Introduction to Simulation:
Modeling and Monte Carlo Simulation

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**Simulation (30E00400)
Lecture 1, Fall 2022**

Example: Overbooking

- How many flight reservations should an airline company take if they know that an uncertain number of passengers will not arrive to the flight
- Airline company wants to optimize their economic gain
- Every ticket not sold is an opportunity loss
- Every overbooking is costly since compensation needs to paid to those who do not fit to the plane
- Can simulate but need a mathematical model involving probabilities of the demand, showups

Example: Food Safety

- Food Safety Organization is interested in the effects of harmful content in certain foods
- Harmful content X in foods
  - The amount varies per unit
- People eat various kinds and amounts of food
- How much is harmful varies from person to person
- Question: How much is too much?
- We are interested in
  - Intakes
  - Health effects
  - Economic effects

Quantification

- All of the above need quantitative measures
- They are very rarely constant, but vary
- Lots of uncertainty
- Need probabilistic models

Probabilistic modelling

- Statistical Variability
- Uncertainty
- Quantitative models
- Probabilistic (stochastic) models
- Randomness
- Random variable
- Probability distribution

Statistical Variability

- Variation in a system or process that is affected by chance
- Examples
  - Outcome when tossing a coin: heads or tails
  - Number of passengers show up on a flight
  - Weight of an egg
  - Measures in production process
  - Food intake
  - Stock price etc….

Uncertainty

- Degree of beliefs that something is true
- Examples
  - Belief of the probability of ”heads” when tossing a coin
  - Probability that it will rain tomorrow
  - Probability that Finland will qualify for the next World cup in football
  - Average content X in certain food

Quantitative model

- A model is a logical description of a phenomenon or how a system performs
- Quantitative model is a mathematical or numerical representation of a phenomenon or a system

Probabilistic model

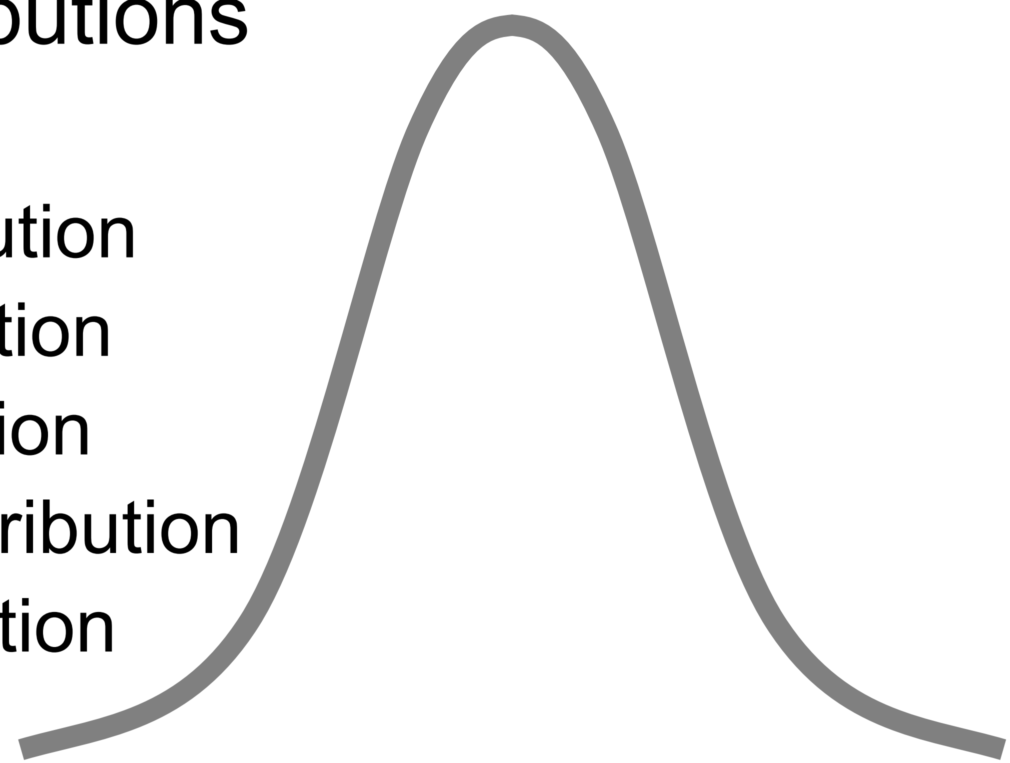
- Quantitative model where probabilities are used
- Used to model variability and uncertainty as randomness

Randomness and Random Variables

- Randomness is described through random variables
- Random variables assign probabilities to possible events
- Examples
  - result from tossing a coin: heads/tails
  - sum of two dice
  - number of goals in a soccer game
  - weight of an egg
  - income
  - stock price
  - unemployment rate
  - etc. etc.

Probability distribution

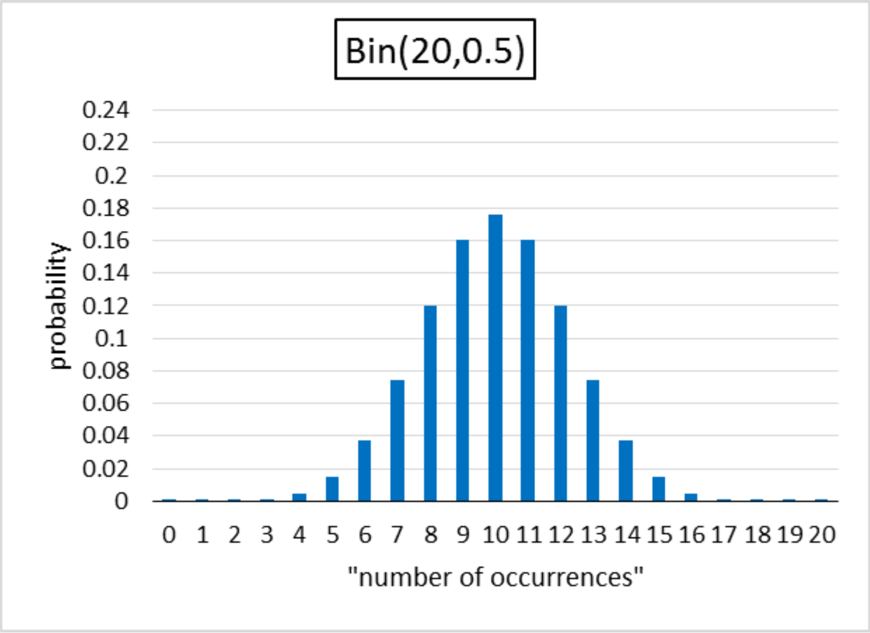
- Random variables and their corresponding probabilities can be described through probability distributions
- Examples
  - Binomial distribution
  - Uniform distribution
  - Normal distribution
  - Exponential distribution
  - Poisson distribution



