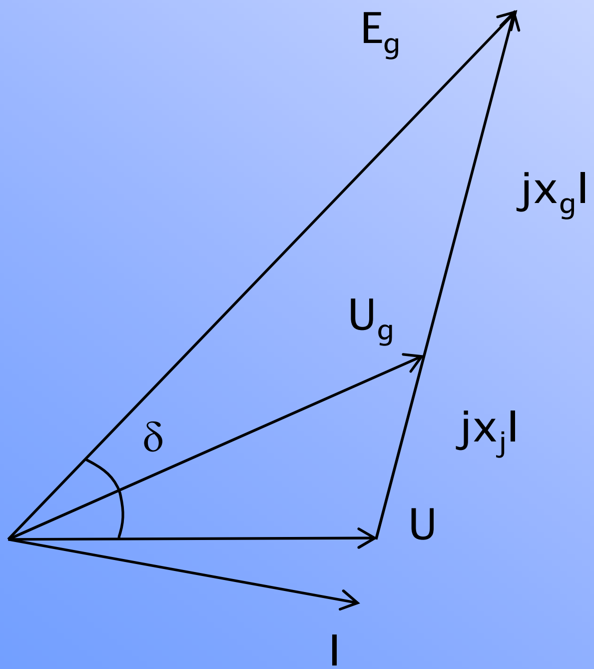
$$U_1 = \sqrt{3}\,U_{1P}$$
$$U_2 = \sqrt{3}\,U_{2P}$$

$$\begin{cases} P = \dfrac{U_1 U_2}{X}\sin\delta \\ Q = \dfrac{U_1^2}{X} - \dfrac{U_1 U_2}{X}\cos\delta \end{cases}$$

