Maxwellin yhtälöt

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot \bar{D} = \rho \qquad \nabla \cdot \bar{B} = 0$$

$$\bar{D} = \varepsilon_0 \bar{E} \qquad \bar{B} = \mu_0 \bar{H}$$

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\ \frac{As}{Vm} \qquad \mu_0 = 4\pi \cdot 10^{-7}\ \frac{Vs}{Am}$$

(TYHJIÖSSÄ)

Sähköstaattiset kentät

$$\frac{\partial}{\partial t} = 0$$

$$\nabla \times \bar{E} = -\cancel{\frac{\partial \bar{B}}{\partial t}}$$

$$\nabla \cdot \bar{D} = \rho$$

$$\nabla \times \bar{E}(\bar{r}) = 0 \qquad \bar{E} = -\nabla \phi$$

$$\nabla \cdot \bar{D}(\bar{r}) = \rho(\bar{r})$$

$$\bar{D}(\bar{r}) = \varepsilon_0 \bar{E}(\bar{r})$$

$$\nabla \cdot \varepsilon_0 (-\nabla \phi) = \rho \quad \Rightarrow \quad \nabla^2 \phi(\bar{r}) = -\frac{\rho(\bar{r})}{\varepsilon_0}$$

POISSONIN YHTÄLÖ

Skalaaripotentiaali

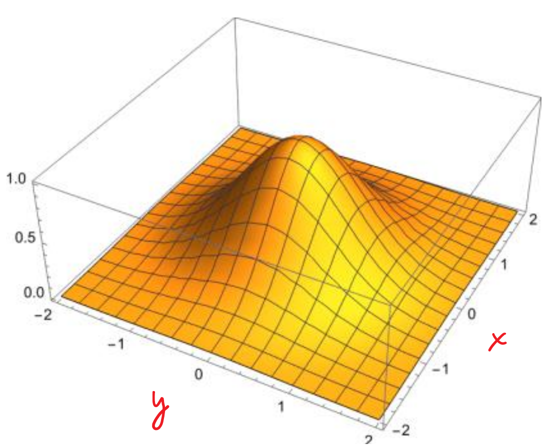
$$\rho \rightarrow \phi \rightarrow \bar{E}$$

Poissonin yhtälö

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$$

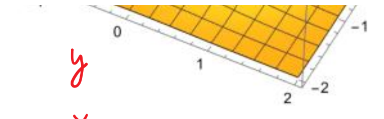
Laplacen yhtälö

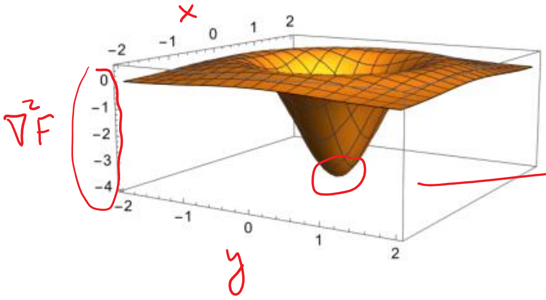
$$\rho = 0 \quad \Rightarrow \quad \nabla^2 \phi(\bar{r}) = 0$$



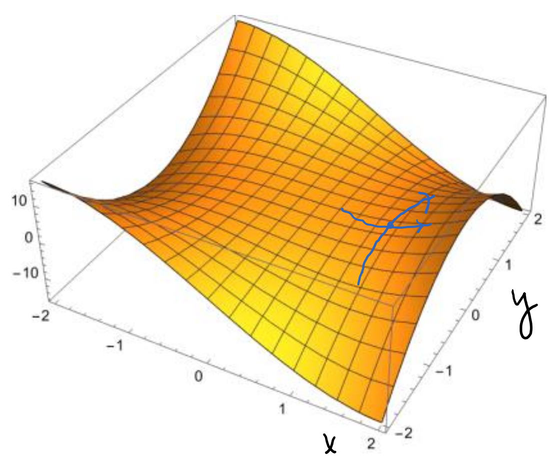
$$F(x,y) = e^{-x^2 - y^2}$$

$$\nabla^2 F = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) F(x,y)$$





negatiivinen kaarevuus



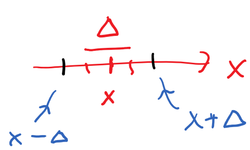
$$G(x,y) = x^3 - 3xy^2$$

$$\frac{\partial^2}{\partial x^2} G = \frac{\partial}{\partial x}\left(3x^2 - 3y^2\right) = 6x$$