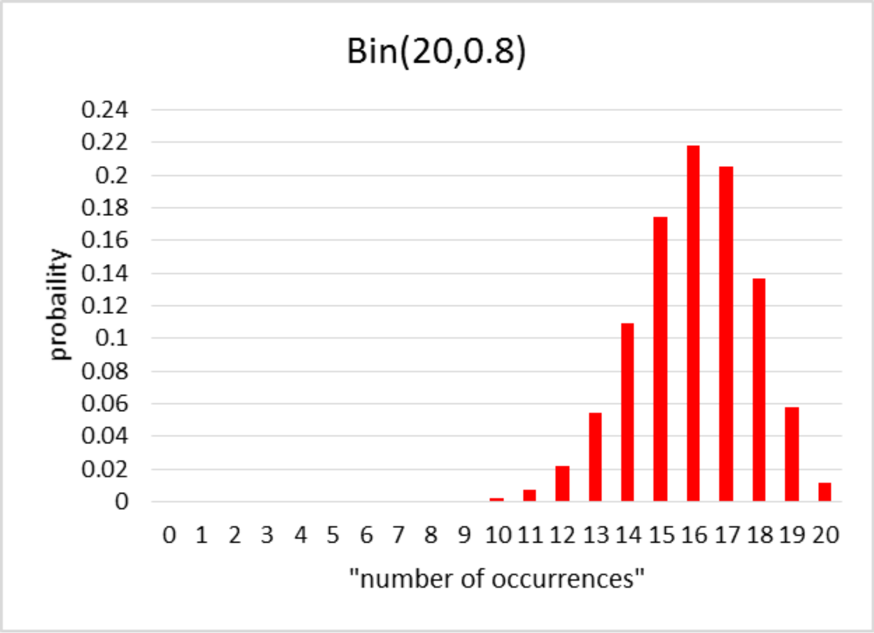
Binomial distribution Bin(n,p)

Number of occurrences when
n=´number of trials'
p='probability of success'

An example of a discrete probability distribution
n and p are parameters



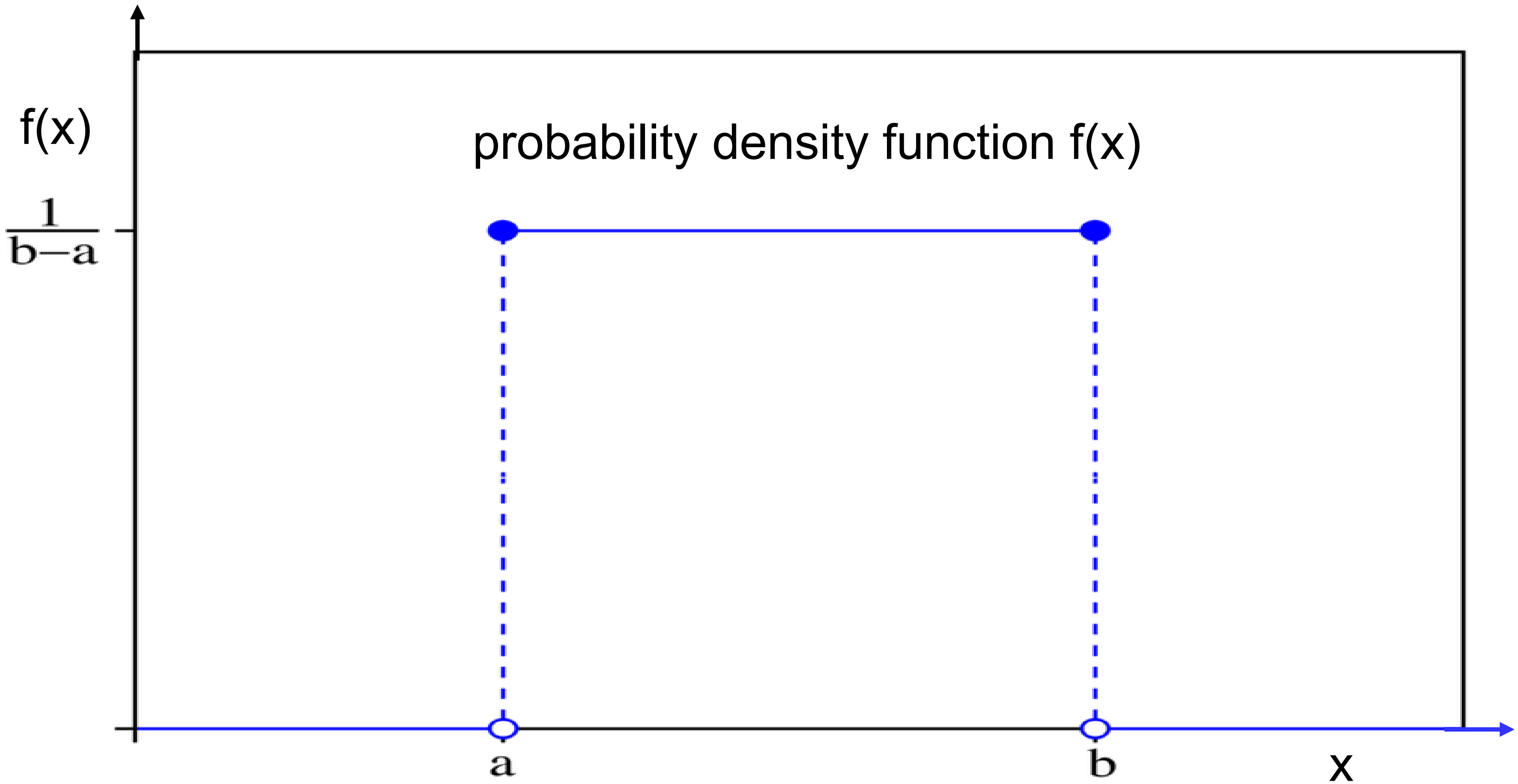
n=20, p=0.5



n=20, p=0.8

Uniform distribution U(a,b)

