Problem 1.

You measure the *Insertion Loss* (IL) of a filter with a signal generator with 50 Ω output impedance and a spectrum analyzer with 1 MΩ input impedance. You have used this setup without impedance matching network and you realize this fact a few days later. What can you do in order to correct the results?

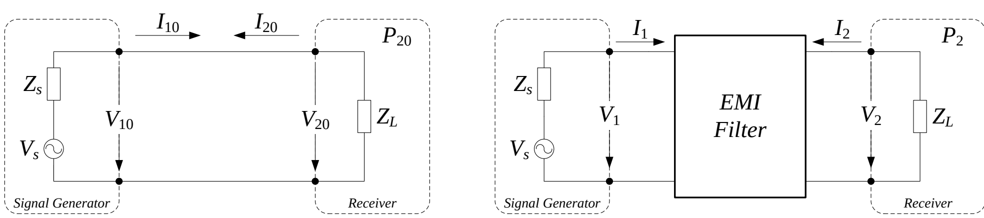
Problem 2.

Consider the setup for measuring insertion loss $IL = 10 \cdot \lg\left(\frac{P_{20}}{P_2}\right)$ as shown in figure below. The test item is a noise suppression capacitor with reactance *X*. The impedances of the signal generator and measuring equipment are resistive and equal to 50 Ω. Calculate and draw the insertion loss as a function of frequency in the following cases:

a) *X* is an ideal capacitor C = 10 μF.

b) *X* is an ideal inductor L = 1 μH.

c) *X* is a real capacitor with C = 10 μF and series inductance L = 1 μH.



Problem 3

A single-stage *LC*-filter, shown in the figure below, is used to suppress *common mode* (CM) noise from a power supply. Filter component values are: C_X = 47 nF, C_Y = 2.2 nF and L = 0.13 mH. Calculate the CM *IL* of the filter at 150 kHz and 1 MHz. Assume three different measurement resistances: 50 / 50 Ω, 0.1 / 100 Ω, and 100 / 0.1 Ω.

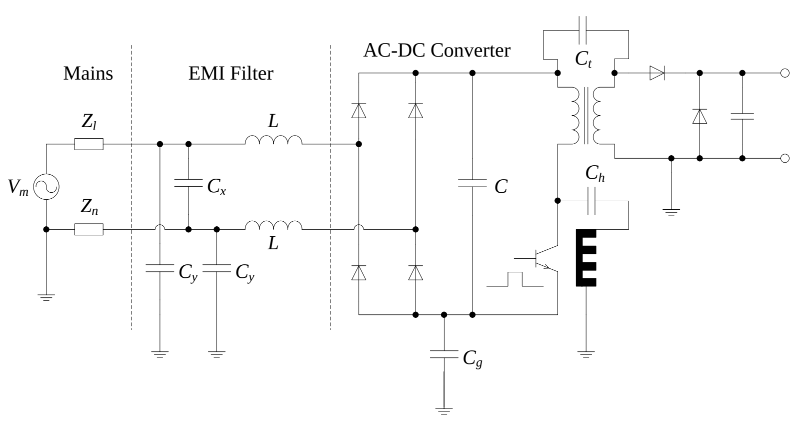


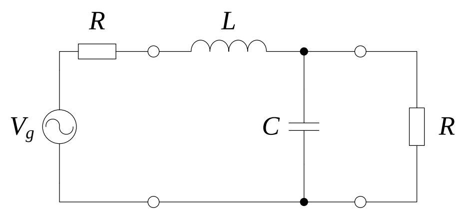
Fig. 1. A switch-mode power supply (SMPS) with input EMI filter.

Problem 4

This continues the previous problem: For the SMPS in Problem 3, draw the *differential mode* (DM) noise equivalent circuit of the filter and calculate the value of *DM insertion loss* (IL_{DM}) of that filter at 150 kHz and 1 MHz. Use the same values as in Problem 3 and compare the results with the previous ones for CM noise.

Problem 5

Design an *LC*-filter, which provides insertion loss of 50 dB at 150 kHz. The allowed 50 Hz voltage drop is 5 V when the current is 15 A and R = 5 Ω. Sketch the diagram of insertion loss versus frequency.