Static stability



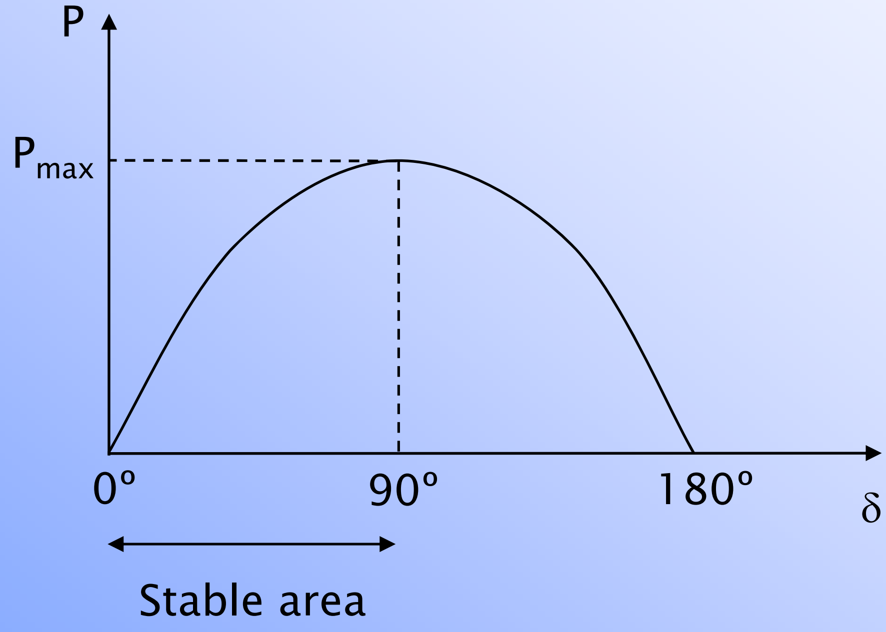




$$P = \frac{UE_q}{X_g + X_j} \sin \delta$$

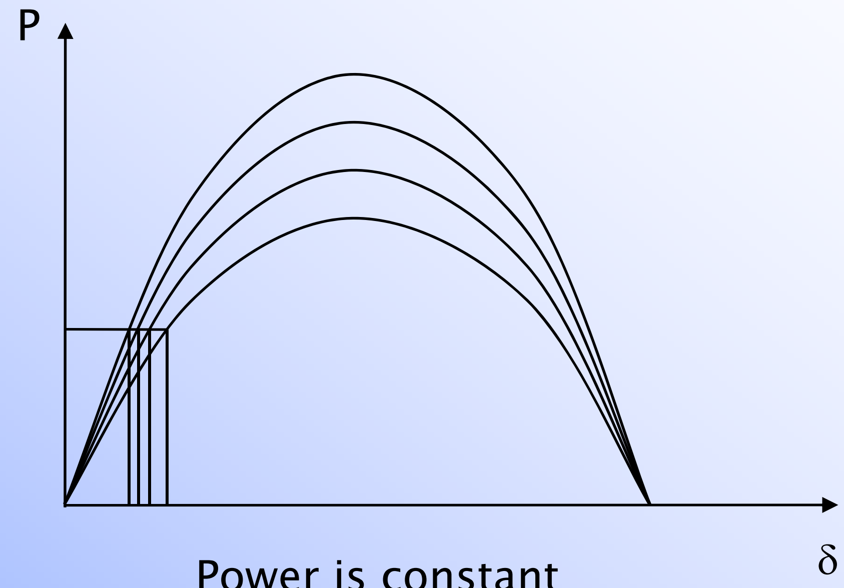
The power-angle equation

$$P = \frac{U_1 U_2}{X} \sin\delta$$



Pmax is the biggest stable power

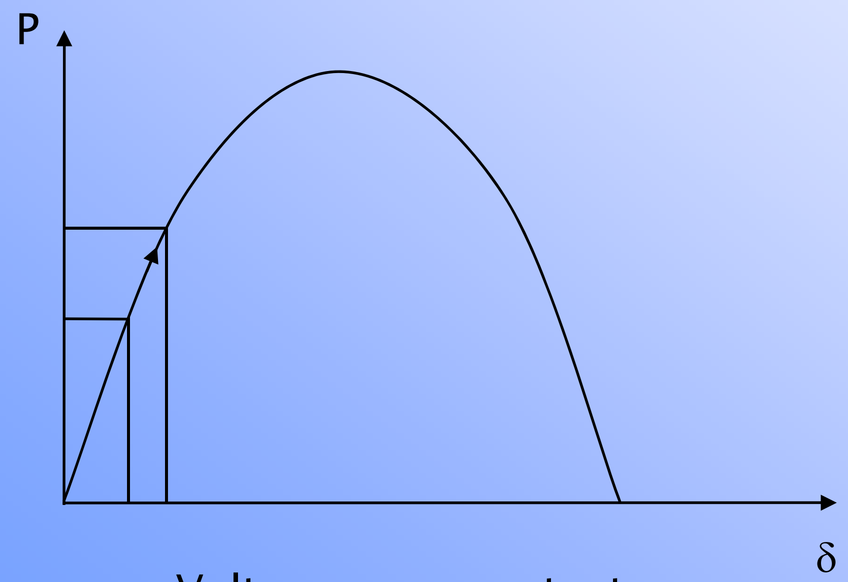
The effect of voltage control



$$P = \frac{E_1 E_2}{\sum X} \cdot \sin \delta$$

Power is constant

Phase angle versus power



Voltages are constant

Static stability and maximum power

$$\delta = \delta_{max} = 90°$$

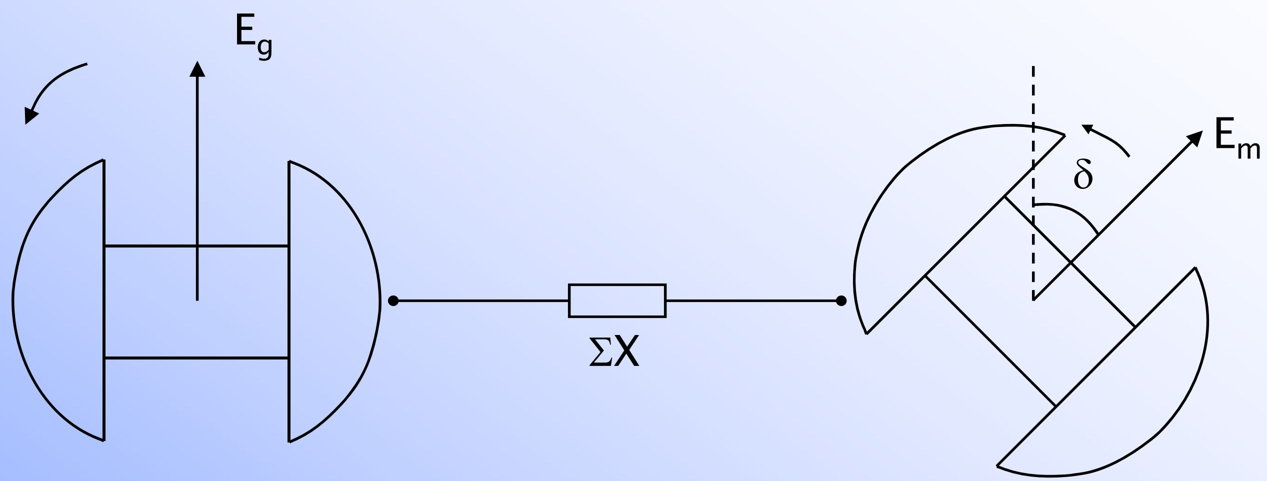
$$P_{max} = \frac{E_1 E_2}{\sum X} \sin 90°$$

$$\boxed{P_{max} = \frac{E_1 E_2}{\sum X}}$$

In practice the max phase angle $\delta_{max} \approx 45° < 90°$, since:

- reactive power production limitations in generators
- margin for dynamic changes in network
- margin for voltage control

Synchronizing power



When load increases the angular frequency tends to decrease.
This leads to the increase in power-angle δ.