An example of a continuous probability distribution

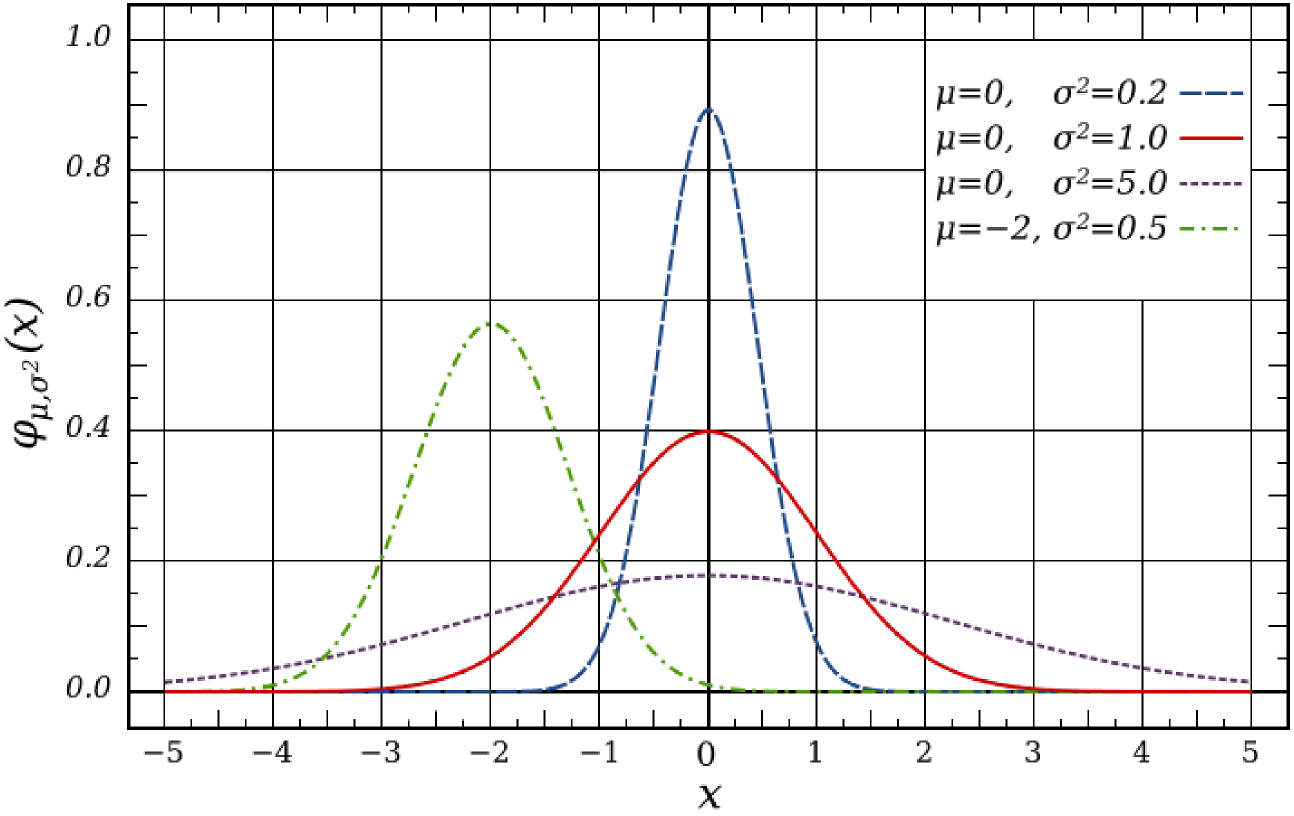


Parameters a and b; how do interpret them in this case?

Normal distribution N(μ,σ^2)

μ='average' σ='standard deviation'

μ and σ are parameters

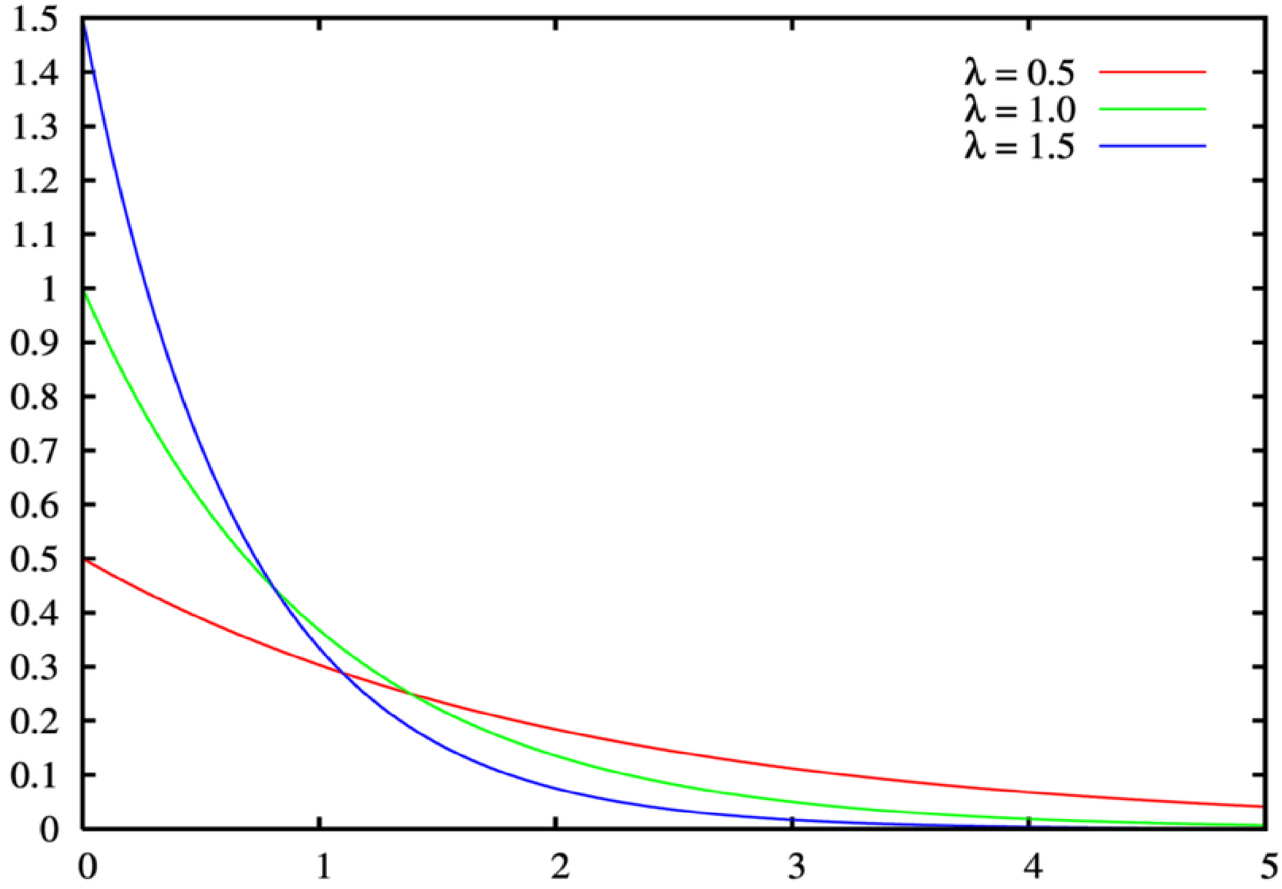


Exponential distribution Exp(λ)

Typically: Waiting time when

λ='average number of occurrences per time unit'

