# Exercise 3 – Solutions

**27.9.2022**

## #1 Behavioral aspects of utility assessment

c) Indifference between lotteries 1 and 2 imposes constraint $U(400€) = p \cdot U(1000€) + (1-p) \cdot U(0€)$. Fixing $U(0€) = 0$ and $U(1000€) = 1$ gives $U(400€) = p$. Similarly, indifference between lotteries 3 and 4 imposes constraint
$0.5 \cdot U(400€) + 0.5 \cdot U(0€) = q \cdot U(1000€) + (1-q) \cdot U(0€) \Leftrightarrow\ 0.5 \cdot U(400€) = q\ \Leftrightarrow$
$U(400€) = 2q$.

d) For most people, $p\ >\ 2q$; for instance, $p = 50\%$ but $q = 20\%$. This can be attributed to the *certainty effect*, whereby individuals tend to be more risk averse when provided with an alternative yielding a reward for certain than when provided only with alternatives none of which yields a reward for certain (cf. Allais paradox). When confronted with their inconsistency, some individuals revise their choices.

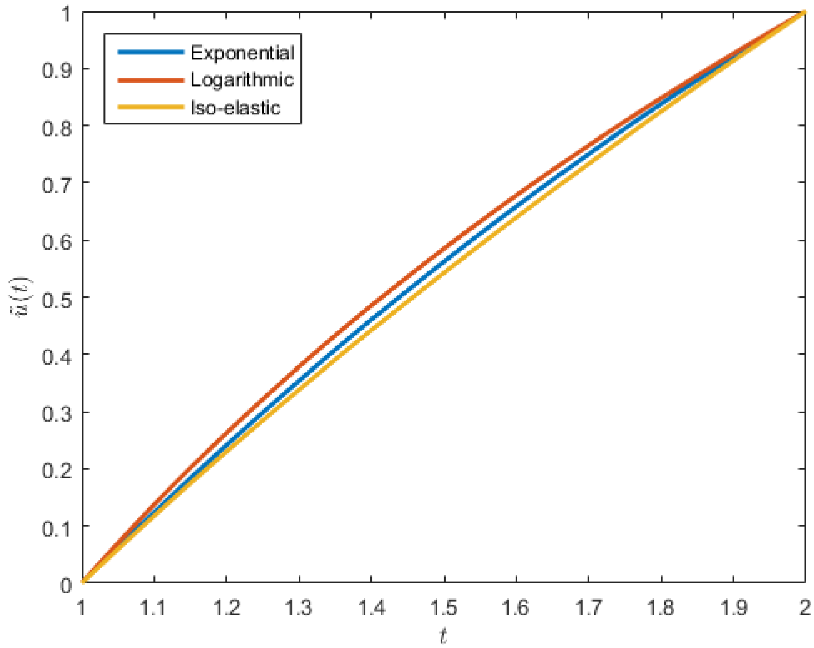
## #2 Risk attitude

a) Let $u(t) = 1 - e^{-\alpha t}, \quad \alpha > 0$. Then, $u'(t) = \alpha e^{-\alpha t} > 0$ so that $u(t)$ is increasing. Moreover, $u''(t) = -\alpha^2 e^{-\alpha t} < 0$, whereby $u(t)$ is concave and therefore corresponds to risk aversion.
Similarly: $u(t) = \log(t) \Rightarrow\ u'(t) = \frac{1}{t} > 0\ \Rightarrow\ u''(t) = \frac{-1}{t^2} < 0$ and
$u(t) = \frac{t^{1-\rho}}{1-\rho},\ \rho > 0, \rho \neq 1 \Rightarrow\ u'(t) = t^{-\rho} > 0 \Rightarrow\ u''(t) = -\rho t^{-\rho-1} < 0.$

b) Using the definitions, we get
   - Exponential utility: $R_a(t) = -\frac{u''(t)}{u'(t)} = \frac{\alpha^2 e^{-\alpha t}}{\alpha e^{-\alpha t}} = \alpha$ = constant.
   - Logarithmic utility: $R_r(t) = -t\frac{u''(t)}{u'(t)} = t\frac{1}{t^2}\frac{t}{1} = 1$ = constant.
   - Iso-elastic utility: $R_r(t) = -t\frac{u''(t)}{u'(t)} = t\frac{\rho t^{-\rho-1}}{t^{-\rho}} = \rho$ = constant.

