

Exercise 4 – Solutions

4.10.2022

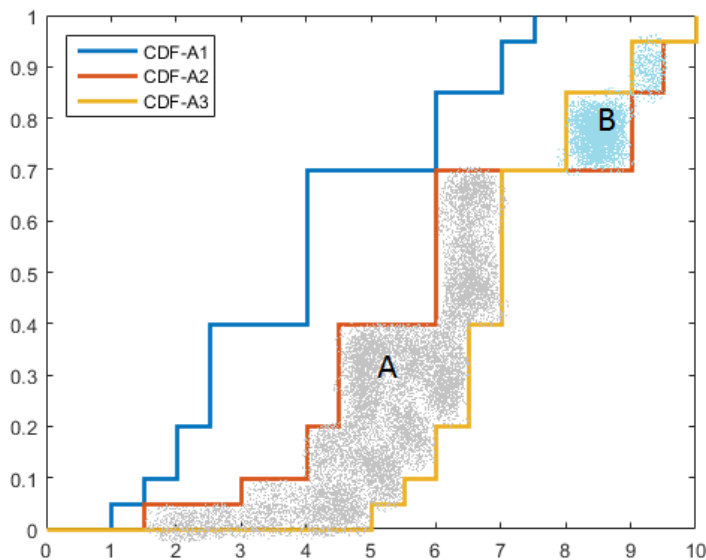
#1 Risk measures

VaR-10% is t such that $F(t)=0.1$. Thus, the VaR-10% values of the alternatives can be deduced based on the PMF table of the exercise sheet or read directly from the CDF plot formed already in Exercise 3 (see the plot again below). Now,

- A1: VaR-10%=1.5
- A2: VaR-10%=3
- A3: VaR-10%=5.5

CVaR-10% is $E[X|X \leq \text{VaR-10\%}]$ so that

- A1: $\text{CVaR}_{10\%} = \frac{0.05(1)+0.05(1.5)}{0.1} = 1.25$
- A2: $\text{CVaR}_{10\%} = \frac{0.05(1.5)+0.05(3)}{0.1} = 2.25$
- A3: $\text{CVaR}_{10\%} = \frac{0.05(5)+0.05(5.5)}{0.1} = 5.25$



In Exercise 3, it was concluded that A1 is FSD dominated by both A2 and A3. This is now reflected by the VaR-10% value of A1 being smaller than those of A2 and A3. The fact that A1 is also SSD dominated by A2 and A3 is reflected by the $\text{CVaR}_{10\%}$ value of A1 being smaller than those of A2 and A3. The SSD dominance of A3 over A2 is reflected by the $\text{CVaR}_{10\%}$ value of the former being larger than that of the latter.

#2 Risk measures with Matlab

- a) See the model solution file “Ex_4_task2_solution.m”. The function “makedist” is part of Statistics and Machine Learning Toolbox so the code might not work if you are missing this Toolbox. To install the Toolbox, see “installation_guide_toolbox.pdf” provided in the exercise folder.
- b) See Figure 2.1. The code for the figure can be seen from the model solution file “Ex_4_task2_solution.m”.
- c) The results are saved in vectors where each component represent one investment opportunity. When looking at the “Ex_4_task2_solution.m” code and after running it the expected values are in variable “EV”, the three VaR values are in variables “Var1”, “Var5” and “Var10” and the three CVaR values are in variables “CVar1”, “CVar5” and “CVar10”
- d) See Figure 2.2. The code for the figure can be seen from the model solution file.
- e) When looking at the options we can see that some options give both the better expected value and risk measure values that others (look at pair A2 and A4 for example) so A2 should be preferred over A4. Also, other such combinations exist. However, there is a tradeoff between expected value and risk meaning that some options are better in terms of risk (for example A1) and some in terms of expected value (for example A3). The DM needs to choose between different alternatives.
- f) The CVaR values of A4 and A5 are always less than of A1 and A2 so A1 SSD A4, A1 SSD A5, A2 SSD A4 and A2 SSD A5. Thus, a risk averse DM should avoid opportunities A4 and A5. Notice here that the CVar 100% is equal to the expected value of the investment opportunity.
- g) See the model solution file “Ex_4_task2_solution.m”
- h) See Figure 2.3. It now shows the SSD between the investment opportunities. CDF of A1 starts below CDF of A5 and the area between curves is larger before the intersection of the curves than after it.
- i) See Figure 2.4. Now CDF of A2 starts below CDF of A3 but the area between curves is larger after the intersection of the curves than before it. Thus, no SSD between A2 and A3.

