Review of Basic Probability

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PROBABILITY THEORY

Random experiment = an activity that has an uncertain outcome.

Rolling two dice, throwing darts, taking an exam, stock price at time t

Sample Space S is the set of all possible outcomes

Event consists of one or more possible outcomes of a random experiment.

Ex. "Rolling a 7", "throwing dart at bull's eye", "failing in exam", "stock price>30 €"

Notation: P(A) = 'probability of event A'

Different **definitions** for probability

1. **Classical** definition (statistical variability)
– each outcome is equally likely
– can use mathematical reasoning

Throwing dice, card games

2. Relative **frequency** definition (statistical variation)
- based on historical data

Stock Market, product demand

3. Subjective probability (uncertainty)
- belief that a particular event will occur

Result of a football game, politics

Notation for sets

- Give all the elements of a set within brackets: $\{a_1,a_2,\ldots a_n\}$
- Mathematically thinking **sets define events** in probability theory
- Example: Throwing one die

  Sample Space S={1,2,3,4,5,6} " all events"

  Event A={1,3,5}

  Event B={4,5,6}

- Sometimes a set could be infinite:

  e.g. non-negative integers {0, 1, 2, …}
- Sets can also be "continuous", e.g. real numbers R or an interval [a,b]

Terminology and notation for combined events

- **Joint probability** is the probability that both of two events A **and** B occur: $P(A\cap B)$

- Sets A and B are **disjoint** if $A\cap B=\varnothing$ (empty set)
- Events A and B are **mutually exclusive** if $P(A\cap B)=0$
- If two sets are disjoint, their corresponding events are mutually exclusive, since $P(\varnothing)=0$
- Note: it is possible that $P(A)=0$ even if $A\neq\varnothing$

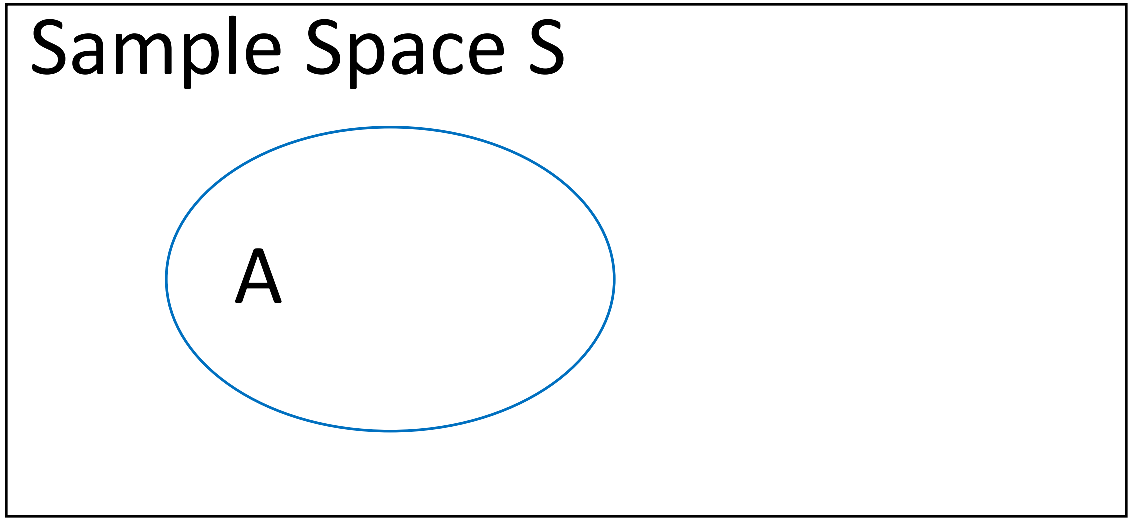
- Probability that event A **or** event B happens: $P(A\cup B)$