only one parameter λ

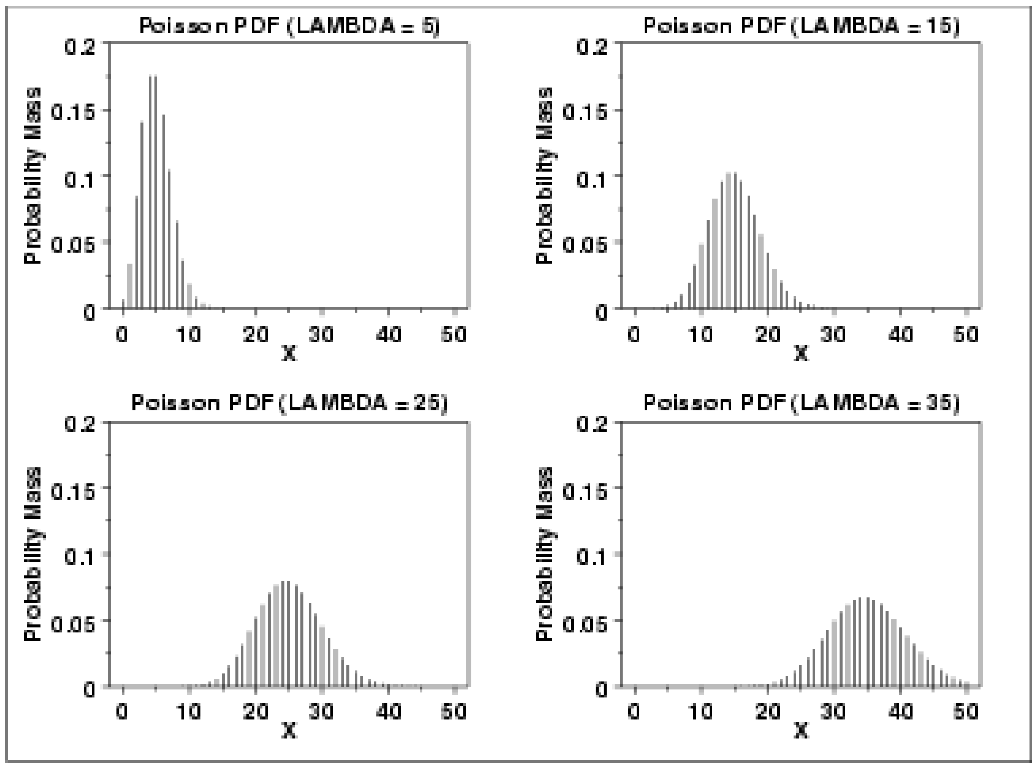


Poisson distribution Poisson(λ)

Number of occurrences when

λ='average number of occurrences per time unit'

only one parameter λ



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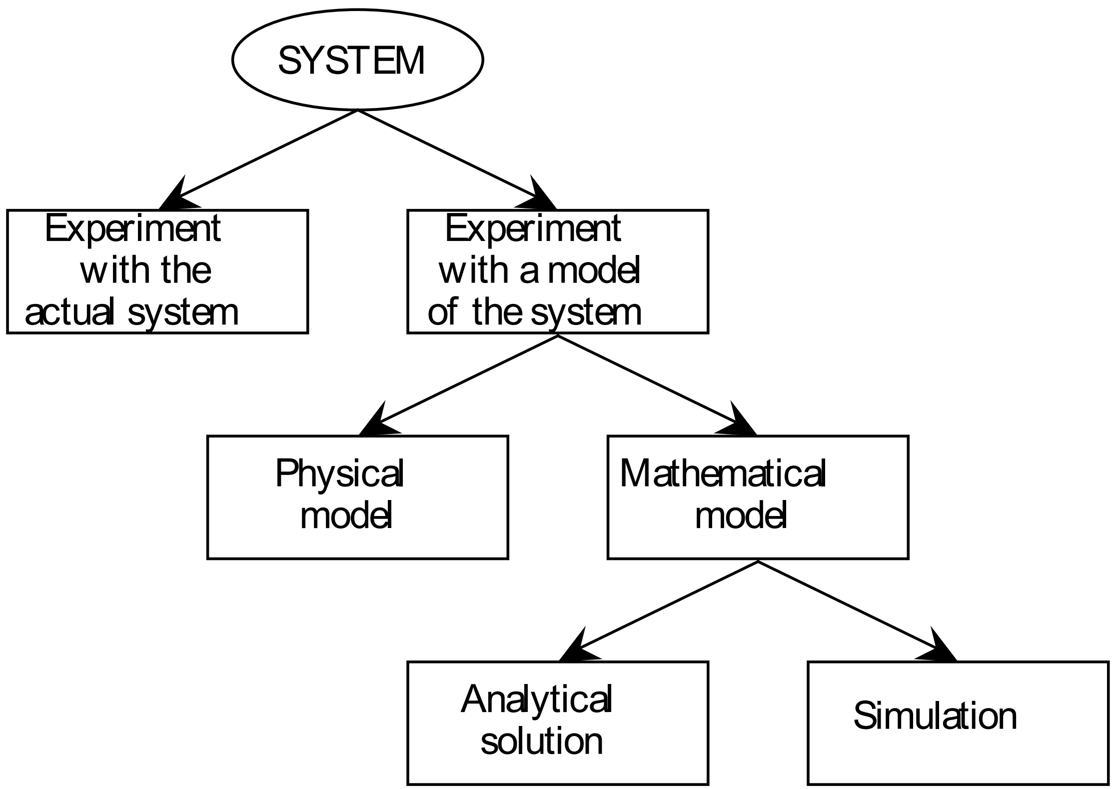
Monte Carlo Simulation

- **Simulation** is a method that is used to give new knowledge of the system of interest by imitating a system of interest artificially, often with the help of a computer.
- **Monte Carlo Simulation** is a simulation technique that uses random numbers and probability to solve problems involving variability and uncertainty but it can also be used to deterministic problems

Examples

- Testing cars and airplanes
- Practice of astronauts
- Traffic engineering and disaster response planning
- Production and inventory simulation
- Business Games
- Finance: e.g. simulation of stock prices
- Risk and reliability
- Population growth
- How does corona virus spread?
- Optimization
- Approximating intractable integrals

How to study a system



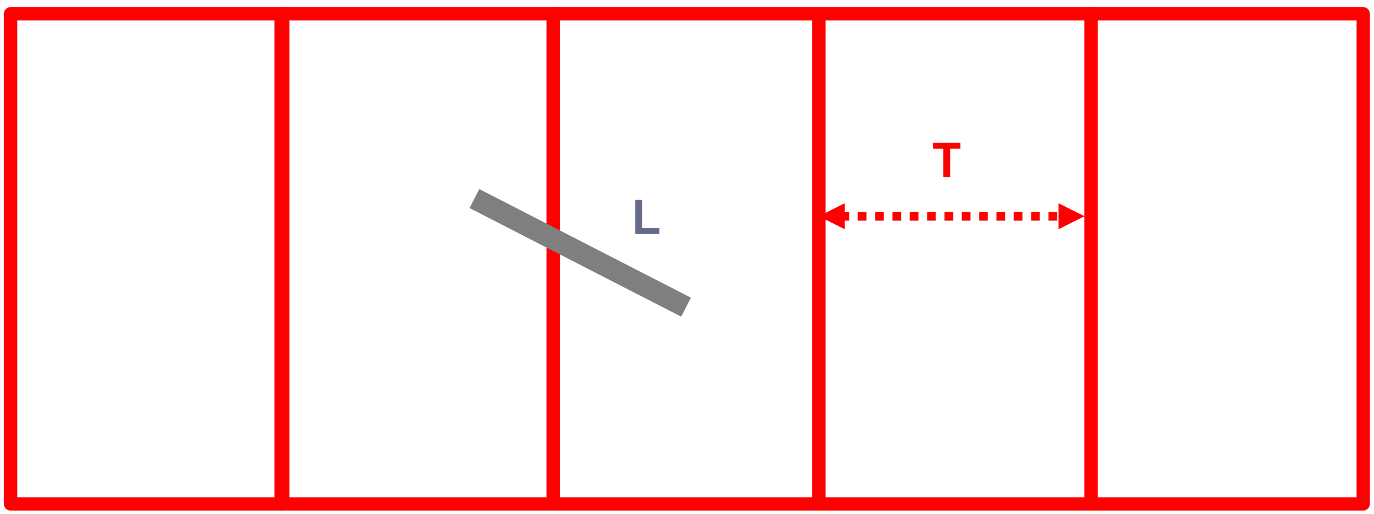
Some History of Monte Carlo Simulation

- **Monte Carlo Sampling** was a code name for the Manhattan Project at Los Alamos for the atom bomb during the second world war.
- In this project von Neumann and Ulam used simulation for the probabilistic problems concerned with random neutron diffusion in fissile material
- However, already in the second half of the nineteenth century simulation experiments were performed by throwing needles randomly on the table to approximate the value of PI (Buffon's needle problem)

Buffon's needle problem

https://en.wikipedia.org/wiki/Buffon%27s_needle_problem
https://www.youtube.com/watch?v=sJVivjuMfWA

Drop a needle randomly on floor with parallel lines with equal distance to each other. What is the probability p that the needle will lie across a line?
L=the length of a needle
T=distance between two lines



It turns out that that if L≤T, the probability $p = \frac{2L}{\pi T}$

If this is simulated enough times, we get a numerical estimate of p, and can calculate an estimate of π

Simulation modeling

- Simulation is based on a model, that is an abstraction, or simplification of the real system, but includes all the relevant aspects of the system. The model is a proxy to the real system.

