Principles of Economics I 2022
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Problem Set 2 (Due 23.9.2022 at 10:00)

1. Let's practice a bit more with indifference curves and budget sets. Draw the indifference curves for a consumer with the following types of preferences over different goods.

   (a) A consumer that considers green and red t-shirts as interchangeable (a green t-shirt is equally good as a red one) but prefers a larger number of shirts to a smaller number.

   (b) A consumer that likes pairs of shoes (left and right) but has no use for unpaired left or right shoes.

   (c) Draw the budget set for a consumer that spends her budget I on food and housing. Put food x on the x-axis and housing y on the y-axis in a plane. Let p_x denote the price of food and p_y the price of housing so that the cost of consuming x, y is $p_x x + p_y y$. Draw the budget set for $I = 200, p_x = 4, p_y = 2$ draw also the budget set for $I = 100, p_x = 2, p_y = 1$. In the latter case, we say that a unit of housing is the numeraire good (since $p_x = 2$ indicates that a unit of food costs the same as two units of housing and the total budget is equal to 100 units of housing). What do you observe when you compare the two budget sets and how do you explain this?

   (d) As in the previous part, let x be the consumption of food and y the consumption of housing. Consider the consumer whose MRS between food and housing depends on the ratio of her consumption so that MRS $= \frac{y}{x}$. This means simply that for example at $x = 30, y = 60$ she considers each unit of food to be equally desirable as two units of housing. From the two equations $MRS = MRT$ and the budget constraint $p_x x + p_y y = I$, solve the optimal consumption x^*, y^* of food and housing. (Hint: treat p_x, p_y and I as parameters in the problem, i.e. treat them as you would treat fixed numerical values).

2. At current wages of EUR 10 per hour Ann chooses to work for 8h per day. To reward her for good performance, her boss gives her a raise to EUR 15 per hour. Ann is lucky enough to be in a job where she can pick her own working hours.

(a) Can you say with certainty what will happen to Ann's working hours as a result of this raise?

(b) Ann computes that at her old working hours, the boss ends up paying EUR 40 more per day. She is tempted to go to the boss and ask for a different wage contract. A flat payment of EUR 40 per day and the old wage of EUR 10 on top. Draw the budget constraint for the alternative wage contract and for the EUR 15 per hour wage (without flat payments).

(c) What can the employer conclude about Ann's intended working hours if Ann chooses to make this proposal? (I.e. when does Ann benefit from the propose scheme relative to the 15 EUR wage per hour?)

(d) Suppose that the boss wants to induce Ann to work more. Rather than raising the wage, the boss gives a bonus EUR 5 per hour for each extra hour of overtime work (i.e. if Ann works for $t > 8$ hours, then her pay is $EUR80 + (t - 8)15$, for $t \leq 8$, the pay is unchanged). Draw Ann's budget set in this case.

(e) Continuing on the previous problem, draw Ann's indifference curves in such a way that is consistent with the choice of $t = 8$ in the original budget set and with $t = 10$ in the new budget set. What is Ann's average pay per hour in the new wage scheme at her optimally chosen working hours? How would Ann choose her working hours if she got paid this average wage for each of the hours that she works and no overtime bonus?

3. Consider next a simple game theoretic situation. There is a crossroads of two one-way roads. One of the roads runs from south to north while the other runs from west to east. Two drivers come to the crossroads simultaneously from different directions. Ann comes from the south while Bob arrives from the west. Ann and Bob must decide simultaneously whether to continue driving or wait.

   (a) Who are the players, what are the strategies, what are the outcomes and what are reasonable payoffs? In other words, draw a game matrix representing this situation.

   (b) Does either of the drivers have a dominant strategy? Are there Nash equilibria?

   (c) Explain the role of traffic rules for this game. Give a reason why traffic lights are used in some crossings, but not all.

4. Here is a simple example of a classic bargaining situation. Ann and Bob have altogether 10 (identical) apples and they need to decide how to divide them. Both like apples so getting more apples is preferred to fewer apples.

   (a) One possible way of doing this is to let Ann and Bob submit simultaneous demands: let x_A and x_B be the demand of Ann and Bob respectively and restrict the demands to be integers between 0 and 10. If $x_A + x_B \leq 10$, then Ann and Bob get the number of apples that they demanded. If $x_A + x_B > 10$, then the game ends in conflict and both players get 0 apples. Assume that the payoff of each player is the number of apples received at the end of the game. Does either of the players have a dominant strategy? What are the Nash-equilibria of the game? (Hint: you do not have to draw the 10 x 10 matrix to answer this, but just think carefully what you can say about best responses).

   (b) Do you think the material payoff (the number of apples received) is a reasonable preference for such bargaining situations? What other motivations might the payoffs take into account?

   (c) One often likes procedures that result in equal treatment of identical players. Comment on the previous method from this perspective and consider an alternative procedure where Ann divides the apples in two sets of apples and Bob chooses one of the sets. How should Bob choose? How should Ann divide? (This is called the divide and choose method).

5. Each of n players chooses whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$. If the good is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows: (i) any outcome in which the good is provided and she does not contribute, (ii) any outcome in which the good is provided and she contributes, (iii) any outcome in which the good is not provided and she does not contribute, (iv) any outcome in which the good is not provided and she contributes.

   (a) Formulate this situation as a strategic game. Does any of the players have a dominant strategy?

   (b) Is there a Nash equilibrium in which more than k players contribute?

(c) Is there one in which exactly k players contribute?

(d) One in which fewer than k players contribute?

6. (Extra credit) Consider the following modification to the bargaining game of Problem 4. Ann and Bob still submit simultaneous demands x_A, x_B and if $x_A + x_B \leq 10$, then each gets the demand they made. If $x_A + x_B > 10$, the conflict is resolved in arbitration and the rules of the arbitration are as follows: If one of the players was less greedy, she gets her demand (and the other player gets the remaining apples). If the demands were the same, then each player gets 5 apples.

   (a) Can either player receive more apples than she demanded?

   (b) Is there a way for either of the players to secure at least 5 apples?

   (c) What is Ann's best response if $x_B > 6$?

   (d) Can there be Nash equilibria where at least one of the players makes a demand strictly higher than 6?

   (e) What is the division of apples in any Nash equilibrium of the game?

   (f) (No question here) Since you did not have to draw the matrix, you can extend the argument above to any number of apples (say 100 or 1000) with simple modifications to the numbers 5 and 6 in the argument above to get the equivalent conclusion.