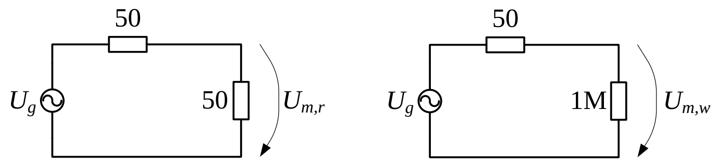


Problem 6

This problem highlights the case of mismatch due to low source impedance. Consider a single stage LC-filter (1 mH / 250 nF). The source impedance Z_g is assumed to be zero. The worst case *IL* is achieved when load impedance Z_L is purely inductive (R_L = 0, $L_L \neq 0$). What is the value of the load inductance L_L to have the worst case at 150 kHz? What is the value of *IL* at 150 kHz, if a 50 Ω resistive load is assumed?

S O L U T I O N S

Solution 1:



a) Measurement with an impedance matching network

b) Measurement without an impedance matching network

Fig. 2. Equivalent circuits for the measurements.

From Fig. 2*a)* the voltage measured by the instrument should be:

$$\frac{U_{m,r}}{U_g} = \frac{50}{50+50} \quad \Rightarrow \quad U_{m,r} = \frac{U_g}{2} \tag{1.1}$$

Instead, due to the wrong set up, the measured voltage is:

$$\frac{U_{m,w}}{U_g} = \frac{10^6}{10^6+50} \quad \Rightarrow \quad U_{m,w} \approx U_g \tag{1.2}$$

Therefore, the wrongly measured result should be divided by two to get the right one, which would have been obtained with an impedance matching network.

Solution 2:

If $Z_s = Z_L = R$ the insertion loss is given by equation (4.47) in the handout:

$$IL = 20 \cdot \lg\left(\frac{V_s}{2V_2}\right) \tag{1.3}$$

When the test item is inserted, the circuit looks like this:

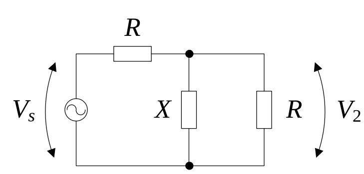


Fig. 3. Equivalent circuit for *IL* measurements of suppression capacitors.

And the voltage ratio in (1.3) is:

$$\frac{V_2}{V_s} = \frac{\dfrac{XR}{X+R}}{R+\dfrac{XR}{X+R}} = \frac{XR}{R^2+2XR} \quad \Leftrightarrow \quad \frac{U_o}{2U_m} = \frac{R+2X}{2X} = 1+\frac{R}{2X} \tag{1.4}$$

a) X is an ideal capacitor $C = 10\ \mu\text{F}$

Substitute equation for capacitor's impedance in (1.4):

$$X = \frac{1}{j\omega C} \quad \Rightarrow \quad \frac{V_s}{2V_2} = 1 + j\frac{\omega CR}{2} \tag{1.5}$$

After inserting (1.5) in (1.3) the insertion loss is:

$$IL = 20 \cdot \lg\left|\frac{V_s}{2V_2}\right| = 20 \cdot \lg\left|1 + j\frac{\omega CR}{2}\right| = 20 \cdot \lg\left[1 + \left(\frac{\omega CR}{2}\right)^2\right]^{\frac{1}{2}} = 10 \cdot \lg\left[1 + \left(\frac{\omega CR}{2}\right)^2\right] \tag{1.6}$$

b) X is an ideal inductor $L = 1\ \mu\text{H}$

Substitute equation for inductor's impedance in (1.4):

$$X = j\omega L \quad \Rightarrow \quad \frac{V_s}{2V_2} = 1 + \frac{R}{2j\omega L} = 1 - j\frac{R}{2\omega L} \tag{1.7}$$

Then the insertion loss is:

$$IL = 20 \cdot \lg\left|\frac{V_s}{2V_2}\right| = 20 \cdot \lg\left|1 - j\frac{R}{2\omega L}\right| = 20 \cdot \lg\left[1^2 + \left(\frac{R}{2\omega L}\right)^2\right]^{\frac{1}{2}} = 10 \cdot \lg\left[1 + \left(\frac{R}{2\omega L}\right)^2\right] \tag{1.8}$$

c) Finally, for a more realistic capacitor, i.e. with its series inductance, from (1.4) it follows:

$$X = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C} = \quad \Rightarrow \quad \frac{V_s}{2V_2} = 1 + j\frac{\omega CR}{2\left(1 - \omega^2 LC\right)} \tag{1.9}$$