Goals of Simulation

- Give an overview of the real system using statistics (e.g. mean, standard deviation or probability distribution)
- Estimating profit or cost
- Forecasting
- Estimating probabilities of events (e.g. risks)
- Testing a model

Advantages of studying the system vs. a model

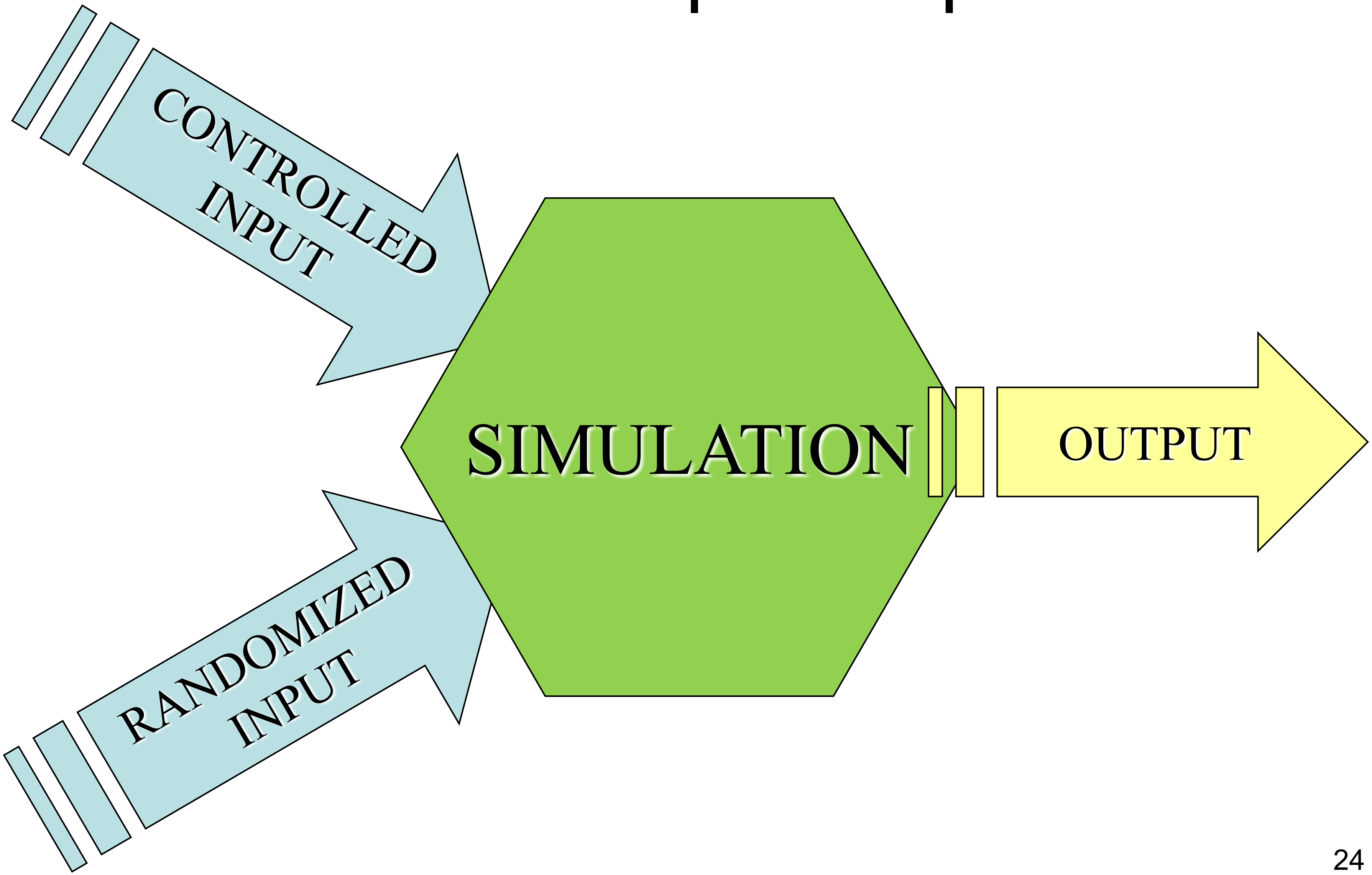
System

- No concerns about validity
- May be impractical or impossible (system may not exist)

Model

- Must be concerned with validity
- Generally much easier to work with
- Can “exercise” it for many more situations than system

Simulation as input-output -model



Types of Systems (and Models)

		Deterministic (Scenarios)	Stochastic (Monte Carlo)
		No randomness Inputs are exact, no uncertainty One model needs only one run	Random inputs—uncertain Inputs are from "known" probability distributions One model needs *more than* one run
Static	No time element	Financial scenarios Cash flows Can use regression models dependent vs. independent variable combinations	Classical "Monte Carlo" simulation Estimate intractable sums or integrals (e.g. average) Get empirical distribution of a new test statistic for some null hypothesis One period asset pricing
Dynamic	Passage of time is important part of model	Cash flows with discounting Differential-equation models of population growth and decay Deterministic forecasting over time Dynamic macroeconomic models	Queueing models representing service, manufacturing, computer, or communications systems Inventory models Multi-period asset pricing Option pricing Forecasting
		Compute "*exactly*" desired output quantities; but no information on their accuracy or uncertainty!	Can only estimate desired output quantities but can consider their accuracy and uncertainty of inputs and outputs!

Example (deterministic model): Intake of harmful content in food

- Amount of harmful content: p=0,150 %
- How much eat (kg): X=2
- Intake of harmful content (g): Y=pX=3
- How much is critical (g)? C=4
- Is it harmful (Y>C)?

