1. $z = Ax + By + c$ $\quad\quad\quad\quad\quad\quad\quad\quad$ $A+B$

$E[z] = \bar{z} = E[Ax + By + c]$

$= E[Ax] + E[By] + E[c]$ $\quad\quad\quad\quad$ $(Mx)^T$

$\bar{z} = A\bar{x} + B\bar{y} + c$ $\quad\quad\quad\quad\quad\quad$ $= x^T M^T$

$\text{cov}(z) = E[(z-\bar{z})(z-\bar{z})^T]$

$= E[(Ax + By + c - A\bar{x} - B\bar{y} - c)(Ax + By + c - A\bar{x} - B\bar{y} - c)^T]$

$= E[(A(x-\bar{x}) + B(y-\bar{y}))(A(x-\bar{x}) + B(y-\bar{y}))^T]$

$= E[(A(x-\bar{x}) + B(y-\bar{y}))((x-\bar{x})^T A^T + (y-\bar{y})^T B^T)]$

$= E[A(x-\bar{x})(x-\bar{x})^T A^T + A(x-\bar{x})(y-\bar{y})^T B^T + B(y-\bar{y})(x-\bar{x})^T A^T + B(y-\bar{y})(y-\bar{y})^T B^T]$

$= E[A(x-\bar{x})(x-\bar{x})^T A^T] + E[A(x-\bar{x})(y-\bar{y})^T B^T] + E[B(y-\bar{y})(x-\bar{x})^T A^T]$
$+ E[B(y-\bar{y})(y-\bar{y})^T B^T]$

$= AE[(x-\bar{x})(x-\bar{x})^T]A^T + AE[(x-\bar{x})(y-\bar{y})^T]B^T + BE[(y-\bar{y})(x-\bar{x})^T]A^T$
$+ BE[(y-\bar{y})(y-\bar{y})^T]B^T$

$\text{cov}(z) = AP_{xx}A^T + AP_{xy}B^T + BP_{yx}A^T + BP_{yy}B^T$

$n_z \times n_x \times n_x \times 1$

2.) Dimension of A, B, c

$n_z \times n_x \times n_z \times 1$

A : $n_z \times n_x$

$z = Ax + By + c$

B : $n_z \times n_y$

c : $n_z \times 1$

$n_z \times 1$

$n_x \times 1$

Exercise 2.

$$x_{k+1} = x_k + v_k \Delta t_k \cos(\theta_k) \leftarrow$$
$$y_{k+1} = y_k + v_k \Delta t_k \sin(\theta_k) \leftarrow$$
$$\theta_{k+1} = \theta_k \frac{\Delta t_k v_k}{L} \tan(\phi_k)$$
$$v_{k+1} = v_k$$
$$\phi_{k+1} = \phi_k$$

$$X_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ v_k \\ \phi_k \end{bmatrix}$$

$$X_{k+1} = f(X_k, t_k)$$

f is a vector valued function.

$$\left.\frac{\partial f}{\partial X}\right|_{X=X_k} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \cdots & \cdots & \\ \vdots & & & & \\ \frac{\partial f_5}{\partial x_1} & \frac{\partial f_5}{\partial x_2} & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -v_k \Delta t_k \sin(\theta_k) & \Delta t_k \cos(\theta_k) & 0 \\ 0 & 1 & v_k \Delta t_k \cos(\theta_k) & & \\ & & & & \end{bmatrix}$$

$$f \qquad \bar{x}_k = E[X_k]$$

Extended Kalman Filter.

$$f(x_k, \dots) = f(\bar{x}_k \dots) + \left.\frac{\partial f}{\partial x}\right|_{x=\bar{x}_k} (x_k - \bar{x}_k) +$$

$$x_{k+1} = f(x_k \dots) = f(\bar{x}_k) + J(x_k - \bar{x}_k)$$

$$\mathrm{cov}(x_{k+1})$$

$$\mathrm{Cov}(X_k) = P_{x_k x_k}$$

$$X_{k+1} = f(\bar{x}_k) + J x_k - J \bar{x}_k$$
$$X_{k+1} = c + J x_k$$
$$\mathrm{cov}(X_{k+1}) = J P_{x_k x_k} J^T$$

$$\int p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,dx_{k-1} = p(x_k|x_{k-2})$$

$$p(x_k, x_{k-1}, x_{k-2}) = \underbrace{p(x_k|x_{k-1}, x_{k-2})}_{\Downarrow}\,p(x_{k-1}, x_{k-2})$$

$$= p(x_k|x_{k-1})\,p(x_{k-1}, x_{k-2})$$

$$p(x_k, x_{k-1}, x_{k-2}) = p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,p(x_{k-2})$$

$$\underbrace{\int p(x_k, x_{k-1}, x_{k-2})\,dx_{k-1}}_{\Downarrow} = \int p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,p(x-2)\,dx_{k-1}$$

$$\underbrace{p(x_k, x_{k-2})}_{\Downarrow} = \int p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,p(x_{k-2})\,dx_{k-1}$$

Bayes: $p(x_k|x_{k-2})\,p(x_{k-2}) = \int p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,p(x_{k-2})\,dx_{k-1}$

$$p(x_k|x_{k-2})\,\cancel{p(x_{k-2})} = \cancel{p(x_{k-2})}\int p(x_k|x_{k-1})\,p(x_{k-1}|x_{k-2})\,dx_{k-1}$$

σ_X : standard deviation.

$\sigma_x = \sqrt{\text{variance}_x}$

$$\rho_{XY} = \frac{\text{cov}(X,Y)}{\sigma_X\,\sigma_Y}$$

$$= \frac{-3}{2\times 3} = -0.5$$

$$\rho_{XY} = 1$$

$$X = \alpha + \beta Y$$