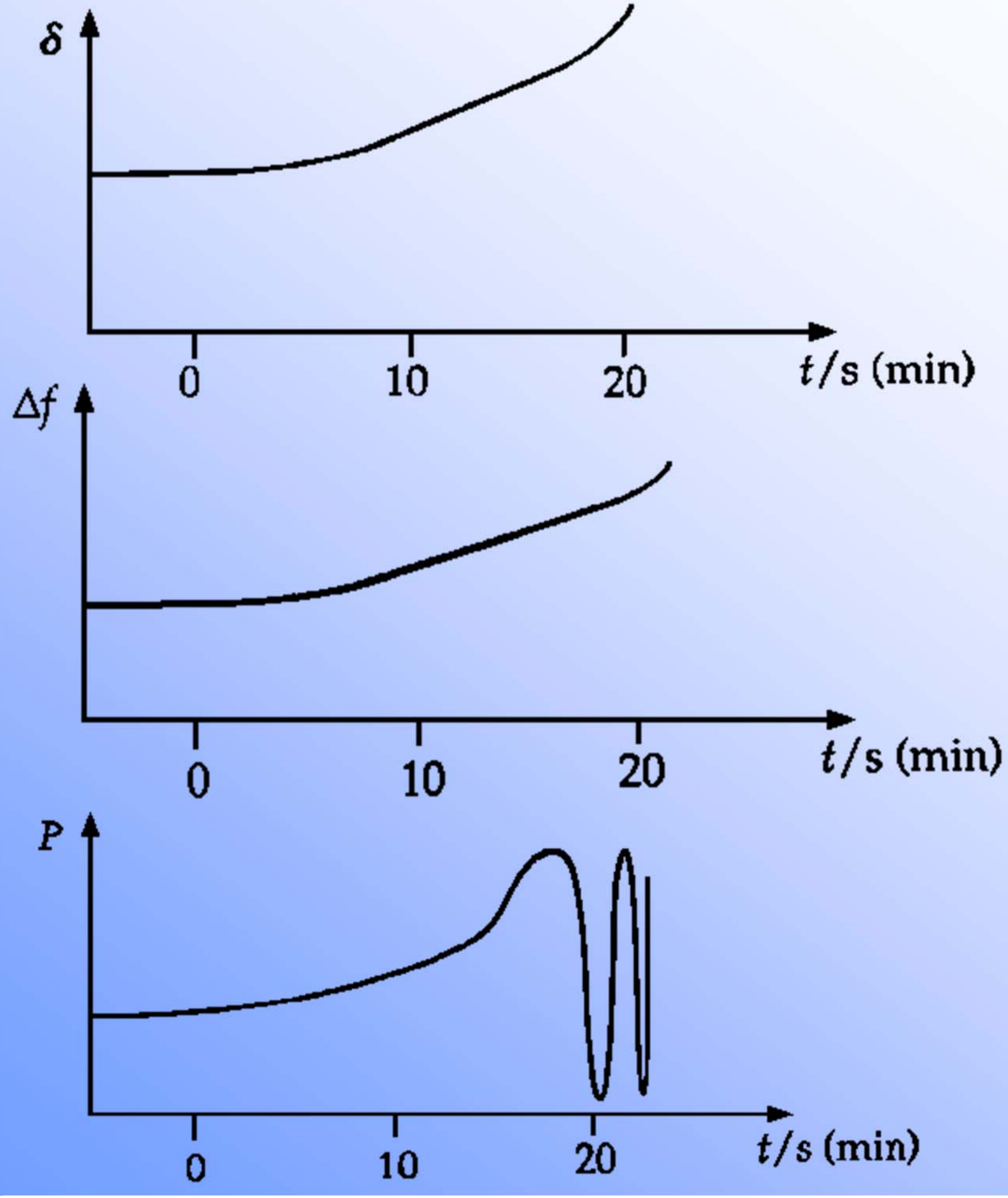
For stable operation, the power transmission must be able to increase when power angle is increased.

$$\frac{\partial P}{\partial \delta} > 0 \qquad P = \frac{E_g E_m}{\sum X} \sin\delta$$

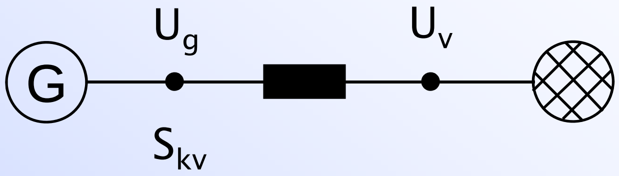
$$\frac{\partial P}{\partial \delta} = \frac{E_g E_m}{\sum X} \cos\delta > 0$$