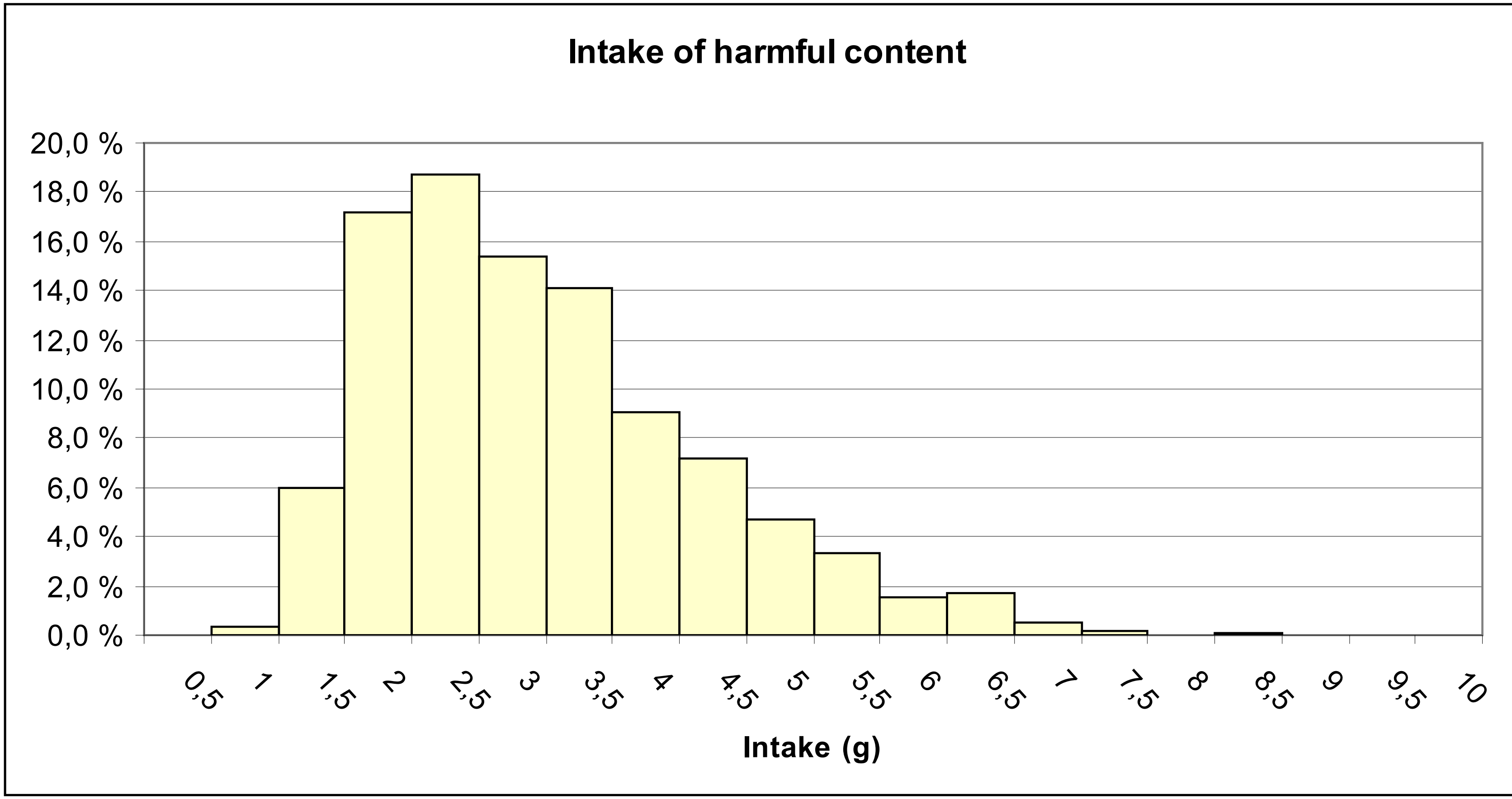
Monte Carlo Simulation model Intake of harmful content

- The model includes stochastic elements
- Amount of harmful content: $p \sim N(0.15\%, (0.03\%)^2)$
- How much eat (kg): $X \sim Beta(1,2,1,4)$
- Intake of harmful content (g): $Y = pX$
- How much is critical? $C \sim N(4, 0.8^2)$
- Is it harmful ($Y > C$)? OUTPUT

Results of simulation (estimates)

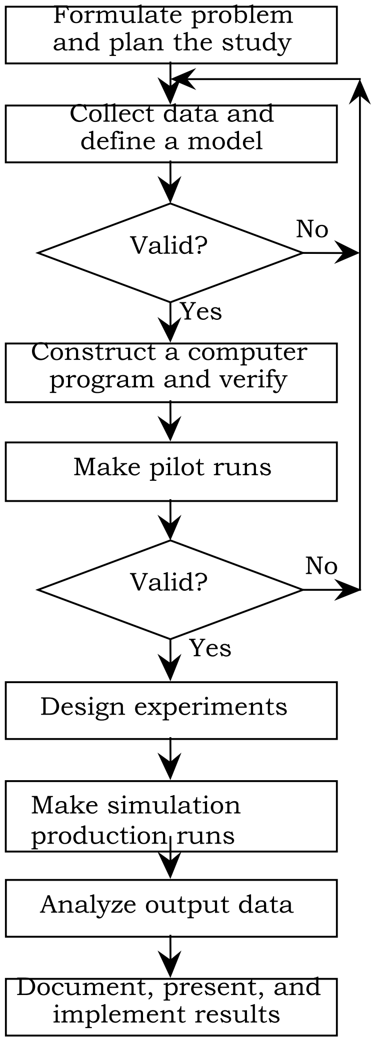
	Amount of harmful content:	How much eat (kg):	Intake (g)	How much is critical (g)?	Is it harmful (Y>c)?
	p	X	Y=p*X	c	YES/NO
min	0.059 %	1.001	0.790	1.830	0
max	0.237 %	3.850	7.827	7.116	1
average	0.151 %	1.986	3.006	3.990	0.250
median	0.152 %	1.830	2.719	4.004	0
st.dev	0.030 %	0.708	1.263	0.793	0.433
st.error of avg	0.001 %	0.022	0.040	0.025	0.014

Estimated distribution from simulation



Steps in a Simulation Project

(Law and Kelton)



Things to consider when building a simulation model

1. **Structural factors of the model**

- Physical/logical relationships among components
  - Variables of the model
  - “Rules” of the model
  - Possible decisions and their consequences
  - Feedback

2. **Quantitative factors of the model**

- Specific numerical assumptions of the variables
  - Possible values
  - Probability distributions used
  - Statistical dependencies between the variables (e.g. correlation)

Elements of both structural and quantitative components can become variables (or factors) in the design of simulation experiments

Advantages and Disadvantages of Simulation

Compared to experimenting with the actual system:

+ Often the only possibility, because the actual system cannot be studied - or it does not even exist
+ Much more flexibility to try things out before building the actual system
+ Flexibility to control for different variables
+ Helps to understand the actual system
- Simulation never corresponds fully to the actual system: validity and uncertainty