Then the insertion loss is:

$$IL = 20 \cdot \lg\left|\frac{V_s}{2V_2}\right| = 20 \cdot \lg\left|1 + j\frac{\omega CR}{2\left(1 - \omega^2 LC\right)}\right| = 10 \cdot \lg\left[1 + \left(\frac{\omega CR}{2\left(1 - \omega^2 LC\right)}\right)^2\right] \tag{1.10}$$

The IL for the three cases considered above is plotted in Fig. 4. Note that the parasitic series inductance reduces the effectiveness of the noise suppression capacitor above resonant frequency.

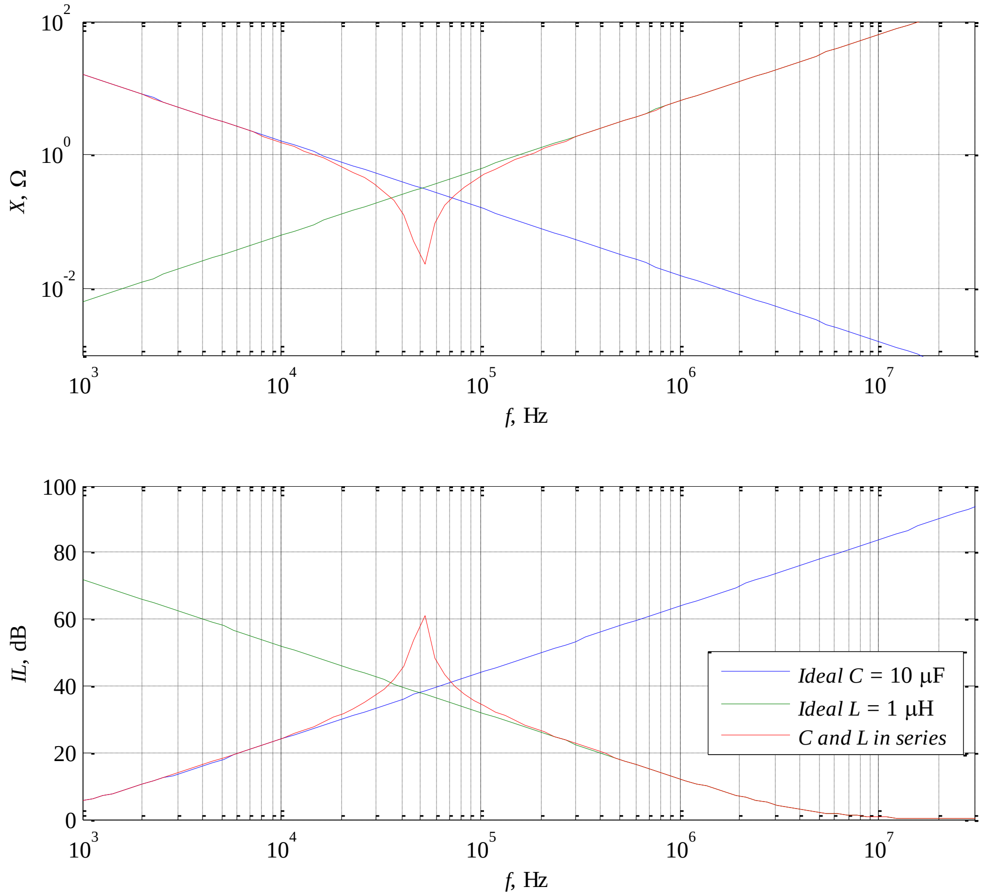


Fig. 4. Impedance and Insertion Loss plots illustrating Problem 2.

Solution 3:

Fig. 5*a)* shows the CM equivalent circuit of the filter in Problem 3.

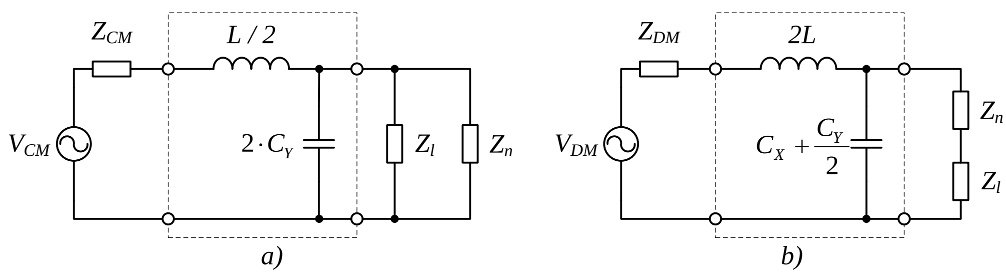


Fig. 5. Equivalent circuits of the filter in Problem 3 a) for CM, b) for DM noise.