$$\text{when } \delta < 90^\circ$$

The loss of static stability



An example



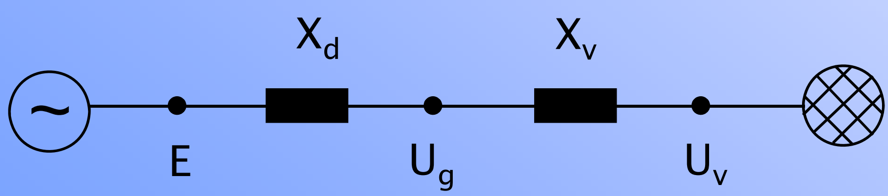
25 MVA
105 %
10 kV

$S_{kv} = 50$ MVA
$U_g = U_v = 1.0$

Maximum power with regard static stability ? P_{max}

Let's select $S_b = 25$ MVA $\quad U_b = 10$ kV

$$\Rightarrow X_d = 1{,}05 \text{ p.u.} \qquad X_v = \frac{25}{50} = 0{,}5 \text{ p.u.}$$



$$\begin{cases} \dfrac{X_v}{X_d} = \dfrac{a}{b} \\ a + b = U_v = 1{,}0 \end{cases}$$

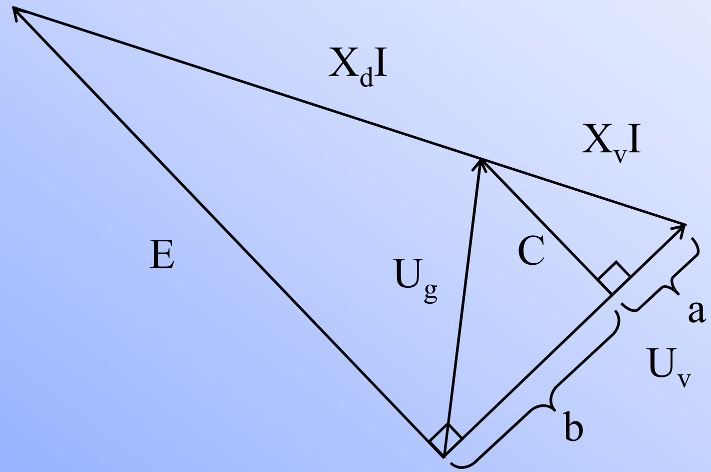
$$\Rightarrow a = 0{,}323 \;;\; b = 0{,}677$$

$$\Rightarrow c = \sqrt{U_g^{\,2} - b^2} = 0{,}736$$

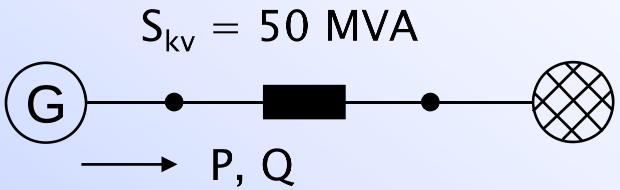
$$\Rightarrow \frac{E}{c} = \frac{X_d + X_v}{X_v} \Rightarrow E = 2{,}282$$

$$\Rightarrow P_{max} = \frac{E\,U_v}{X_v + X_d} = \frac{2{,}282 \cdot 1{,}0}{0{,}5 + 1{,}05} \text{ p.u.}$$

$$= 1{,}472 \text{ p.u.} \;\hat{=}\; 36{,}8 \text{ MW}$$



Example 2



$U_g = U_v = 1.0$

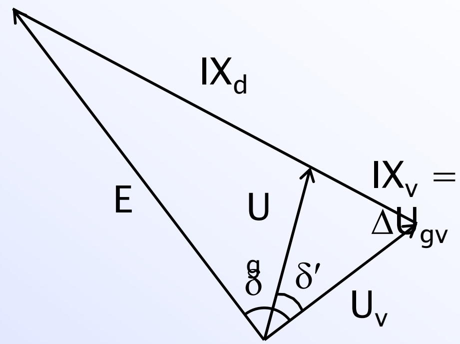
25 MVA
105 %

$S_b = 25$ MVA

P = 15 MW = 0,6 p.u.

Compute Q, E, δ', δ

($X_v = 0{,}5$ p.u. $X_d = 1{,}05$ p.u.)



$$P = \frac{U_g U_v}{X_v} \sin \delta'$$

$$\Rightarrow \sin \delta' = \frac{X_v P}{U_g U_v} \Rightarrow \underline{\delta' = 17{,}46°}$$

$$Q = \frac{U_g^2}{X_v} - \frac{U_g U_v}{X_v} \cos \delta' = 0{,}092 \text{ p.u.} \mathrel{\hat{=}} \underline{2{,}3 \text{ MVAr}}$$

$$\underline{E} = \frac{X_d + X_v}{X_v} \Delta \underline{U}_{gv} + \underline{U}_v$$

$$= \frac{X_d + X_v}{X_v} (\underline{U}_g - \underline{U}_v) + \underline{U}_v$$

$$= \frac{X_d + X_v}{X_v} (U_g \cos \delta' - U_v + j U_g \sin \delta') + U_v$$

$$= \left(\frac{X_d + X_v}{X_v} U_g \cos \delta' - \frac{X_d}{X_v} U_v \right) + j \frac{X_d + X_v}{X_v} U_g \sin \delta'$$

$$= 0{,}8572 + j\,0{,}9301 = 1{,}265 \angle 47{,}3°$$

$$\begin{cases} E = 1{,}265 \text{ p.u.} = 12{,}65 \text{ kV} \\ \delta = 47{,}3° \end{cases}$$

