Exercise 3

27.9.2022

#1 Behavioral aspects of utility assessment

a) Find the value for *p* that makes you indifferent between Lotteries 1 and 2:
   - Lottery 1: Win 1000€ with probability *p*
     Win 0€ with probability 1-*p*
   - Lottery 2: Win 400€ for sure

b) Now find the *q* that makes you indifferent between Lotteries 3 and 4:
   - Lottery 3: Win 400€ with probability 0.5
     Win 0€ with probability 0.5
   - Lottery 4: Win 1000€ with probability *q*
     Win 0€ with probability 1-*q*

c) According to your assessment in part a, u(400€)=p. In part b, u(400€)=2q. Explain why this is the case.

d) To be consistent, your assessments should be such that *p=2q.* Were your assessments consistent? Would you change them? Which assessment do you feel most confident about? Why?

#2 Risk attitude

The level of risk aversion can be measured by *absolute risk aversion* $R_a(t)$ and *relative risk aversion* $R_r(t)$, defined as

$$R_a(t) = -\frac{u''(t)}{u'(t)} \qquad \text{and} \qquad R_r(t) = -t\frac{u''(t)}{u'(t)}.$$

Absolute risk aversion measures aversion to loss in absolute terms, whereas relative risk aversion measures aversion to loss relative to the DM's wealth.

The most used utility function forms to model risk aversion for $t > 0$ are

- Exponential utility $u(t) = 1 - e^{-\alpha t}, \quad \alpha > 0$
- Logarithmic utility $u(t) = \log(t),$
- Iso-elastic utility $u(t) = \frac{t^{1-\rho}}{1-\rho}, \qquad \rho > 0, \rho \neq 1.$

a) Verify that the above utility functions are increasing and model risk averse preferences.

b) Show that exponential utility function represents constant absolute risk aversion, whereas logarithmic and iso-elastic utility functions represent constant relative risk aversion.

c) Let $\alpha = \rho = 0.5$. Scale the utility functions such that each scaled function $\tilde{u}(t)$ fulfils $\tilde{u}(1) = 0$ and $\tilde{u}(2) = 1$. Plot these functions $\tilde{u}(t)$ for $t \in [1, 2]$.

d) Let the probabilities of outcomes be $f_X(1) = f_X(2) = 0.5$. Compute the CE and RP for the scaled logarithmic utility function $\tilde{u}(t)$.

#3 Stochastic dominance and risk measures

A team of managers at a company is considering between three investment opportunities A1, A2, and A3. To support decision making, they have assessed discrete probability distributions for revenues (in M€) resulting from each alternative investment opportunity:

Probability	0.05	0.05	0.1	0.2	0.3	0.15	0.1	0.05
A1	1	1.5	2	2.5	4	6	7	7.5
A2	1.5	3	4	4.5	6	9	9.5	10
A3	5	5.5	6	6.5	7	8	9	10

a) Plot the CDFs of the three investment opportunities. Which alternatives dominate which in the sense of first-degree stochastic dominance? How about in the sense of second-degree stochastic dominance?

b) Which alternatives would you recommend to managers that prefer more to less? What about if the managers are also known to be risk averse?