The CM insertion loss (IL_{CM}) is measured using the test method shown in Fig. 7*a)*. Because there are three different combinations of source and load impedances (Z_g and Z_m respectively) it is better to obtain a general expression for the IL_{CM} of an *LC*-cell as a function of Z_g and Z_m. From Fig. 6 the ratio between the measured voltage V_m and that of the signal generator V_g is:

$$\frac{V_m}{V_g}=\frac{V_2}{V_g}=\frac{Z_m\|X_{C_e}}{Z_m\|X_{C_e}+X_m+Z_g}=\frac{Z_m}{Z_m+Z_g-\omega^2C_eL_eZ_m+j\omega\left(L_e+C_eZ_mZ_g\right)} \tag{1.11}$$

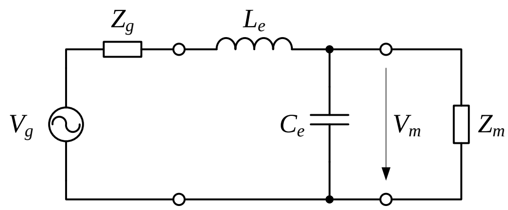


Fig. 6. Equivalent circuit when measuring the *IL* from a single *LC*-cell.

The same ratio without filter is:

$$\frac{V_m}{V_g}=\frac{V_1}{V_g}=\frac{Z_m}{Z_m+Z_g} \tag{1.12}$$

From (1.11) and (1.12) the ratio of V_1 and V_2 can be found. Insert it in (1.3) to get the *IL*:

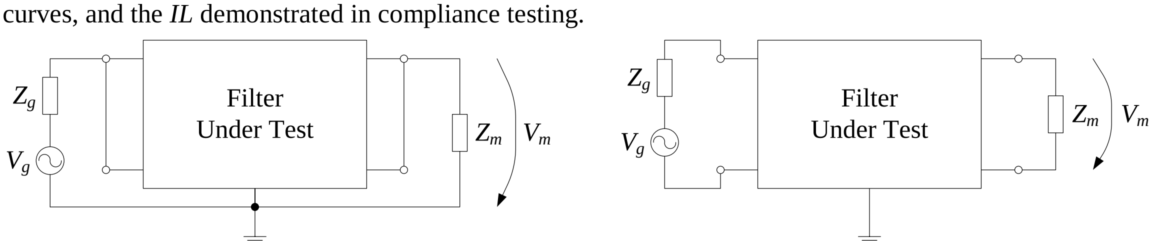
$$IL=20\cdot\lg\left|\frac{V_1}{V_2}\right|=10\cdot\lg\left[\left(1-\frac{\omega^2L_eC_eZ_m}{Z_m+Z_g}\right)^2+\left(\frac{\omega\left(L_e+C_eZ_mZ_g\right)}{Z_m+Z_g}\right)^2\right] \tag{1.13}$$

For CM, $L_e=\frac{L}{2}=0.065\text{ mH}\quad C_e=2\cdot C_Y=4.4\text{ nF}$, and IL_{CM} for 150 kHz and 1 MHz is:

$$\begin{aligned} &IL_{CM}\Big|_{50\Omega/50\Omega}^{150\text{ kHz}}=1.05\text{ dB} \quad IL_{CM}\Big|_{0.1\Omega/100\Omega}^{150\text{ kHz}}=1.38\text{ dB} \quad IL_{CM}\Big|_{100\Omega/0.1\Omega}^{150\text{ kHz}}=-0.31\text{ dB}\\ &IL_{CM}\Big|_{50\Omega/50\Omega}^{1\text{ MHz}}=16.47\text{ dB} \quad IL_{CM}\Big|_{0.1\Omega/100\Omega}^{1\text{ MHz}}=12.46\text{ dB} \quad IL_{CM}\Big|_{100\Omega/0.1\Omega}^{1\text{ MHz}}=20.9\text{ dB} \end{aligned} \tag{1.14}$$

Note that in compliance tests, thanks to the LISN, the load impedance is Z_m = 25 Ω (two 50 Ω resistors in parallel). However, the CM noise source impedance Z_{CM} (in Fig. 5*a*) is unknown and different from the standard source resistances used (0.1 Ω, 50 Ω, or 100 Ω). Therefore, the performance of this filter in compliance tests will differ, from the *IL* shown by filter manufacturers.