$$\frac{\partial^2}{\partial y^2} G = \frac{\partial}{\partial y}\left(-6xy\right) = -6x$$

$$\nabla^2 G = 0$$

$$f(x) \Rightarrow \quad f'(x) \cong \frac{1}{\Delta}\left[f(x+\tfrac{\Delta}{2}) - f(x-\tfrac{\Delta}{2})\right]$$



$$f''(x) = \frac{1}{\Delta^2}\left(\underbrace{f(x+\Delta) - f(x) - \left(f(x) - f(x-\Delta)\right)}_{=0}\right)$$

$$f''(x) = 0$$

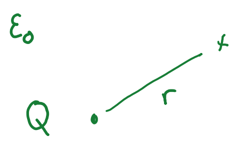
$$\searrow \quad f(x+\Delta) + f(x-\Delta) = 2f(x)$$

$$f(x) = \frac{1}{2}\left(f(x+\Delta) + f(x-\Delta)\right)$$

(KESKIARVO)

3D: $$f(x,y,z) = \frac{1}{6}\big(f(x+\Delta,y,z) + f(x-\Delta,y,z) + f(x,y+\Delta,z) + f(x,y-\Delta,z) + f(x,y,z+\Delta) + f(x,y,z-\Delta)\big)$$

Pistevarauksen potentiaali



$$\bar{E}(\bar{r}) = \frac{Q}{4\pi\varepsilon_0 r^2}\,\bar{u}_r$$

$$\phi(\bar{r}) = ?$$

$$\bar{E} = -\nabla\phi$$

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0}\,\frac{\bar{u}_r}{r^2}$$

$$= -\bar{u}_r\,\frac{\partial\phi}{\partial r}$$

$$\Rightarrow \phi(\bar{r}) = \frac{Q}{4\pi\varepsilon_0 r}$$

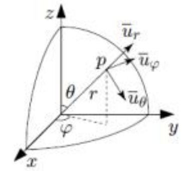
Pallokoordinaatisto

$$\nabla f(\bar{r}) = \bar{u}_r\frac{\partial}{\partial r}f + \cancel{\bar{u}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}f} + \cancel{\bar{u}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f}$$

$$\nabla\times\bar{f} = \frac{1}{r^2\sin\theta}\begin{vmatrix}\bar{u}_r & r\bar{u}_\theta & r\sin\theta\,\bar{u}_\varphi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ f_r & rf_\theta & r\sin\theta\,f_\varphi\end{vmatrix}$$

$$\nabla\cdot\bar{f} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,f_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f_\varphi$$

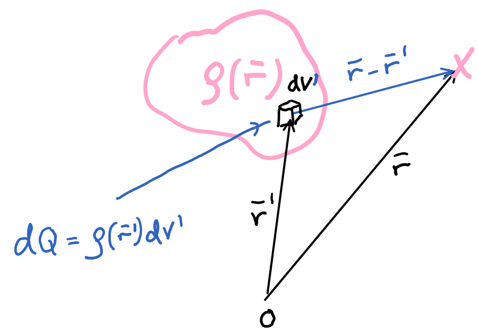
$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$



