$$J(x) = \sum_{n=1}^{N} (y_n - g\cdot x)^2$$

$$x^* = \arg\min_{x} J(x)$$

$$\frac{\partial J}{\partial x} = 0$$

$$\frac{\partial J}{\partial x} = \frac{\partial}{\partial x} \sum_{n=1}^{N} (y_n - g\cdot x)^2$$

$$= \sum_{n=1}^{N} \frac{\partial}{\partial x} (y_n - g\cdot x)^2$$

$$= \sum_{n=1}^{N} (-g)\cdot 2\cdot (y_n - g\cdot x)$$

$$= \sum_{n=1}^{N} \left[-2g\cdot y_n + 2g^2 x\right]$$

$$= -2g\left[\sum_{n=1}^{N}\left[y_n - gx\right]\right]$$

$$= -2g\underbrace{\left[\left(\sum_{n=1}^{N} y_n\right) - N\cdot gx\right]}_{=0} = 0$$

$$\Rightarrow \sum_{n=1}^{N} y_n = N\cdot g x^*$$

$$x^* = \frac{1}{g\cdot N}\sum y_n \qquad \left[y_n = gx + r_n\right]$$

$\hat{x}$ in slides

$$E[x^*] = \frac{1}{g\cdot N}\sum_{n=1}^{N} E[y_n]$$

$g\cdot x$ as $E[r_n] = 0$

$$= \frac{1}{g \cdot N} \sum \overbrace{E[g \cdot x + r_n]}$$

$$= \frac{1}{g \cdot N} \sum_{n=1}^{N} g x = \frac{1}{\cancel{g \cdot N}} \cancel{N \cdot g} x = x$$

$$Var[x^*] = Var\left[\frac{1}{g \cdot N} \sum_{n=1}^{N} y_n\right]$$

$$= \frac{1}{g^2 N^2} Var\left[\sum_{n=1}^{N} y_n\right]$$

$$= \frac{1}{g^2 N^2} \sum_{n=1}^{N} Var[y_n]$$

$$= \frac{1}{g^2 N^2} \sum_{n=1}^{N} \underbrace{Var[g x + r_n]}_{Var[r_n] = \sigma^2_{r,n}}$$

$$= \frac{1}{g^2 N^2} \sum_{n=1}^{N} \sigma^2_{r,n} \qquad \{ \; \sigma^2_{r,n} = \sigma^2$$

$$\Rightarrow \frac{1}{g^2 N^{\cancel{2}}} \cdot \cancel{N} \sigma^2 = \frac{\sigma^2}{g^2 N}$$

$$std = \sqrt{Var} = \frac{\sigma}{|g| \cdot \sqrt{N}}$$

$$y_n = \underbrace{\frac{2}{c}}_{g} \cdot \underbrace{p^x}_{x} + r_n$$

$$x_1^* = \hat{x}_1 = \frac{c}{2N} \sum_{n=1}^{N} y_n$$

$$y_1 = g_1 \cdot x + r_1$$

$$y_2 = g_2 \cdot x + r_2$$

$$\vdots$$

$$\vec{y} = \begin{pmatrix} y_0 \\ \vdots \\ y_N \end{pmatrix}$$

$$\vec{y} = G\vec{x} + \vec{r} \quad , \quad \vec{z} \cdot \vec{z} = \sum z_i^2 = \vec{z}^T \vec{z}$$

$$J(\vec{x}) = (\vec{y} - G\vec{x})^T (\vec{y} - G\vec{x})$$

$$\nabla J(\vec{x}) = \frac{\partial J}{\partial \vec{x}} = \begin{pmatrix} \partial J / \partial x_1 \\ \partial J / \partial x_2 \\ \vdots \\ \partial J / \partial x_K \end{pmatrix}$$

notation oops.

