## KURSSIN SISÄLTÖ

Periodi I:

Johdanto
Vektorianalyysi ja -algebra
Nabla
Sähköstatiikka
Varausjakautumien kentät, Laplacen yhtälö, dipoli
Magnetostatiikka
Sähkövirran magneettivaikutus
Dynaamiset sähkömagneettiset kentät


Periodi II:

Aikaharmoniset sähkömagneettiset kentät
Kompleksivektorit
Tasoaallot
Eteneminen ja vaimeneminen
Heijastuminen rajapinnoista ja läpäisy
Sähkömagneettisten aaltojen synnyttäminen
Antennit ja radioyhteydet


KENTTÄTEORIA

Kentät

$$F(\underbrace{x,y,z}_{\bar{r}};t)$$

$$\bar{E}(\bar{r},t)$$

Skalaarikentät ja vektorikentät

SI-yksiköt

kg m s A

(V W T)

johdannaisyksikköjä

SÄHKÖKENTÄN VOIMAKKUUDEN YKSIKKÖ

Laadut

Sähkökentän voimakkuus $\bar{E}$ $[\bar{E}] = \frac{V}{m}$

Magneettikentän voimakkuus $\bar{H}$ $[\bar{H}] = \frac{A}{m}$

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Magneettikentän voimakkuus $\bar{H}$ $[\bar{H}] = \frac{A}{m}$

Sähkövuon tiheys $\bar{D}$ $[\bar{D}] = \frac{As}{m^2}$

Magneettivuon tiheys $\bar{B}$ $[\bar{B}] = \frac{Vs}{m^2} = T$ (tesla)

TYHJÖ
TYHJIÖ

Sähkömagnetiikan vakiot

TYHJIÖN PERMITTIIVISYYS $\varepsilon_0 = 8{,}854 \cdot 10^{-12} \frac{As}{Vm}$

—"— PERMEABIILISUUS $\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

$$\begin{cases} \bar{D} = \varepsilon_0 \bar{E} \\ \bar{B} = \mu_0 \bar{H} \end{cases}$$

Sähkön lähteet

$Q$ $C = As = [Q]$

ALKEISVARAUS $e = -1{,}602 \cdot 10^{-19}$ As

TILAVUUSVARAUS (VARAUSTIHEYS)
$\varrho$ $\frac{C}{m^3} = [\varrho]$

PINTAVARAUS
$\varrho_s$ $\frac{C}{m^2} = [\varrho_s]$

VIIVAVARAUS $q$ $\frac{C}{m} = [q]$

VIRRANTIHEYS $\bar{J}$ $\frac{A}{m^2} = [\bar{J}]$

Maxwellin yhtälöt

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

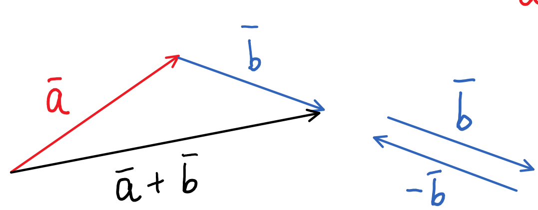
$$\nabla \cdot \bar{D} = \varrho$$

$$\nabla \cdot \bar{B} = 0$$

Vektorialgebraa

lihavoitu

$\bar{a}$ $\mathbf{a}$ a



$$\bar{a} - \bar{b} = \bar{a} + (-\bar{b})$$

### Yksikkövektori

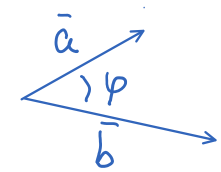
$$\bar{a} = |\bar{a}|\,\bar{u} \qquad \bar{u} = \frac{\bar{a}}{|\bar{a}|} \qquad |\bar{a}| = a$$

### Skalaarilla kertominen

$$\alpha \bar{a} = \alpha |\bar{a}|\,\bar{u} = |\bar{a}|\,\alpha\,\bar{u}$$
$$= \bar{a}\,\alpha$$