$$\frac{d}{dr}\left(\frac{1}{r}\right) = \frac{d}{dr}r^{-1} = -1\cdot r^{-2}$$

Varausjakautuman potentiaali

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$



$$\phi(\bar{r}) = ?$$

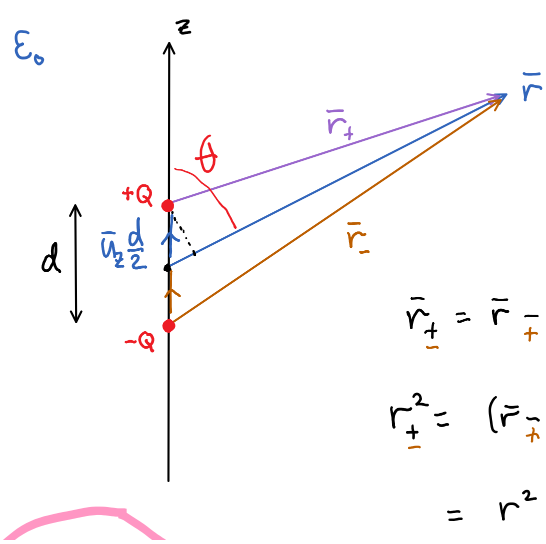
$$\bar{E}(\bar{r}) = ?$$

$$\phi(\bar{r}) = \int_V \frac{\rho(\bar{r}')\,dv'}{4\pi\varepsilon_0|\bar{r}-\bar{r}'|}$$

(Sähköstaattinen) dipoli

$+Q \quad -Q$

ε_0



$$\phi(\bar{r}) \rightarrow \bar{E}(\bar{r}) = -\nabla\phi(\bar{r})$$

$$\phi_d = \frac{+Q}{4\pi\varepsilon_0 r_+} + \frac{-Q}{4\pi\varepsilon_0 r_-}$$

$$\bar{r}_\pm = \bar{r} \mp \bar{u}_z\frac{d}{2} \qquad r_\pm = \sqrt{\bar{r}_\pm\cdot\bar{r}_\pm}$$

$$r_\pm^2 = \left(\bar{r}\mp\bar{u}_z\frac{d}{2}\right)\cdot\left(\bar{r}\mp\bar{u}_z\frac{d}{2}\right) = r^2 \mp \bar{r}\cdot\bar{u}_z\frac{d}{2} \mp \bar{u}_z\cdot\bar{r}\frac{d}{2} + \bar{u}_z\cdot\bar{u}_z\left(\frac{d}{2}\right)^2$$

$$= r^2 \mp rd\cos\theta + (d/2)^2 = r^2\left(1 \mp \frac{d\cos\theta}{r} + \cancel{\left(\frac{d}{2r}\right)^2}\right)$$

$$= r^2 \mp r d \cos\theta + (d/2)^2 = r^2\left(1 \mp \frac{d\cos\theta}{r} + \cancel{\left(\frac{d}{2r}\right)^2}\right)$$

$r >> d$

$$r_\pm = r\sqrt{1 \mp \frac{d\cos\theta}{r}} \simeq r\left(1 \mp \frac{d\cos\theta}{2r}\right) = r \mp \frac{d\cos\theta}{2}$$

$$\phi_d(\bar{r}) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r\left(1-\frac{d\cos\theta}{2r}\right)} - \frac{1}{r\left(1+\frac{d\cos\theta}{2r}\right)}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 r}\left(\frac{1}{1-\frac{d\cos\theta}{2r}} - \frac{1}{1+\frac{d\cos\theta}{2r}}\right)$$

$\left(\frac{1}{1-x} \simeq 1+x\right)$

$(|x|<<1)$

$$\simeq \frac{Q}{4\pi\varepsilon_0 r}\left(1 + \frac{d\cos\theta}{2r} - \left(1 - \frac{d\cos\theta}{2r}\right)\right) = \frac{Q}{4\pi\varepsilon_0 r}\cdot\frac{d\cos\theta}{2r}2$$

$$= \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\overbrace{p\cos\theta}^{p\,\bar{u}_z\cdot\bar{u}_r}}{4\pi\varepsilon_0 r^2}$$