Moreover, in real life operation the load impedance (a combination of Z_l and Z_n), is also different from the LISN's resistors. Therefore, the real life noise reduction is different from both – the filter manufacturer's *IL* curves, and the *IL* demonstrated in compliance testing.



(a) *(b)*

Fig. 7. Meaurement set up for: a) CM *IL*; b) DM *IL*.

Solution 4:

The equivalent circuit for DM was shown in Fig. 5*b)*. It is similar to that of CM, found in the previous problem, but L_e and C_e are different:

$$L_e = 2 \cdot L = 260\ \mu\text{H} \quad C_e = C_X + \frac{C_Y}{2} = 48.1\ \text{nF} \tag{1.15}$$

Although the measurement set up for DM, shown in Fig. 7*b)*, is different from that for CM, the equivalent circuit of the filter is again an LC-cell, as it was for CM (Fig. 6). Then the DM *IL* for the frequencies in question can be calculated when the values from (1.15) are used in (1.13):

$$\begin{aligned} IL_{DM}\Big|_{50\Omega/50\Omega}^{150\text{ kHz}} &= 15.26\text{ dB} \quad IL_{DM}\Big|_{0.1\Omega/100\Omega}^{150\text{ kHz}} = 8.44\text{ dB} \quad IL_{DM}\Big|_{100\Omega/0.1\Omega}^{150\text{ kHz}} = 20.33\text{ dB} \\ IL_{DM}\Big|_{50\Omega/50\Omega}^{1\text{ MHz}} &= 47.85\text{ dB} \quad IL_{DM}\Big|_{0.1\Omega/100\Omega}^{1\text{ MHz}} = 24.27\text{ dB} \quad IL_{DM}\Big|_{100\Omega/0.1\Omega}^{1\text{ MHz}} = 53.85\text{ dB} \end{aligned} \tag{1.16}$$

The *IL* characteristics for both CM and DM are plotted in Fig. 8.