Advantages and Disadvantages of Monte Carlo Simulation

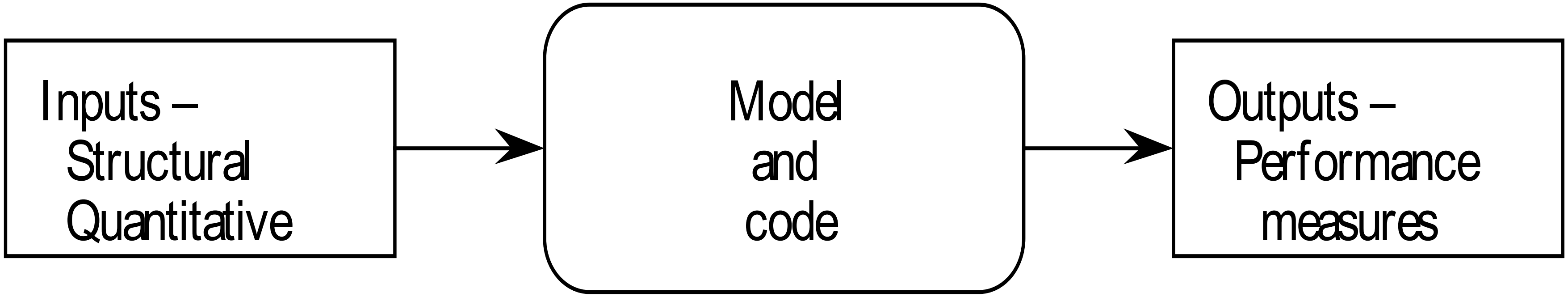
Compared to the exact analytical or mathematical model:

- \+ Makes it possible to study more complicated models, which do not have an analytical solution (or solution is difficult)
- \+ Don’t have to make as many simplifying assumptions—get more flexible models that can be more valid
- \+ Can include randomness in a controlled way
- \+ Correlations and other inter-depencies can be included
- \+ Changes to the model are quite easy and quick to do
- \- Don’t get simple formulas, which could help to understand the system
- \- Don’t get exact answers—only estimates, which Include uncertainty - that can and should also be estimated

Building "good" simulation models

- Verification—Code is correct (in whatever programming language or program)
- Validation—Model (as expressed in the verified code) faithfully mimics the system to study; can use model/code as surrogate for system to make decisions
- Statistical reliability—Accuracy. How much is the error due to random variation?
- Credibility—The valid model is accepted by decision makers; critical for implementation success

“Machine” View of What a Simulation Does:



What is needed to be able to simulate?

- In simulation you need a **model**, which represents reality with a reasonable accuracy
- You need to know the **relationships** between different parts of the model
- Many of the relationships are given with mathematical **formulas** or functions; they often include randomness
- You need to know which parts (and relationships) of the model are assumed random (stochastic) and which are assumed **fixed**; this may be a choice of the modeller/decision maker
- You need the numerical values of the **parameters** appearing in the model
- You need the **probability distributions** of the random parts of the model as well as their parameters
- Often in simulation several models are combined to represent a more complicated **system** or process

When simulation is **not** appropriate (Banks & Gibson 1997)

1. Simulation should not be used when the problem can be solved by common sense.
2. **Simulation is not appropriate if the problem can be solved analytically**.
3. Simulation is not reasonable when the real-world experiment is more easily (or less costly etc.) done than using simulation.
4. Simulation is not appropriate if the cost exceeds savings when using simulation.
5. There should also be resources available, and if there is not, simulation is not appropriate.
6. Simulation is not useful if there is not enough time to use the simulation results.
7. For simulating, there needs to be enough data, and if there is not, another way than simulation should be used.
8. Simulation should not be used if the simulation model can't be verified or validated.
9. Simulation is not appropriate if the project expectations cannot be met.
10. Simulation is not appropriate if the system to be investigated is too complex to be simulated.

When simulation is useful (Banks 2005)

1. Simulation enables the study of, and experimentation with, the internal interactions of complex system.
2. Informational, organizational, and environmental changes can be simulated, and the effect of these can be observed.
3. The knowledge developed when designing the simulation model can be used for improving the system under investigation.
4. Changing simulation inputs and observing the outputs can produce valuable insight of determining the interaction of the variables.
5. Simulation can be used as a pedagogical device to reinforce analytical solution methodologies.
6. Simulation can be used to experiment with new designs or policies before implementation, so as to prepare for what might happen.
7. Simulation can be used to verify analytical solutions.
8. Simulating different capabilities for a machine can help to determine the requirements on it.
9. Simulation models designed to training can ease the learning process with less cost and disruption.
10. Simulation can be used to visualize the behaviour of the system.
11. The modern system is so complex that its internal interactions can only be treated through simulation.

Some related concepts

Virtual_reality

- Related to the interactive senso-motoric experience in a simulated environment

Augmented_reality

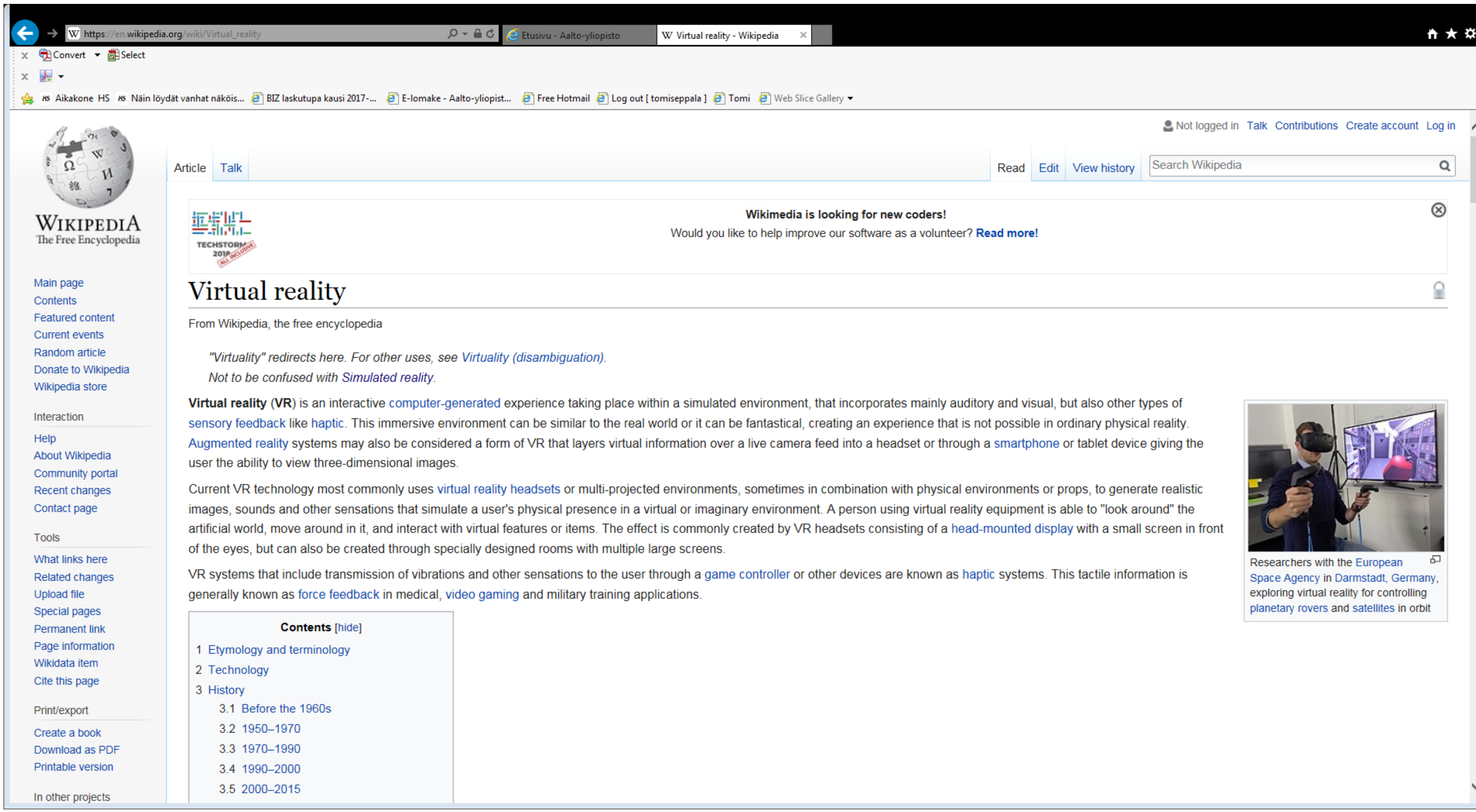
- Adds digital components to the real world