$(p = Qd)$

$[p] = \mathrm{Asm}$

$$\bar{E}_d(\bar{r}) = -\nabla\phi_d = -\nabla(\quad)$$

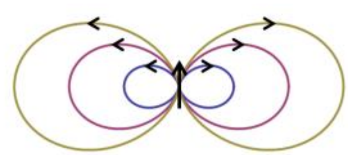
$$\frac{\partial}{\partial r} r^{-2} = -2r^{-3}$$

$$\frac{\partial}{\partial\theta}\cos\theta = -\sin\theta$$

$$= -\left(\bar{u}_r\frac{\partial\phi_d}{\partial r} + \frac{1}{r}\bar{u}_\theta\frac{\partial\phi_d}{\partial\theta}\right)$$

$$= \frac{p}{4\pi\varepsilon_0}\left(\bar{u}_r\, 2r^{-3}\cos\theta + \frac{1}{r}\bar{u}_\theta\sin\theta\, r^{-2}\right)$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\bar{u}_r + \sin\theta\,\bar{u}_\theta\right)$$



$R(\theta) = k\sin^2\theta$

z

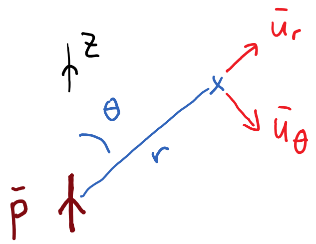
$\bar{p} = \bar{u}_z p$

$\bar{p} = \bar{u}p$

$$\phi = \frac{\bar{p}\cdot\bar{u}_r}{4\pi\varepsilon_0 r^2}$$

z $\quad$ x $\quad$ $\bar{u}_r$

$\theta = 0 \Rightarrow 2\bar{u}_r = 2\bar{u}_z$

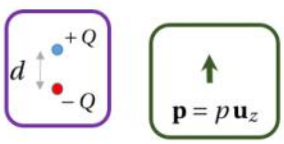


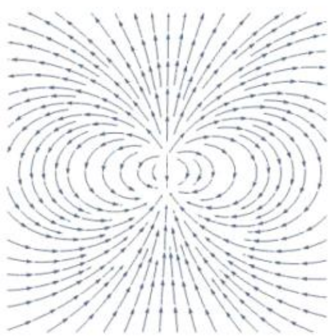
$$\theta = 0 \Rightarrow 2\bar{u}_r = 2\bar{u}_z$$
$$\theta = 90^\circ \Rightarrow 1\bar{u}_\theta = -\bar{u}_z$$
$$\theta = 180^\circ \Rightarrow -2\bar{u}_r = 2\bar{u}_z$$

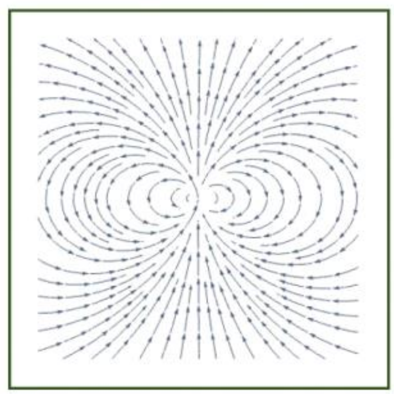
Taylorin sarjojen tarkkuudesta

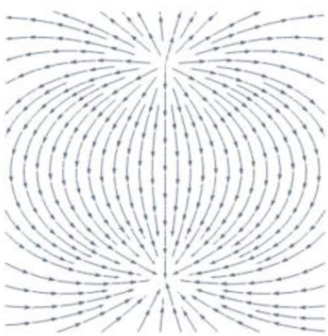
x	$\frac{1}{1+x}$	$1-x$	virhe		$\sqrt{1+x}$	$1+\frac{x}{2}$	virhe
0,1	0,90909090909	0,9	1 %		1,0488088482	1,05	0,114 %
0,01	0,99009900990	0,99	10^{-4}		1,0049875621	1,005	$1{,}24 \cdot 10^{-5}$
0,001	0,99900099900	0,999	10^{-6}		1,0004998751	1,0005	$1.25 \cdot 10^{-7}$
0,0001	0,999900009999	0,9999	10^{-8}		1,0000499988	1,00005	$1.25 \cdot 10^{-9}$

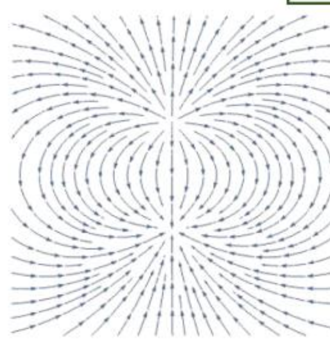
Kaksi pistevarausta
vai dipoli?