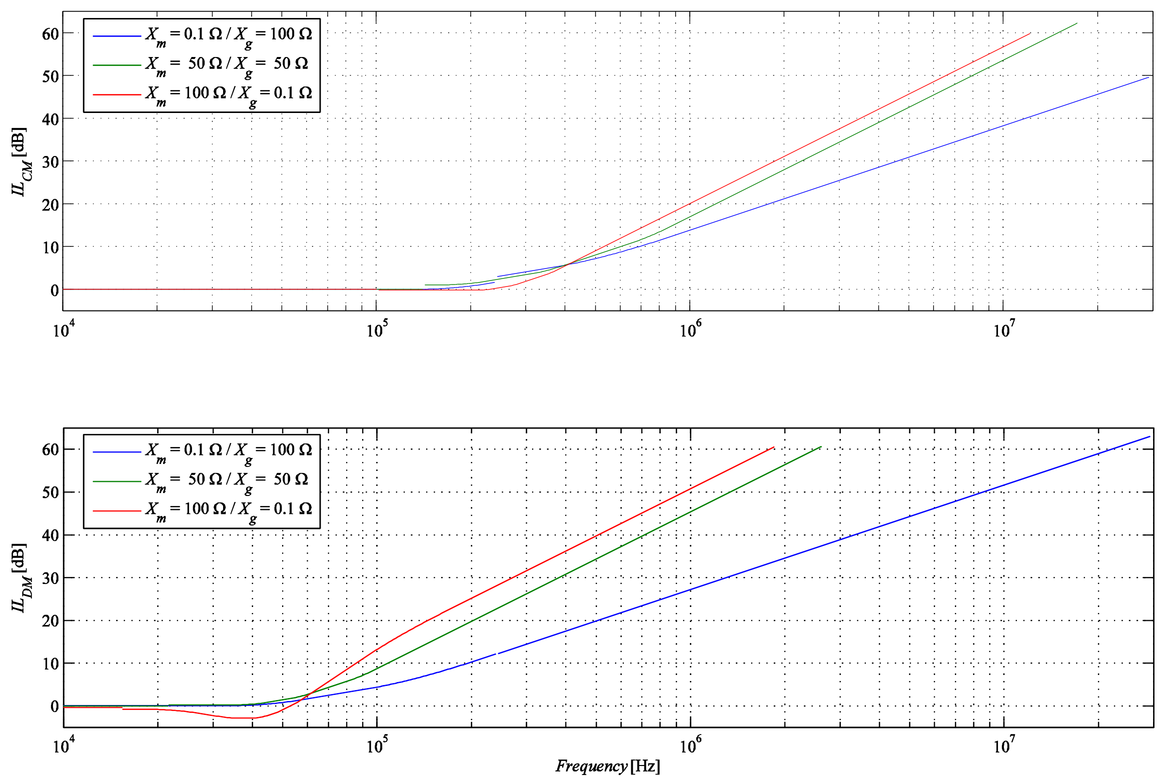


Fig. 8. CM and DM *IL* characteristics for the filter in Problems 3 and 4.

Note that in compliance tests, thanks to the LISN, the DM load impedance is Z_m = 100 Ω (two 50 Ω resistors in series). As in the CM case the noise source impedance Z_{DM} (Fig. 5*b*) is largely unknown and the same considerations apply

Solution 5:

The required insertion loss is 50 dB at 150 kHz.

1) Determine the cut-off frequency (f_o) from Fig. 9.3 in the textbook:

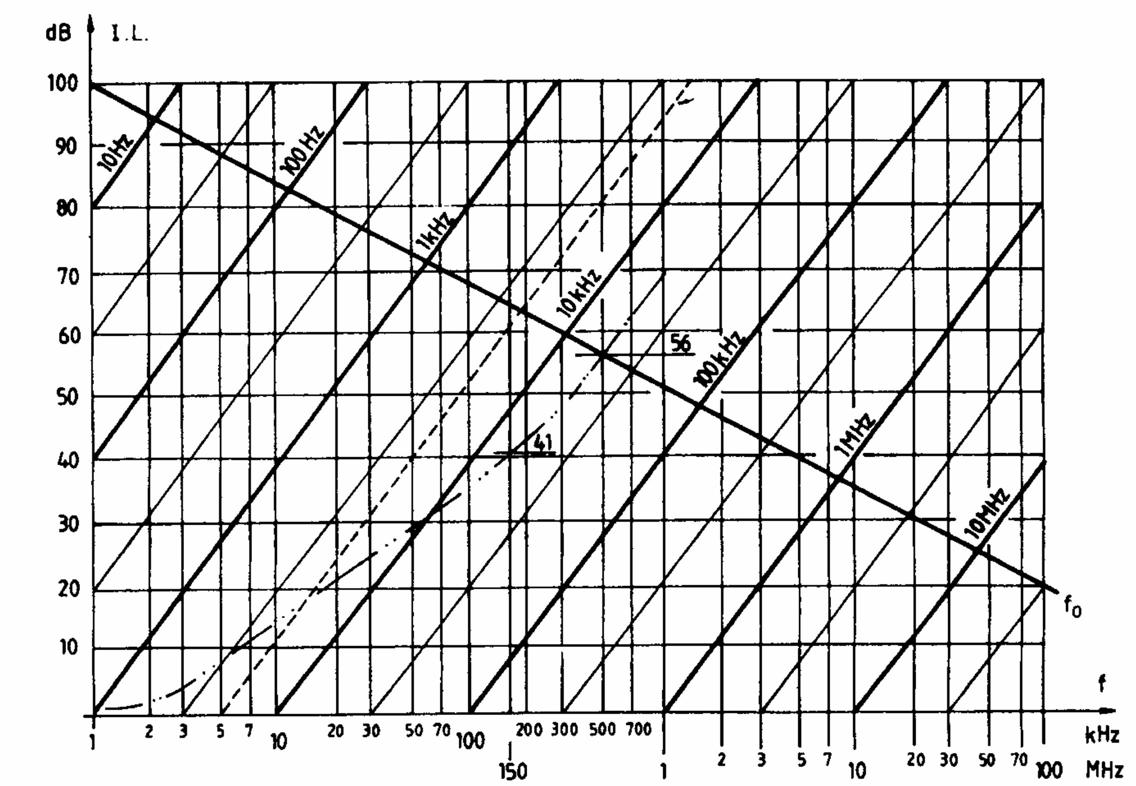


Fig. 9. Figure 9.3 from the textbook. Butterworth response, i.e. the *IL* for ideally damped LC-configuration with resistive source and load impedances.