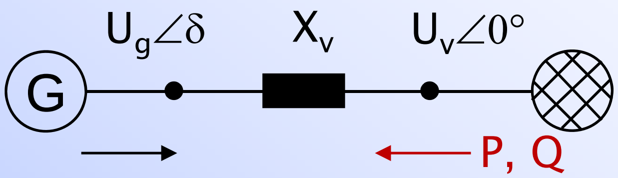
Check :

$$P = \frac{E\,U_v}{X_d + X_v} \sin \delta$$

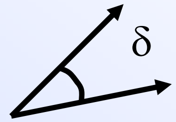
$$= \frac{1{,}265 \cdot 1{,}0}{1{,}05 + 0{,}5} \sin 47{,}3°$$

$$= 0{,}6\ !$$

Example 3



$U_g = U_v = 1.0$



$\delta = 17{,}46°$

$P = 0{,}6 \;\widehat{=}\; 15\ \text{MW}$

$Q = 0{,}092 \;\widehat{=}\; 2{,}3\ \text{MVAr}$

Loads in the end of line $P, Q = ?$

$$P = \frac{U_v U_g}{X_v}\sin(-\delta) = -0{,}6 \;\widehat{=}\; -15\ \text{MW}$$

$$Q = \frac{U_v^{\;2}}{X_v} - \frac{U_v U_g}{X_v}\cos(-\delta) = 0{,}092 \;\widehat{=}\; 2{,}3\ \text{MVAr}$$

$\Rightarrow$ to maintain the voltage, reactive power must be fed also into the end of the line

Transient stability

· **the ability to maintain operation during power swings**

– in unstable situation two or more network areas lose the mutual synchronization with the consequence:

* large power swings between the areas
* overload risk for the connecting lines
* risk of damage to generators
* voltage control / power control try to stabilize

If synchronization is lost the network is split into parts

Some of the parts will lose voltage

Power swings

- In steady state, the mechanical power P_m and electrical power of the system P_s are equal.

- when the mechanical power of the turbine is constant, but the electrical power is changed by faults, disconnection etc., the difference in power balance will increase the kinetic energy of the turbine / generator

$$W_{kin} = \frac{1}{2} J\omega^2$$

The change of kinetic energy:

$$P_m - P_s' = \frac{\partial W_{kin}}{\partial t} = J\omega \frac{\partial \omega}{\partial t} = J\omega \frac{\partial^2 \delta'}{\partial t^2}$$

$$\omega = \omega_s + \Delta\omega \qquad \Delta\omega = \frac{\partial^2 \delta'}{\partial t^2}$$

In the single machine case:

$$P_m = \frac{E_1' E_2}{X_{d1}' + X} \sin \delta' + J\omega \frac{\partial^2 \delta'}{\partial t^2}$$

The variation of power angle δ′ during a power swing

$$P_m - P_s' = J\omega \frac{\partial^2 \delta'}{\partial t^2}$$

No electrical power transm. during the fault:

$$P_s' = 0$$

$$J\omega \frac{\partial^2 \delta'}{\partial t^2} = P_m$$

$$J\omega \frac{\partial \delta'}{\partial t} = P_m \cdot t + C_1$$

In start, the angular frequency is nominal:

$$\Delta\omega(0) = \delta'(0) = 0 \quad \Rightarrow \quad C_1 = 0$$

$$J\omega \frac{\partial \delta'}{\partial t} = P_m \cdot t$$

$$J\omega\delta' = \frac{1}{2} P_m \cdot t^2 + C_2$$

Steady state power angle in the beginning:

