ELEC-E8107 - Stochastic models, estimation and control

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Exercises Session 1

Exercise 1

Given the random variables x and y of dimensions n_x and n_y, with means $\bar{x}$ and $\bar{y}$ respectively, and with covariances matrices P_{xx}, P_{yy} and P_{xy}:

1. Find the mean and covariances of the n_z-dimensional vector $z = Ax + By + c$, where A and B are matrices of appropriate dimensions and c is a vector.

2. Indicate the dimensions of A, B and c.

Exercise 2

A nonlinear system dynamic model of the car shown in Fig 1 is given by the following equation.

$$\begin{cases} x_{k+1} = x_k + cos(\theta_k)\Delta t_k v_k \\ y_{k+1} = y_k + sin(\theta_k)\Delta t_k v_k \\ \theta_{k+1} = \theta_k \frac{\Delta t_k v_k}{L} tan(\Phi_k) \end{cases} \tag{1}$$

Where v is the speed of the vehicle, θ is the heading and Φ the steering angle. The state of the vehicle can be define as the vector $X_k = \left[x_k, y_k, \theta_k, v_k, \Phi_k\right]^T$. The equation (1) can be written as $X_{k+1} = f(X_k, t_k)$.

1. Compute the Jacobian of the function $f(X_k, t_k)$ with respect to the state vector X_k

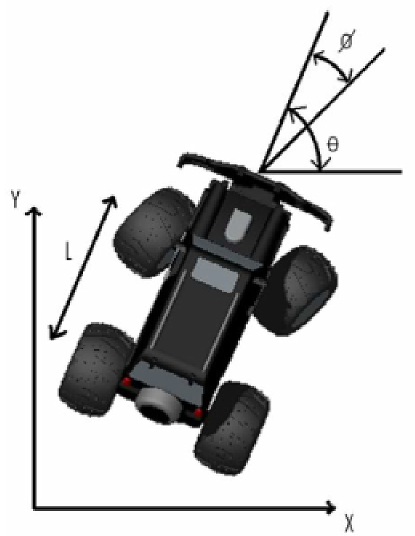

Figure 1: Simple car kinematic model markings: The distance between the axles of the vehicle is described by L, the direction is described by θ and the steering angle by Φ. The vehicle navigation point is in the center of the rear axle.

Exercise 3

Prove that the following equation holds for a discrete time Markov process

$$\int p(x_k|x_{k-1})p(x_{k-1}|x_{k-2})dx_{k-1} = p(x_k|x_{k-2})$$

Exercise 4

The covariance matrix of random variables X and Y happens to be:

$$Q = \begin{bmatrix} 4 & -3 \\ -3 & 9 \end{bmatrix}$$

1. Find the variance of X and Y?
2. Compute the correlation coefficient between the two random variables.