### Pistetulo

$$\bar{a} \cdot \bar{b}$$
$$= |\bar{a}||\bar{b}|\cos\varphi$$



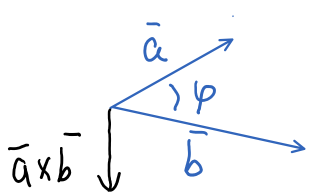

VAIHDANNAINEN: $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$

$$\bar{a} \cdot \bar{a} = |\bar{a}||\bar{a}|\underbrace{\cos 0}_{+1} = |\bar{a}|^2$$
$$\Rightarrow |\bar{a}| = \sqrt{\bar{a} \cdot \bar{a}}$$

$$\bar{a} \cdot \bar{b} = a_x b_x + a_y b_y + a_z b_z$$
$$\bar{a} = a_x \bar{u}_x + a_y \bar{u}_y + a_z \bar{u}_z$$
$$\bar{b} = b_x \bar{u}_x + b_y \bar{u}_y + b_z \bar{u}_z$$

### Ristitulo

$$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\varphi$$




VASTAVAIHDANNAINEN
$$\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$$

### bac-cab-sääntö

$$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{b}\,(\bar{a} \cdot \bar{c}) - \bar{c}\,(\bar{a} \cdot \bar{b})$$

$$(\bar{a} \times \bar{b}) \times \bar{c} \neq \bar{a} \times (\bar{b} \times \bar{c})$$

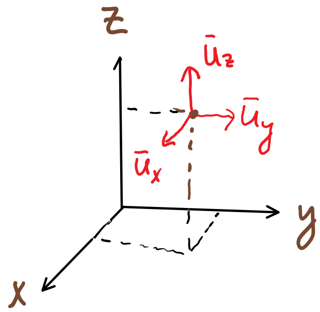
$\bar{b} \to \bar{a}$: $\quad \bar{a} \times \bar{a} = 0 \qquad \bar{a}\,(\bar{a} \cdot \bar{c}) - \bar{c}\,(\bar{a} \cdot \bar{a}) \neq 0$

### Koordinaatistot

## Koordinaatistot

### Karteesinen koordinaatisto

$x, y, z$




$$\bar{a} = a_x \bar{u}_x + a_y \bar{u}_y + a_z \bar{u}_z$$

$$a_x = \bar{u}_x \cdot \bar{a}$$

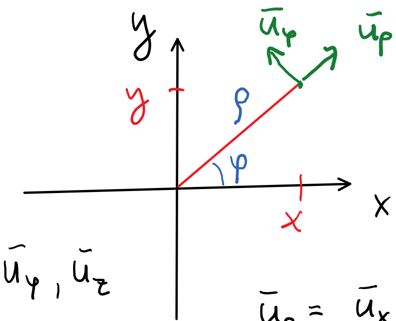
$$\bar{u}_x \cdot \bar{u}_x = 1 \qquad \bar{u}_x \cdot \bar{u}_y = 0$$

$$\bar{u}_x \times \bar{u}_y = \bar{u}_z \qquad \bar{u}_z \times \bar{u}_y = -\bar{u}_x$$

$$\bar{u}_y \times \bar{u}_x = -\bar{u}_z$$

etc.

### Sylinterikoordinaatisto $(\rho, \varphi, z)$




$$\rho = \sqrt{x^2 + y^2}$$

$$\tan\varphi = y/x$$

$\bar{u}_\rho, \bar{u}_\varphi, \bar{u}_z$

$$\bar{u}_\rho = \bar{u}_x \cos\varphi + \bar{u}_y \sin\varphi$$

$$\bar{u}_\varphi = -\bar{u}_x \sin\varphi + \bar{u}_y \cos\varphi$$

$$\bar{u}_\rho = \overset{f_\rho}{1} \cdot \bar{u}_\rho + \overset{f_\varphi}{0} \cdot \bar{u}_\varphi + \overset{f_z}{0} \cdot \bar{u}_z$$

