

Exercise 0

6.9.2022

#1 Binomial distribution, expected value

You are playing a game in which you draw five cards one-by-one from a regular deck of 52 cards such that after each draw, the card is returned to the deck. Your profit distribution is as follows:

# Spades	0	1	2	3	4	5
Profit (€)	0	4	10	20	50	100

- What is the probability of getting exactly three spades?
- What is the probability of getting at most one spade?
- What is your expected profit?

How much would you be willing to pay to participate in this game?

#2 Conditional probability, law of total probability, Bayes' theorem

Assume that 95% of people have been vaccinated against a given disease. Assume further that 7% of unvaccinated people will catch the disease, as will 0.4% of the vaccinated people.

- Consider a randomly selected person. What is the probability that this person has the disease?
- Consider a person who has caught the disease. What is the probability that this person is unvaccinated?
- How do the results change, if only 50% of people have been vaccinated?

#3 Monte Carlo simulation using Excel

A nail manufacturing company specializes in making 12 cm nails. Their machine produces nails the length of which is normally distributed with mean 12 cm and standard deviation 0.05 cm. Any nail that is more than 1mm too short or too long will be rejected.

- Simulate 200 nails produced by the machine using Excel.
- Based on your simulations, determine the average nail length and its standard deviation.
- Based on your simulations, how many nails out of 200 will be rejected?
- How do the average length, standard deviation, and the share of rejected nails correspond to their theoretical values?

#4 Monte Carlo simulation using Matlab

A small company that builds summer cottages is trying to predict the number of orders for cottages next year. The probability distribution for the orders estimated by the company manager is shown below.

# orders	0	1	2	3
Probability	0.1	0.4	0.3	0.2

- a) Simulate 1000 replications for the number of orders using Matlab.
- b) Compute the average number of orders and the standard deviation and compare them to the theoretical values.