$$\vec{x}^* = \hat{\vec{x}} = \arg\min_{\vec{x}} J(\vec{x})$$

$$\frac{\partial J}{\partial \vec{x}} = 0$$

$$J(\vec{x}) = (\vec{y} - G\vec{x})^T (\vec{y} - G\vec{x})$$

$$= \vec{y}^T \vec{y} - \underbrace{\vec{y}^T G}_{= \vec{a}^T} \vec{x} - \vec{x}^T \underbrace{G^T \vec{y}}_{\vec{a}} + \vec{x}^T G^T G \vec{x}$$

$$\frac{\partial}{\partial \vec{x}}\left(\vec{a}^T\vec{x}\right) = \vec{a} \qquad \{\ \vec{a}^T\vec{x} = \vec{x}^T\vec{a}$$

$$\frac{\partial}{\partial \vec{x}}\left(\vec{x}^T\vec{a}\right) = \vec{a}$$

symm. A

$$\frac{\partial}{\partial \vec{x}}\left(\vec{x}^T A \vec{x}\right) = (A + A^T)\vec{x} = 2A\vec{x}$$

$$\frac{\partial J}{\partial \vec{x}} = 0 - G^T\vec{y} - G^T\vec{y} + 2G^TG\vec{x} = 0$$

$$\Rightarrow \quad \hat{\vec{x}} = (G^TG)^{-1}G^T\vec{y}$$

$$\left\{ \begin{array}{l} -2G^T\vec{y} + 2G^TG\vec{x} = 0 \\ G^T\vec{y} = G^TG\vec{x} \\ G^TG\vec{x} = G^T\vec{y} \quad \left|\ (G^TG)^{-1}\cdot\right. \\ \hat{\vec{x}} = (G^TG)^{-1}G^T\vec{y} \end{array}\right.$$

$$E[\hat{\vec{x}}] = E[(G^TG)^{-1}G^T\vec{y}]$$
$$= E[(G^TG)^{-1}G^T(G\vec{x} + \vec{r})]$$
$$= E[(G^TG)^{-1}G^TG\vec{x}] + \underbrace{E[(G^TG)^{-1}G^T\vec{r}]}_{0}$$
$$= \cancel{(G^TG)^{-1}}\,\cancel{(G^TG)}\,\vec{x}$$
$$= \vec{x}$$

$$Cov[\hat{\vec{x}}] = Cov[(G^TG)^{-1}G^T\vec{y}]$$
$$= Cov[(G^TG)^{-1}G^T(G\vec{x} + \vec{r})]$$
$$= Cov[(G^TG)^{-1}G^T\vec{r}]$$
$$= (G^TG)^{-1}G^TRG(G^TG)^{-1}$$

$$Cov[x + r] = Cov[r]$$

$$\vec{y} = G\vec{x} + \vec{r}, \quad E[\vec{r}] = 0, \quad \mathrm{Cov}(\vec{r}) = R$$

$$J(\vec{x}) = (\vec{y} - G\vec{x})^T \underbrace{W}_{= R^{-1}} (\vec{y} - G\vec{x})$$

$$= (\vec{y} - G\vec{x})^T R^{-1} (\vec{y} - G\vec{x})$$

$\vec{x}^T\vec{a}$

$$= \underline{\vec{y}^T R^{-1} \vec{y}} - \underline{\vec{x}^T G^T R^{-1} \vec{y}}$$

$$- \underline{\vec{y}^T R^{-1} G \vec{x}} + \vec{x}^T G^T R^{-1} G \vec{x}$$

$$\frac{\partial J}{\partial \vec{x}} = -G^T R^{-1} \vec{y} - G^T R^{-1} \vec{y} + 2 G^T R^{-1} G \vec{x} = 0$$

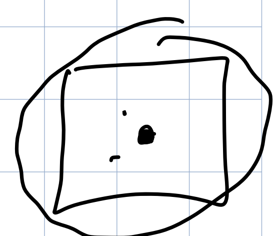
$$-\cancel{2} G^T R^{-1} \vec{y} + \cancel{2} G^T R^{-1} G \vec{x} = 0$$

$$G^T R^{-1} G \vec{x} = G^T R^{-1} \vec{y} \quad \Big| \, (G^T R^{-1} G)^{-1}.$$

$$\vec{x} = (G^T R^{-1} G) G^T R^{-1} \vec{y}$$

$$J(\vec{x}) = (\vec{y} - Gx)^T R^{-1} (\vec{y} - G\vec{x})$$

$$+ (\vec{x} - \vec{m}) P^{-1} (\vec{x} - \vec{m})$$



$\vec{m}$

P