$$\delta'(0) = \delta_0' \quad \Rightarrow \quad C_2 = J\omega\delta_0'$$

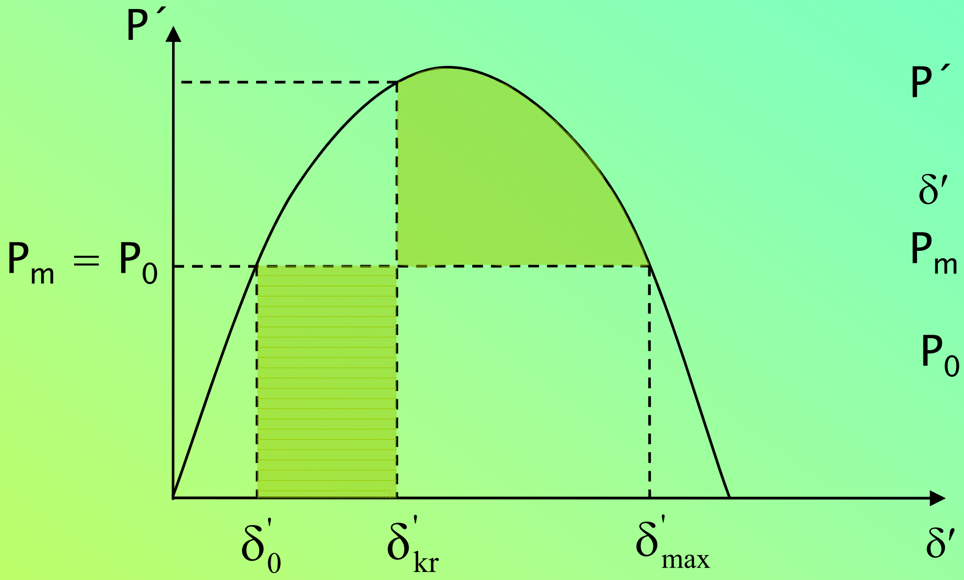
$$J\omega\delta' = \frac{1}{2} P_m \cdot t^2 + J\omega\delta_0'$$

$$\delta' = \delta_0' + \frac{P_m}{2J\omega} t^2$$

$$t = \sqrt{2 \frac{J\omega}{P_m} (\delta' - \delta_0')}$$

Single machine model

Equal area criterion



Assessing the transient stability

P' = transient electrical power

δ' = transient power angle

P_m = mechanical power

P_0 = electrical power in the beginning

Assumptions:

$$\begin{cases} P_m = \text{constant} \\ P_e = \dfrac{E'U}{X'} \sin \delta' \end{cases}$$

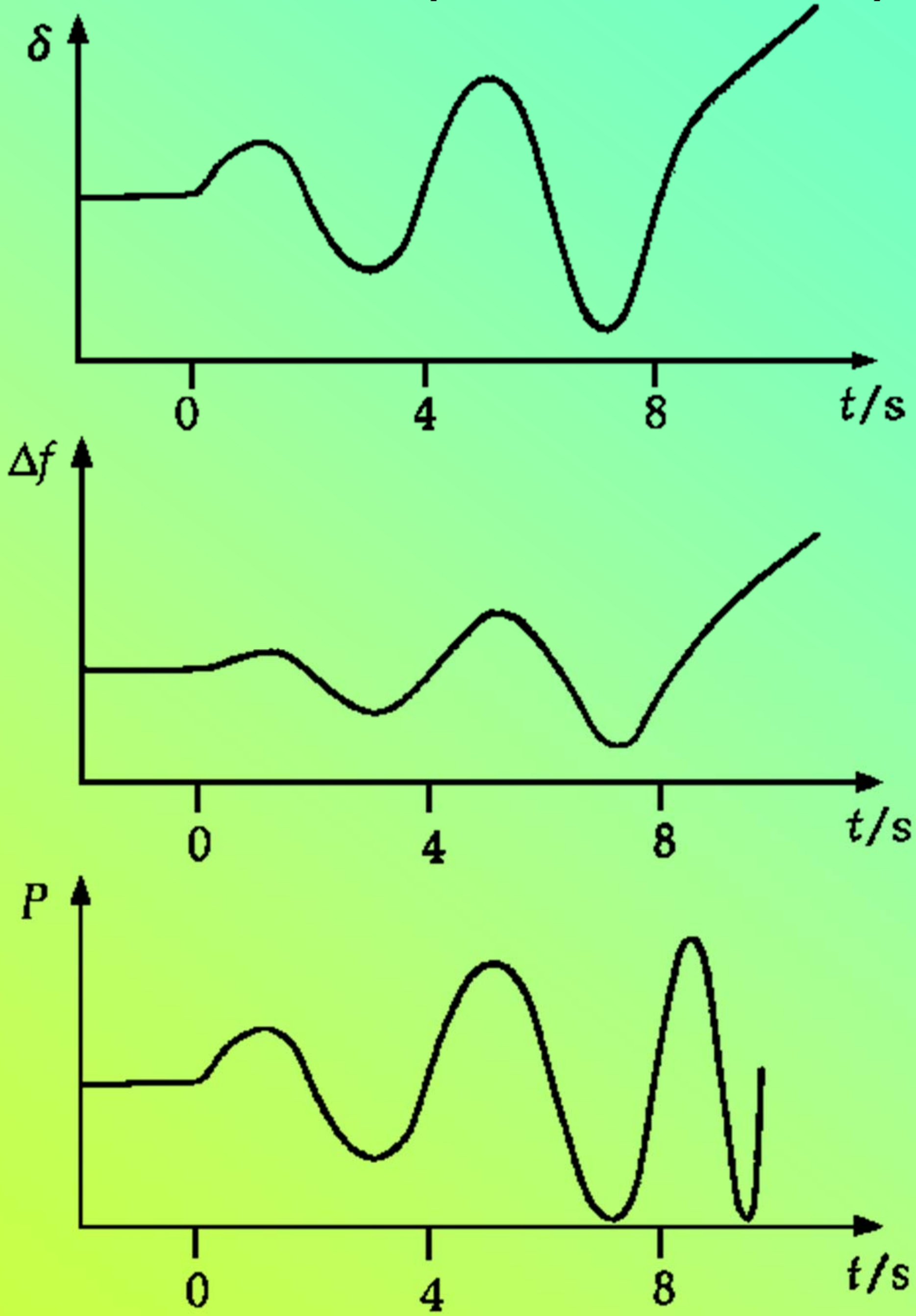
$\Rightarrow \int (P_m - P_e)\, d\delta$ Corresponds to:

- accelerating energy, when $P_m > P_e$
- braking energy, when $P_e > P_m$

$\Rightarrow$ areas must be equal, or the braking area must be bigger

$\Rightarrow \int (P_m - P_e)\, d\delta \leq 0$ (stability condition)

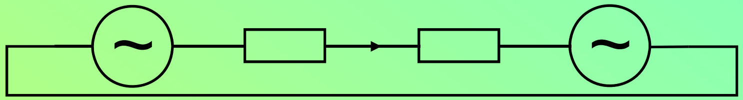
The loss of dynamic stability

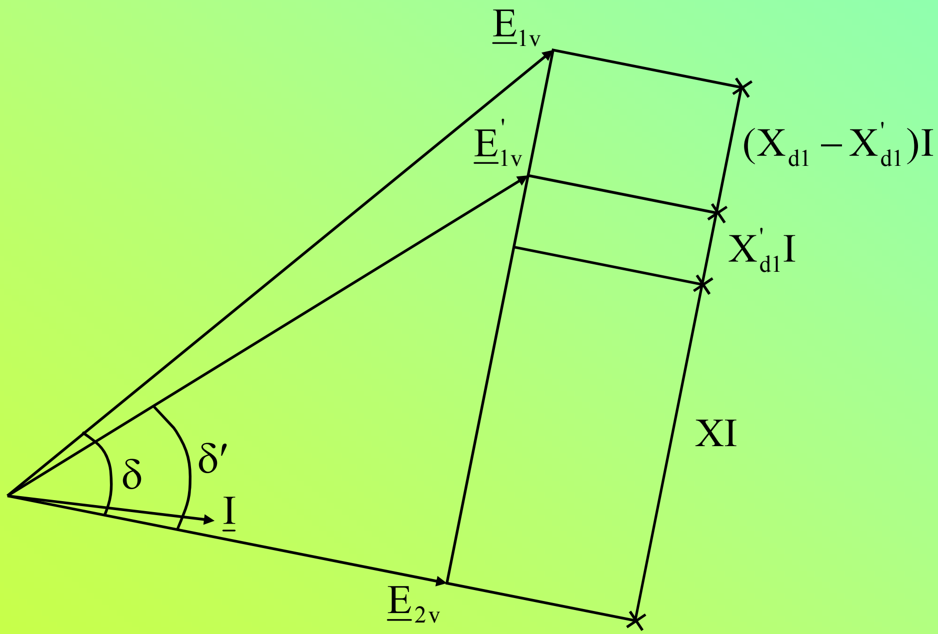


Single machine model

– machine 2 is the stiff grid

$\underline{E}_{1v}$	jX_{d1}	$\underline{I}$	jX	$\underline{E}_{2v}$	Steady state stability
$\underline{E}'_{1v}$	jX'_{d1}	$\underline{I}$	jX	$\underline{E}_{2v}$	Dynamic stability Transient stability





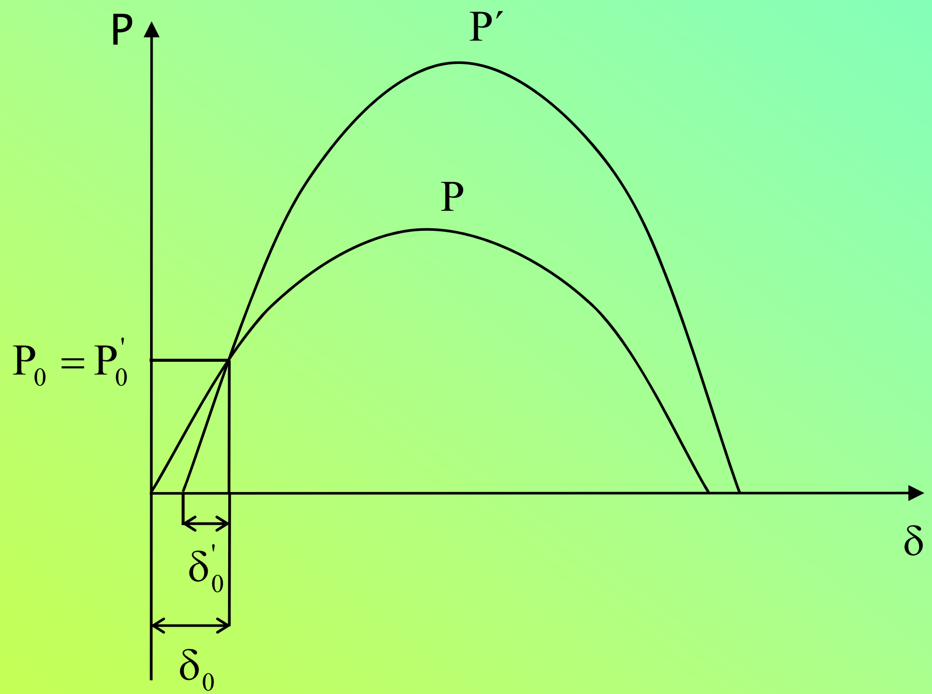
Power angle equation & single machine model