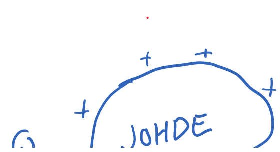






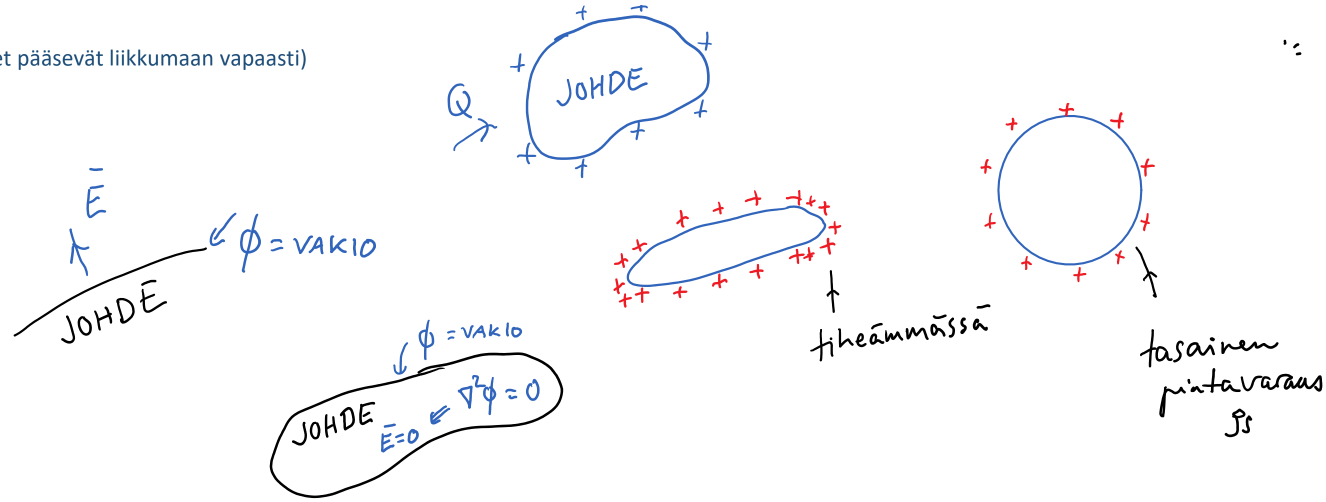
JOHDE

(varaukset pääsevät liikkumaan vapaasti)



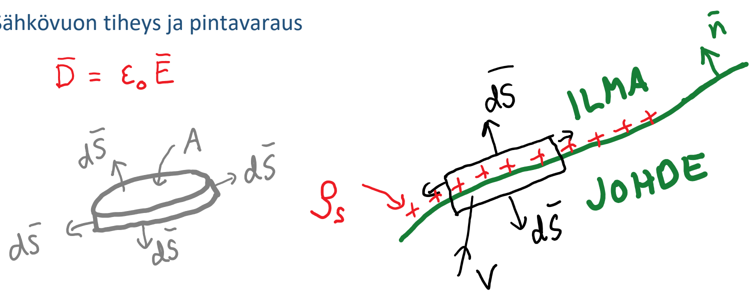
JOHDE

(varaukset pääsevät liikkumaan vapaasti)



Sähkövuon tiheys ja pintavaraus

$\bar{D} = \varepsilon_0 \bar{E}$



$\bar{n}\, dS$

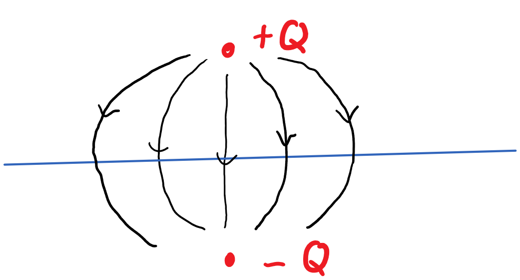
$$\int_V \underbrace{\nabla\cdot\bar{D}}_{\rho}\, dV = \oint_S \bar{D}\cdot d\bar{S}$$