Example: Throwing one die

- Sample space S={1,2,3,4,5,6}
- Event A={1,3,5}
- Event B={4,5,6}
- Event $A \cap B=\{5\}$, $P(A \cap B)=P\{5\}=1/6$
- Event $A \cup B=\{1,3,4,5,6\}$, $P(A \cup B)=P\{1,3,4,5,6\}=5/6$
- If C={2,4,6} then $A \cap C= \varnothing$ and $P(A \cap C)=P(\varnothing)$

Graphical Illustration

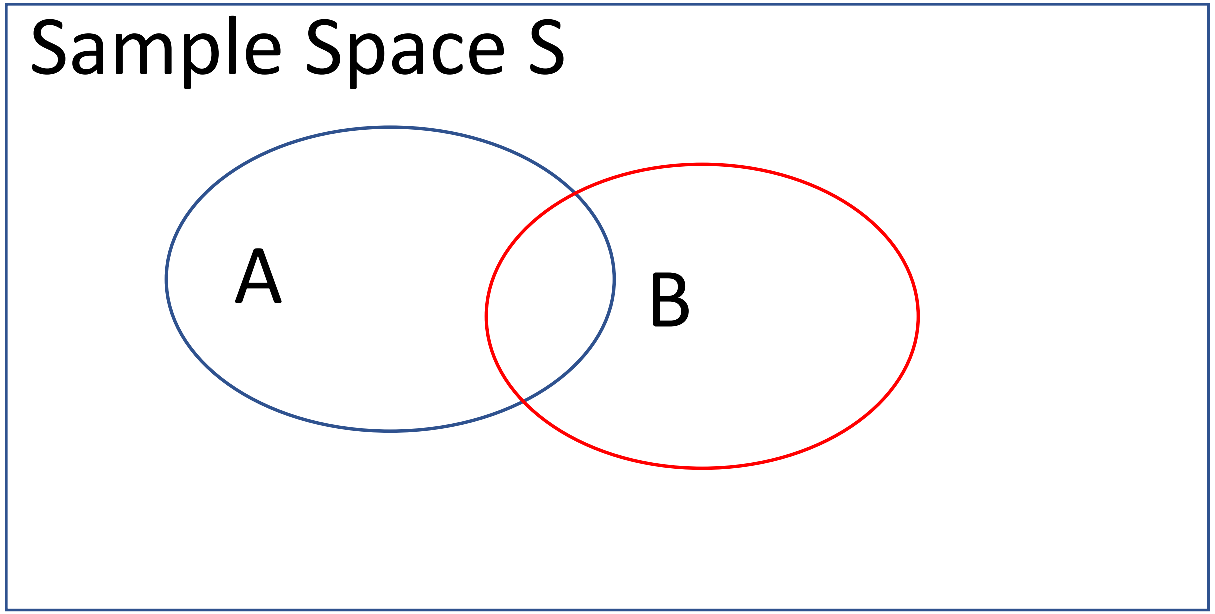
Event A