c) Utility functions are unique up to positive affine transformations. Hence, we can define for each utility function constants $a$ and $b$ such that $\tilde{u}(t) = au(t) + b,\ \tilde{u}(1) = 0,\ \tilde{u}(2) = 1$. This gives
   - Exponential utility: $\tilde{u}(t) = e^{0.5}\left(\frac{1-e^{-0.5t}}{1-e^{-0.5}} - 1\right)$
   - Logarithmic utility: $\tilde{u}(t) = \frac{\log(t)}{\log(2)}$
   - Iso-elastic utility: $\tilde{u}(t) = \frac{\sqrt{t}-1}{\sqrt{2}-1}$.

See the Matlab file “ex3task2c_and_3a.m” for the code plotting the utility functions.




d) $E[X] = 0.5 * 1 + 0.5 * 2 = 1.5$ and

$E[\tilde{u}(X)] = 0.5 * \frac{\log(1)}{\log(2)} + 0.5 * \frac{\log(2)}{\log(2)} = 0.5.$

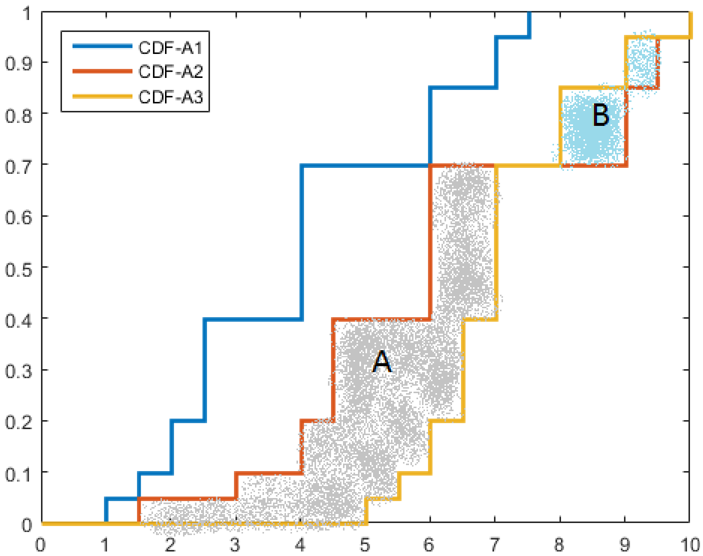
By definition, $\tilde{u}(CE[X]) = E[\tilde{u}(X)] \Leftrightarrow CE[X] = \tilde{u}^{-1}(E[\tilde{u}(X)])$. To obtain the inverse utility function $z = \tilde{u}^{-1}$ we note:

$\tilde{u} = \frac{\log(t)}{\log(2)} \Leftrightarrow \log(t) = \tilde{u}\log(2) \Leftrightarrow t = 2^{\tilde{u}} = z(\tilde{u}).$

Thus, $CE[X] = z(E[\tilde{u}(X)]) = \sqrt{2} \approx 1.41 \Rightarrow RP = E[X] - CE[X] \approx 0.09.$

## #3 Stochastic dominance and risk measures

a) See the Matlab file “ex3tasks_2c_and_3a.m” for the code for plotting the CDFs. Based on the plotted CDFs, A1 is FSD dominated by both A2 and A3. The FSD dominance implies that A1 is also SSD dominated by both A2 and A3. Because area A is larger than area B and to the left of B, A3 SSD dominates A2.




b) Because A1 is FSD dominated, managers who prefer higher revenue should not choose it. Thus, knowing only that the managers prefer more to less, you should recommend choosing between A2 and A3. Because A3 SSD dominates all the other alternatives, risk averse managers should choose it.