- In continuous state: (steady state stability)

$$P = \frac{E_1 E_2}{X_{d1} + X} \sin \delta$$

- In transient state: (dynamic stability, transient stability)

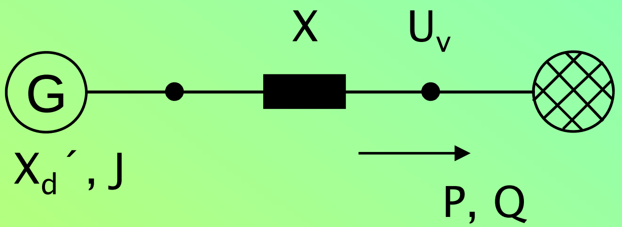
$$P' = \frac{E_1' E_2}{X_{d1}' + X} \sin \delta'$$



In static situation the steady state power equals to the transient state power

Dynamic stability

An example



Outage time
t = 0,3 s

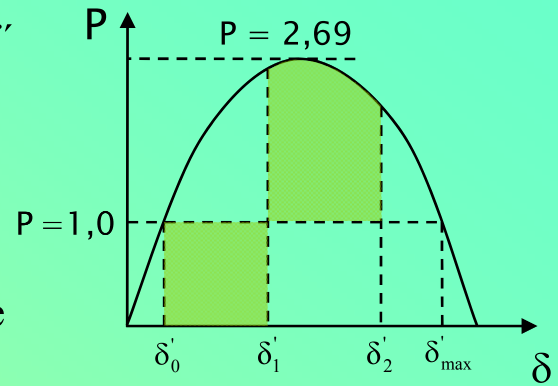
P = 1,0 ; Q = 0 ; U_v = 1,0
X = 0,15 ; X'_d = 0,25 ; ωJ = 0,05 s^2

In the stability maintained ?

Power angle equation in transient case

$$P' = \frac{E' U_v}{X + X_d'} \sin \delta'$$

$$= 2{,}69 \sin \delta'$$



$\delta_{max}' = 180° - \delta_0' = 158{,}2°$

If $\delta_2' < \delta_{max}'$, the system is stable

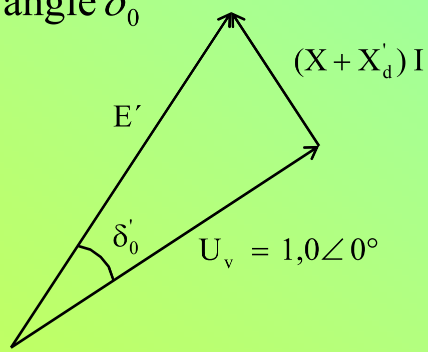
Compute emf E′ and power angle δ_0'

$P = UI \Rightarrow I = 1{,}0$

$\Rightarrow (X + X_d')\, I = 0{,}4$

$\Rightarrow \underline{E' = 1{,}08 \angle 21{,}8°}$

$\underline{\delta_0' = 21{,}8°}$



power angle at switching moment, t = 0,3 s

$$\delta_1' = \frac{1}{2} \frac{P}{\omega J} t^2 + \delta_0' = \underline{73{,}37°}$$

$$\int_{\delta_0'}^{\delta_1'} P_m \, d\delta' = \int_{\delta_1'}^{\delta_2'} (P_s - P_m)\, d\delta'$$

$$= \int_{\delta_1'}^{\delta_2'} (2{,}69 \sin \delta' - P_m)\, d\delta'$$

$P_m = 1{,}0$

$\Rightarrow \delta_1' - \delta_0' = 2{,}69 (\cos \delta_1' - \cos \delta_2') - \delta_2' + \delta_1'$

$\Rightarrow \cos \delta_2' = 0{,}428 - 0{,}372\, \delta_2'$

$\Rightarrow \delta_2' = 104{,}5° < \delta_{max}'$

⇒ the system is stable!