Visually you can think the areas represent the probabilities of the events

Graphical Illustration

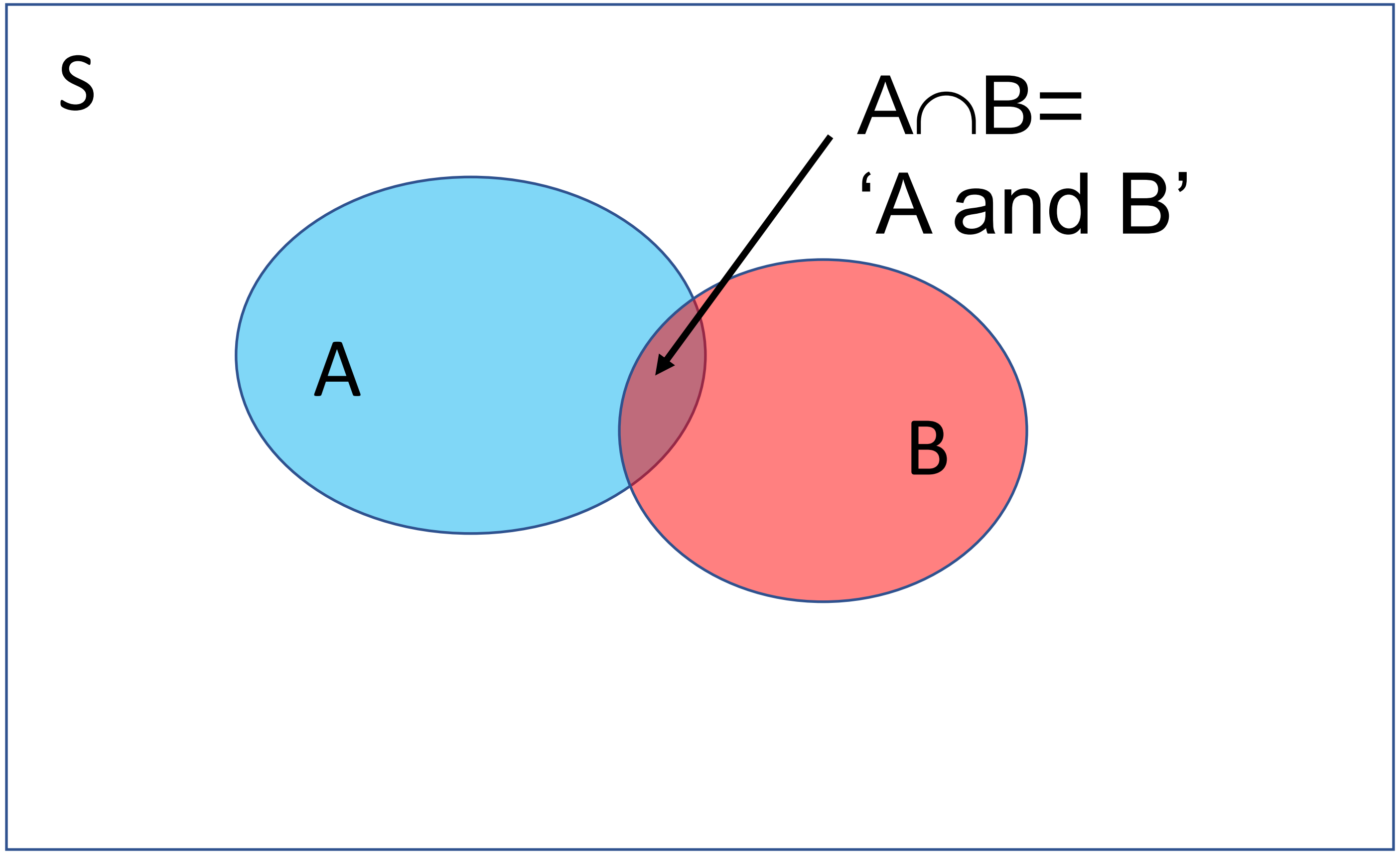
Events A and B

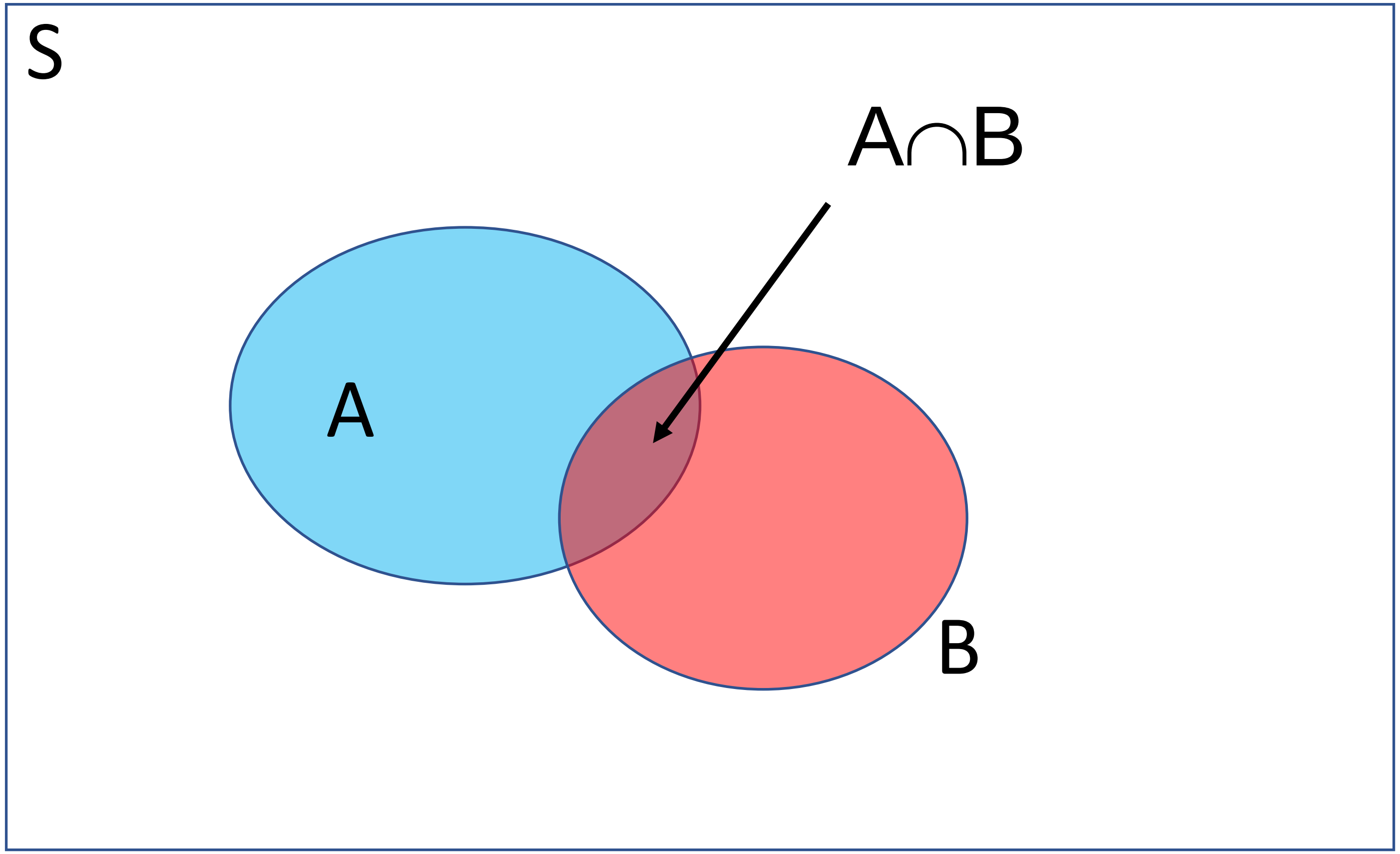


RULES FOR PROBABILITY

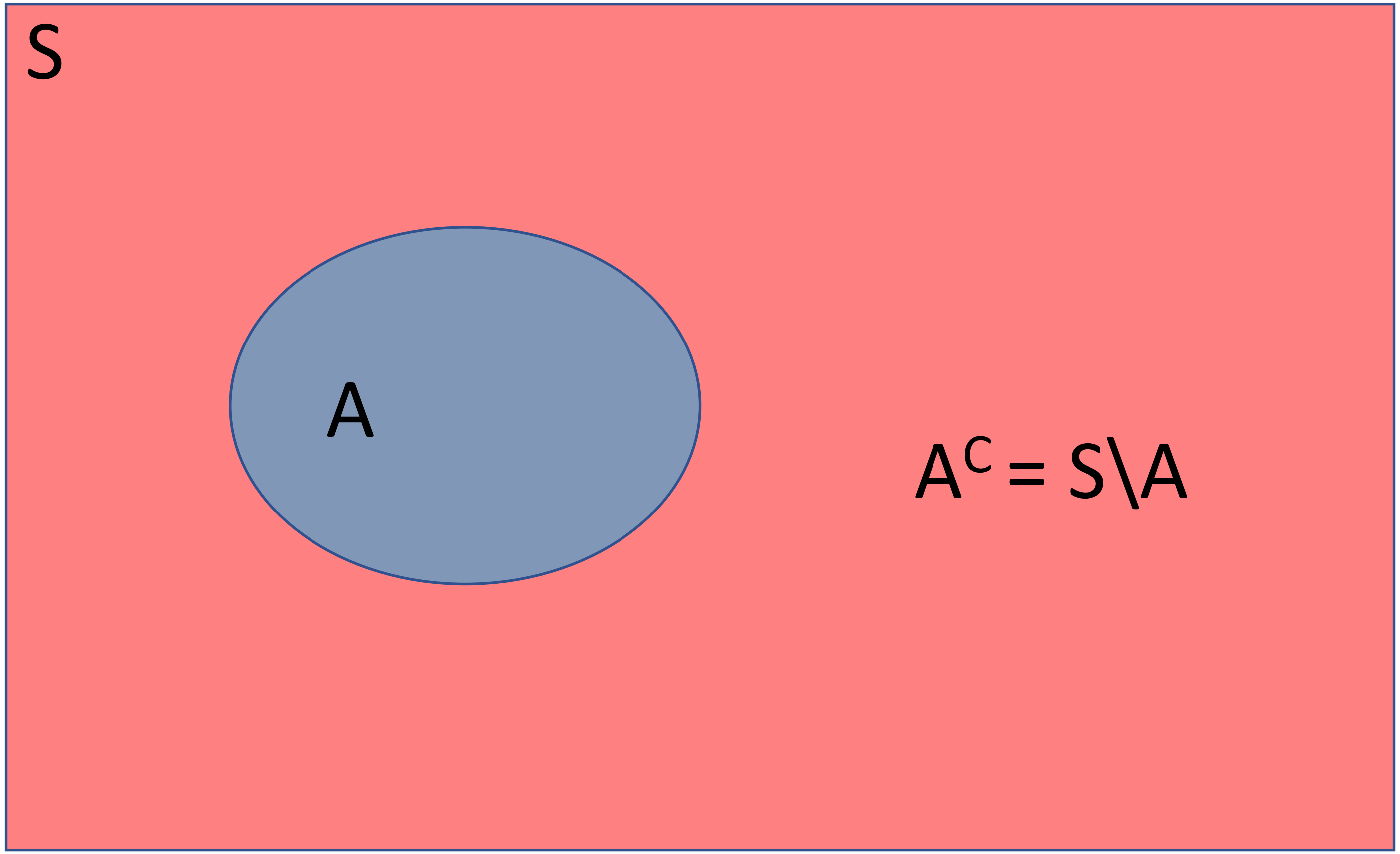
1. $0 \leq P(A) \leq 1$
2. $P(S)=1$, ’certain event’
3. $P(\emptyset)=0$, ’null event’, ‘never happens’
4. $P(A \cup B)=P(A)+P(B) - P(A \cap B)$
5. $P(A^c)=1-P(A)$
6. $P(A \cap B)=P(A)P(B|A)$
7. If $A \subset B$ then $P(A) \leq P(B)$

A complement of event A is A^c = ’A will not occur’. $A \subset B$ means that A is a subset of B (including the possibility that A=B).


S
A∩B=
‘A and B’
A
B



Rule 4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Rule 5. $P(A^C) = 1 - P(A)$

Example: Throwing one die

- Sample space S={1,2,3,4,5,6}. Let A={1,3,5}, B= {4,5,6}
- $P(A \cap B)=P\{5\}=1/6$
- $P(A \cup B)=P\{1,3,4,5,6\}=5/6$ or using rule 4:
- $P(A \cup B)=P(A)+P(B) - P(A \cap B) = P\{1,3,5\}+P\{4,5,6\}- P\{5\}$

  $3/6+3/6-1/6=5/6$
- F="You get a 6", $F^{C=}$ ="You do not get a 6",
- $P(F^C) = 1 - P(F)= 1-1/6=5/6$ using rule 5