Find the intersection point of f = 150 kHz and IL = 50 dB. Draw a 40 dB/decade line through that point. The intersection with x-axis gives the required f_o, which in this case is $f_o \approx 8$ kHz.

2) Determine the maximum inductance L_{max} from the allowed voltage drop at maximum rms current: In this example $V_{L\max}$ = 5 V at $I_{L\max}$ = 15 A, at line frequency 50 Hz. Therefore,

$$V_{L\max} = X_{L\max} \cdot I_{L\max} = \omega L_{\max} \cdot I_{L\max} \quad \Rightarrow \quad L_{\max} = \frac{V_{L\max}}{\omega \cdot I_{L\max}} = \frac{5}{2\pi \cdot 50 \cdot 15} = 1.06 \text{ mH} \tag{1.17}$$

3) Determine capacitor C:

$$\omega_0^2 = \frac{1}{LC} \quad \Rightarrow \quad C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \cdot 8 \cdot 10^3)^2 1.06 \cdot 10^{-3}} \approx 3.73 \cdot 10^{-7} \text{ F = 373 nF} \tag{1.18}$$

Select C = 390 nF and calculate the inductance L:

$$\omega_0^2 = \frac{1}{LC} \quad \Rightarrow \quad L = \frac{1}{(2\pi f_0)^2 C} = \frac{1}{(2\pi \cdot 8 \cdot 10^3)^2 390 \cdot 10^{-9}} \approx 1.01 \cdot 10^{-3} \text{ H = 1.01mH} < L_{\max} \tag{1.19}$$

4) The damping ratio according (9.3) in the textbook is:

$$d = \frac{L}{CR^2} \approx 104.09 \tag{1.20}$$

From Fig. 10, with $a = \lg d \approx 2.017$ and normalized frequency $F = \frac{f}{f_0} = \frac{150}{8} = 18.75$, the $IL \approx 50$ dB.