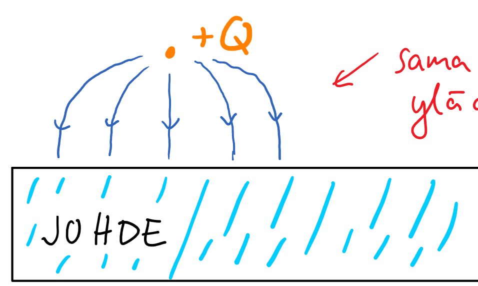
$\bar{n}\, D$

$= \rho_s A \qquad = DA$

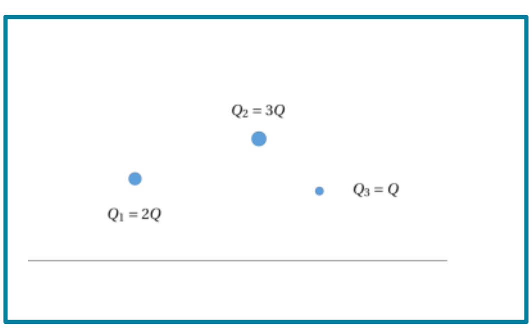
$$\rho_s = D_n$$

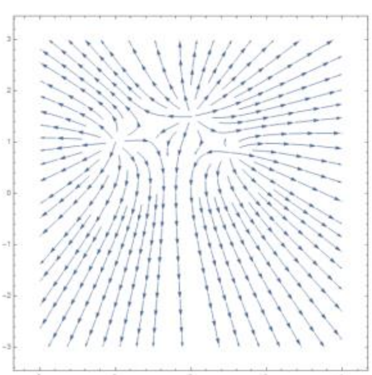
Kuvalähdeperiaate (peilikuvaperiaate)