Simulated_reality

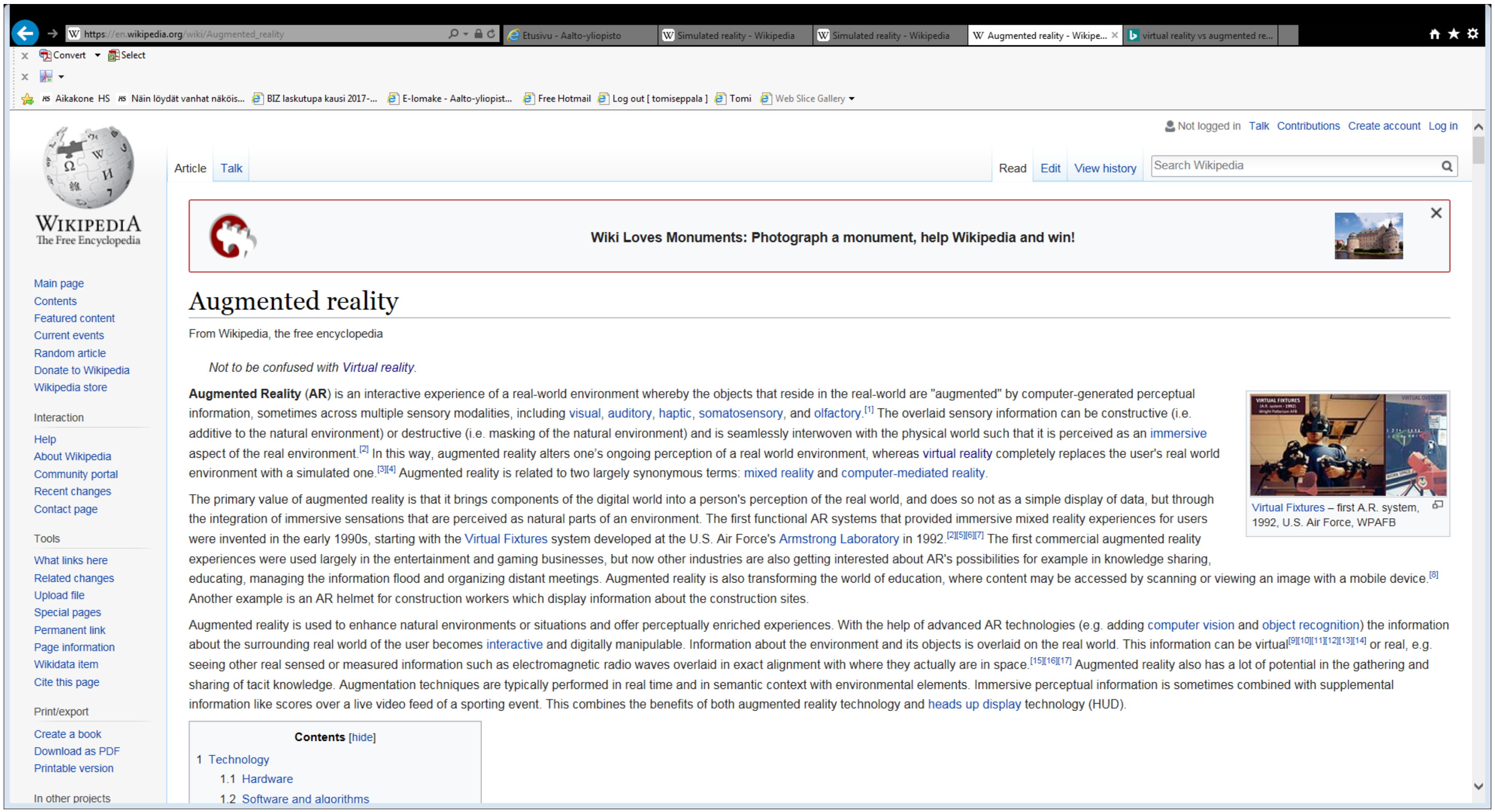
- Related to the philosophical argument or hypothesis claiming that true reality can be artificially exactly simulated

These concepts, although clearly of utmost interest, are not part of this course

Virtual reality



Augmented reality



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Augmented reality

From Wikipedia, the free encyclopedia

Not to be confused with Virtual reality.

Augmented Reality (**AR**) is an interactive experience of a real-world environment whereby the objects that reside in the real-world are "augmented" by computer-generated perceptual information, sometimes across multiple sensory modalities, including visual, auditory, haptic, somatosensory, and olfactory.[1] The overlaid sensory information can be constructive (i.e. additive to the natural environment) or destructive (i.e. masking of the natural environment) and is seamlessly interwoven with the physical world such that it is perceived as an immersive aspect of the real environment.[2] In this way, augmented reality alters one's ongoing perception of a real world environment, whereas virtual reality completely replaces the user's real world environment with a simulated one.[3][4] Augmented reality is related to two largely synonymous terms: mixed reality and computer-mediated reality.

Virtual Fixtures – first A.R. system, 1992, U.S. Air Force, WPAFB

The primary value of augmented reality is that it brings components of the digital world into a person's perception of the real world, and does so not as a simple display of data, but through the integration of immersive sensations that are perceived as natural parts of an environment. The first functional AR systems that provided immersive mixed reality experiences for users were invented in the early 1990s, starting with the Virtual Fixtures system developed at the U.S. Air Force's Armstrong Laboratory in 1992.[2][5][6][7] The first commercial augmented reality experiences were used largely in the entertainment and gaming businesses, but now other industries are also getting interested about AR's possibilities for example in knowledge sharing, educating, managing the information flood and organizing distant meetings. Augmented reality is also transforming the world of education, where content may be accessed by scanning or viewing an image with a mobile device.[8] Another example is an AR helmet for construction workers which display information about the construction sites.

Augmented reality is used to enhance natural environments or situations and offer perceptually enriched experiences. With the help of advanced AR technologies (e.g. adding computer vision and object recognition) the information about the surrounding real world of the user becomes interactive and digitally manipulable. Information about the environment and its objects is overlaid on the real world. This information can be virtual[9][10][11][12][13][14] or real, e.g. seeing other real sensed or measured information such as electromagnetic radio waves overlaid in exact alignment with where they actually are in space.[15][16][17] Augmented reality also has a lot of potential in the gathering and sharing of tacit knowledge. Augmentation techniques are typically performed in real time and in semantic context with environmental elements. Immersive perceptual information is sometimes combined with supplemental information like scores over a live video feed of a sporting event. This combines the benefits of both augmented reality technology and heads up display technology (HUD).

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Simulated reality