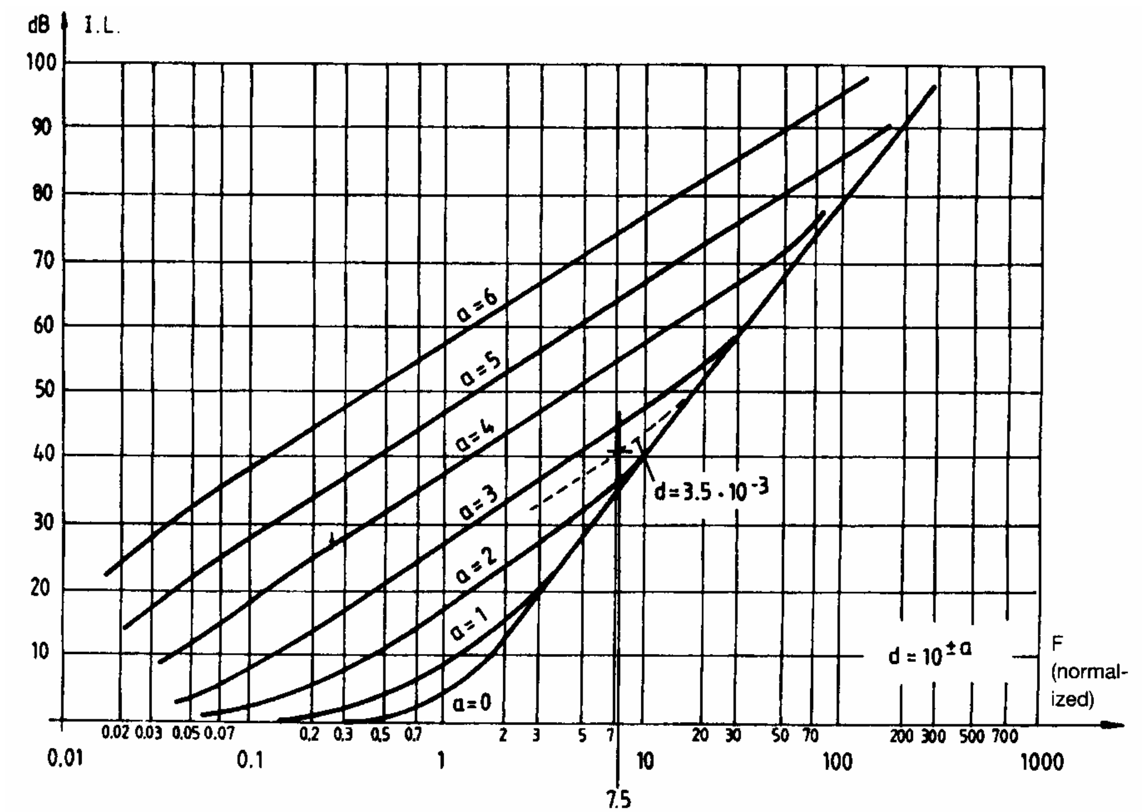


Fig. 10. Figure 9.4 from the textbook. *IL* chart for non-ideally damped LC-configuration with resistive source and load impedances.

An alternative way is to find D from (9.4):

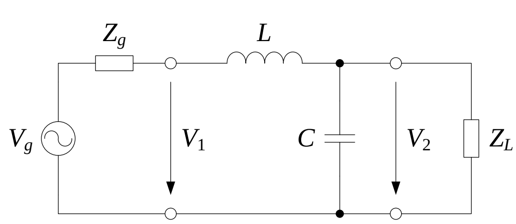
$$D = \frac{1-d}{\sqrt{d}} \approx -10.1 \tag{1.21}$$

Then from (9.1), the *IL* is:

$$IL = 10 \cdot \lg\left(1 + F^2 \frac{D^2}{2} + F^4\right) \approx 51.51\,\text{dB} \tag{1.22}$$

Note that the procedure described in this solution applies for filters terminated by equal and resistive source and load impedances. In any other case the *IL* will differ.

Solution 6:



$$\frac{V_2}{V_1} = \frac{\dfrac{Z_L X_C}{Z_L + X_C}}{X_L + \dfrac{Z_L X_C}{Z_L + X_C}} = \frac{\dfrac{j\omega L_L / j\omega C}{j\omega L_L + 1/j\omega C}}{j\omega L + \dfrac{j\omega L_L / j\omega C}{j\omega L_L + 1/j\omega C}} = \frac{L_L}{L + L_L - \omega^2 L_L LC} \tag{1.23}$$

The worst case *IL* is when the filter and the load resonate. The resonance occurs at frequencies, at which the denominator in (1.23) is zero. The resonance would be at 150 kHz if L_L is:

$$L + L_L - \omega^2 L_L LC = 0 \quad \Rightarrow \quad L_L = \frac{L}{\omega^2 LC - 1} \approx 4.5 \cdot 10^{-6} \text{ H} = 4.5\ \mu\text{H} \tag{1.24}$$

To derive the *IL* in the case when $Z_L = R = 50\ \Omega$ resistance one need the powers dissipated over *R* with and without filter:

$$\left.\begin{aligned} P_1 &= \frac{V_1^2}{R} \\ P_2 &= \frac{V_2^2}{R} \end{aligned}\right\} \quad \Rightarrow \quad IL = 10 \cdot \lg\left(\frac{P_1}{P_2}\right) = 20 \cdot \lg\left|\frac{V_1}{V_2}\right| \tag{1.25}$$

When the Z_L in (1.23) is replaced by *R* the result is:

$$\frac{V_2}{V_1} = \frac{\dfrac{RX_C}{R + X_C}}{X_L + \dfrac{RX_C}{R + X_C}} = \frac{R}{R - \omega^2 LRC + j\omega L} \tag{1.26}$$

Then the *IL* in (1.25) becomes:

$$IL = 20 \cdot \lg\left|\frac{R - \omega^2 LRC + j\omega L}{R}\right| = 20 \cdot \lg\sqrt{\left(1 - \omega^2 LC\right)^2 + \left(\frac{\omega L}{R}\right)^2} \tag{1.27}$$

Equation (1.27) is the exression for *IL* of an LC-filter with resistive load. With the values in this Problem at 150 kHz the *IL* is about 46.92 dB.