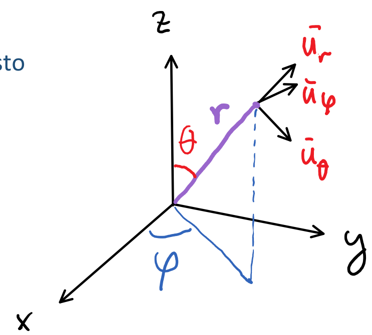
$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix}$$

$\cos\varphi$ $\sin\varphi$ $0$ $1$ $0$ $0$

$$\begin{pmatrix} f_\rho \\ f_\varphi \\ f_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$$

### Pallokoordinaatisto

$r, \theta, \varphi$




$$\bar{u}_r \times \bar{u}_\theta = \bar{u}_\varphi, \qquad \bar{u}_\theta \times \bar{u}_\varphi = \bar{u}_r, \qquad \bar{u}_\varphi \times \bar{u}_r = \bar{u}_\theta$$

$$\bar{u}_r = \bar{u}_z \cos\theta + \bar{u}_x \sin\theta\cos\varphi + \bar{u}_y \sin\theta\sin\varphi$$

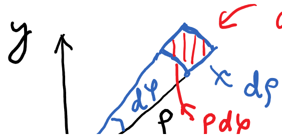
### Muunnoskaavat

### Kaavakokoelma

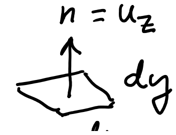
### Viiva-alkio

### Pinta-alkio

$$dS = dx\,dy$$



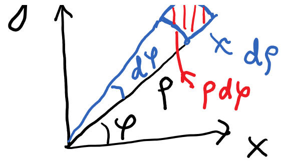

$$dS = \rho\, d\rho\, d\varphi$$

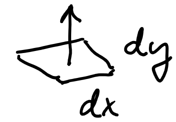


$dS = dx\,dy$

$dy$ □ $dx$

$d\bar{S} = \bar{u}_z\,dx\,dy$



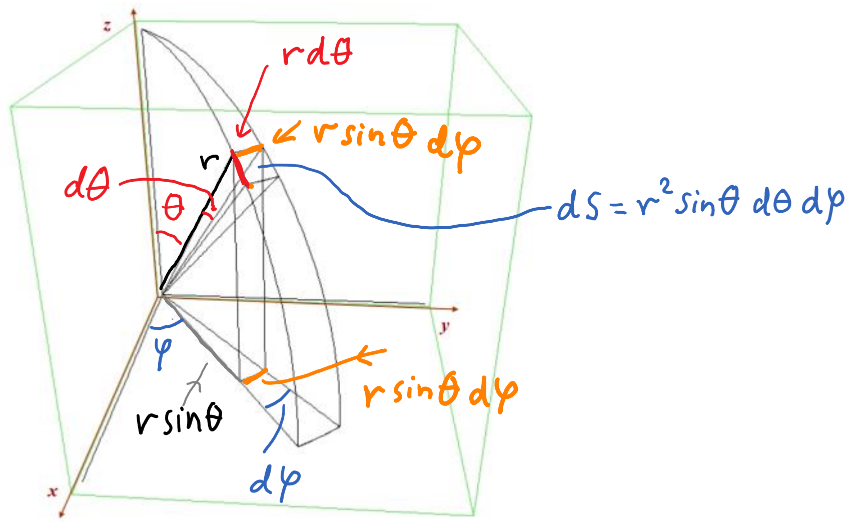

$d\bar{S} = \bar{u}_z\,dx\,dy$

Tilavuusalkio

$dV = dx\,dy\,dz$

$dV = \rho\,d\rho\,d\varphi\,dz$

$dV = r^2 \sin\theta\,dr\,d\theta\,d\varphi$




$$\int_S \underbrace{\bar{J}\cdot\overbrace{d\bar{S}}^{\bar{n}\,dS}}_{|\bar{J}|\,dS\cos\varphi}$$