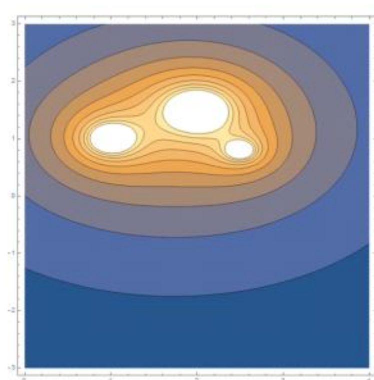


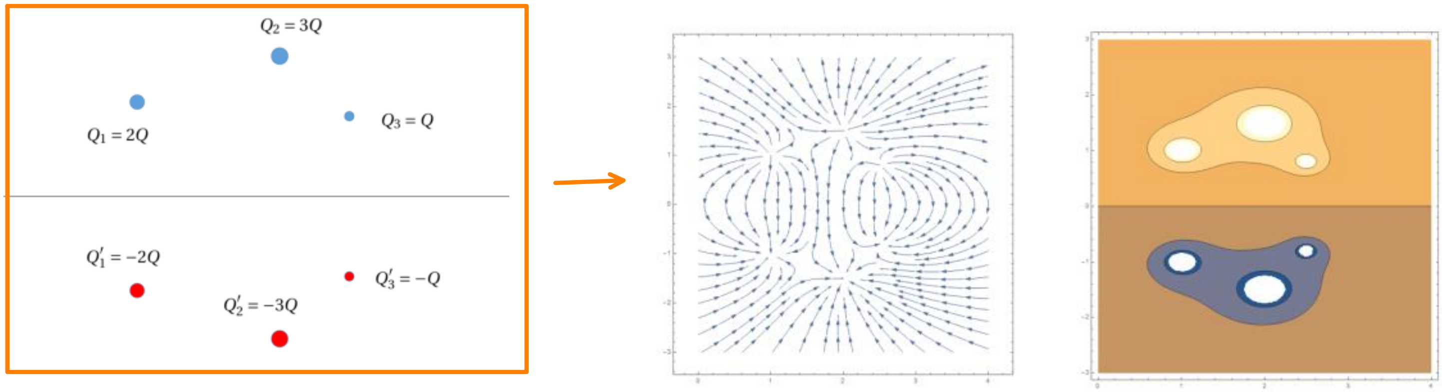


sama kenttä yläalueessa

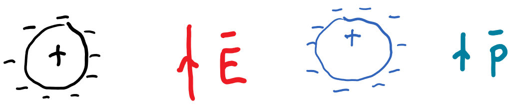








ERISTE



$$\bar{P} = \frac{n\bar{p}}{V} \qquad \bar{p} = Qd\bar{u}$$

Asm

$$[\bar{P}] = \frac{Asm}{m^3} = \frac{As}{m^2}$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$$

$$\bar{P} = \chi_e \varepsilon_0 \bar{E}$$

SUSKEPTIIVISUUS

$$\bar{D} = \varepsilon_0 (1 + \chi_e) \bar{E} = \varepsilon \bar{E}$$

Permittiivisyys $\varepsilon = \varepsilon_0 (1 + \chi_e) = \varepsilon_r \varepsilon_0$

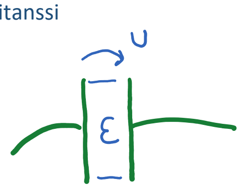
Suskeptiivisuus χ_e

Suhteellinen permittiivisyys ε_r

Permittiivisyys ~~Permittiviteetti~~

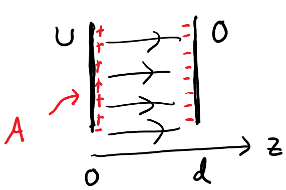
Permeabiilisuus ~~Permeabiliteetti~~

Kapasitanssi



$$C = \frac{Q}{U} \qquad \frac{As}{V} = \text{faradi}$$

$$\nabla^2 \phi = 0 \Rightarrow \phi \to \bar{E} \to \bar{D} \to \rho_s \to Q$$



$$\phi''(z) = 0 \qquad \phi(z) = Az + B \qquad \phi(0) = U$$
$$\phi(d) = 0$$

$$\phi(z) = U(1 - z/d)$$

$$\bar{E} = -\nabla\phi = -\bar{u}_z \frac{\partial}{\partial z} U(1 - z/d) = \bar{u}_z \frac{U}{d} \Rightarrow \bar{D} = \varepsilon \bar{u}_z \frac{U}{d}$$

$$\rho_s = \varepsilon \frac{U}{d} \Rightarrow Q = \varepsilon \frac{U}{d} A \Rightarrow C = \varepsilon \frac{A}{d}$$

Johtavuus σ

$$\bar{J} = \sigma \bar{E}$$ ← OHMIN LAKI

$$[\sigma] = \frac{A/m^2}{V/m} = \frac{A}{Vm} = \frac{S}{m}$$

$\sigma = \mu \rho$ ← VARAUSTIHEYS

(μ: LIIKKUVUUS)

Rajapintaehdot:

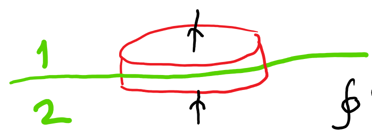
Sähkövuon tiheyden normaalikomponentti jatkuva rajapinnan ylitse
Sähkökentän voimakkuuden tangenttikomponentti jatkuva rajapinnan ylitse

(analogiset rajapintaehdot magneettikenttien puolella)

ε_1, $\bar{n}$, $\bar{E}_1$, $\bar{D}_1 = \varepsilon_1 \bar{E}_1$

ε_2, $\bar{E}_2$, $\bar{D}_2 = \varepsilon_2 \bar{E}_2$

$$D_{n_1} = D_{n_2} \qquad \bar{n} \cdot \bar{D}_1 = \bar{n} \cdot \bar{D}_2$$ (normaali-komponentti)



$$\oint \bar{D} \cdot d\bar{S} = 0$$

(ei varausta rajapinnalla)

$$\bar{E}_{t_1} = \bar{E}_{t_2}$$ (tangenttikomponentti)

$$\bar{n} \times \bar{E}_1 = \bar{n} \times \bar{E}_2$$