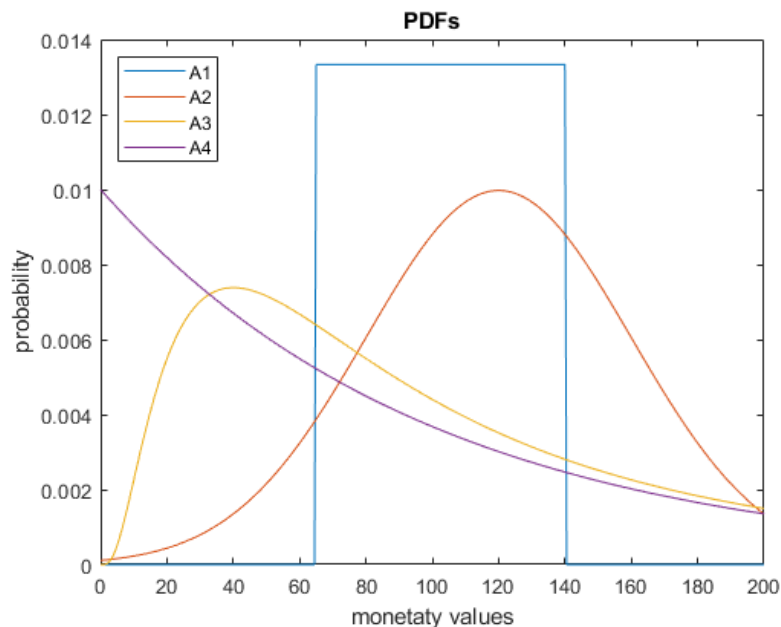


Figure 2.1: PDFs

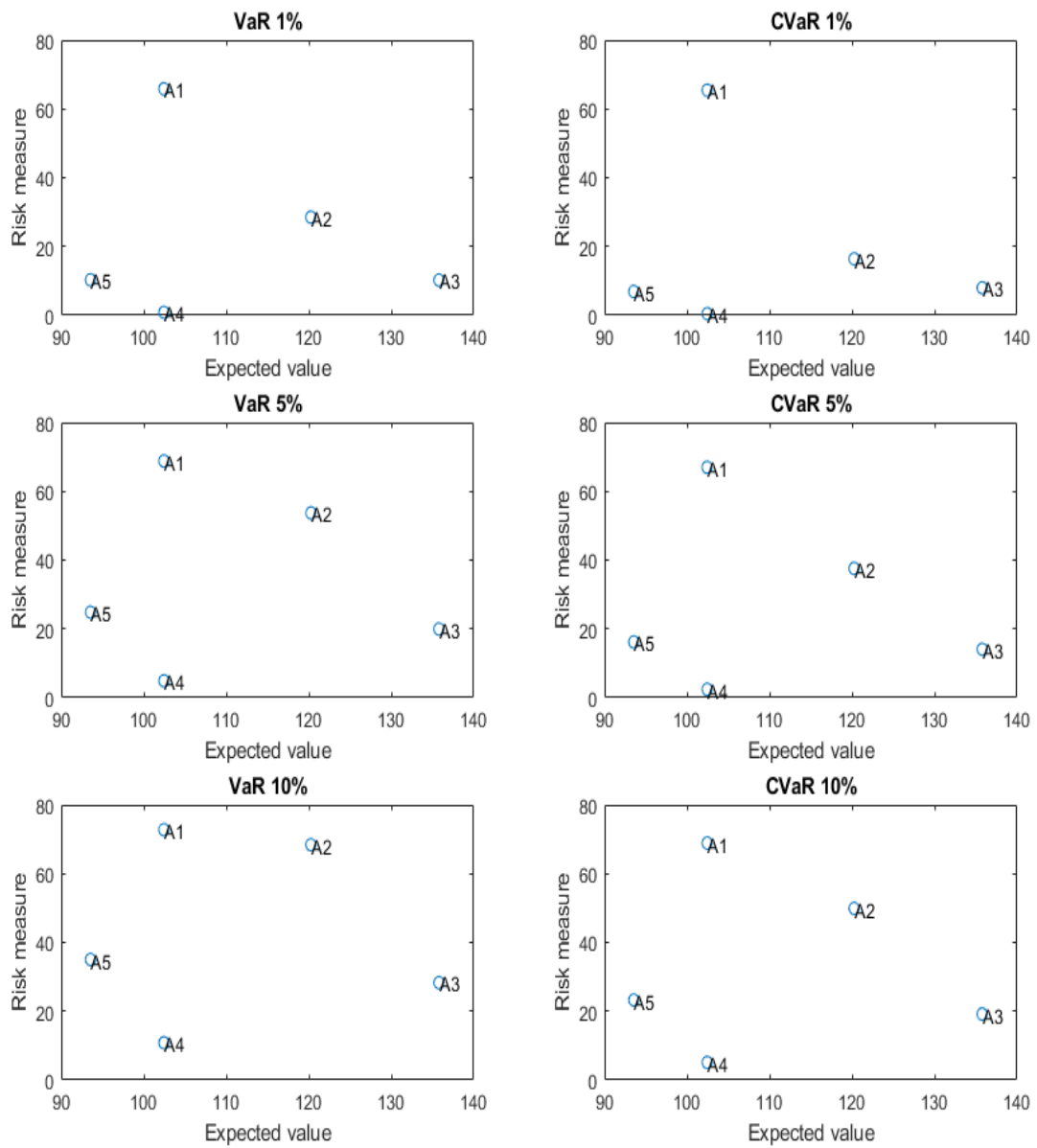


Figure 2.2: Risk measures plotted for each investment opportunity

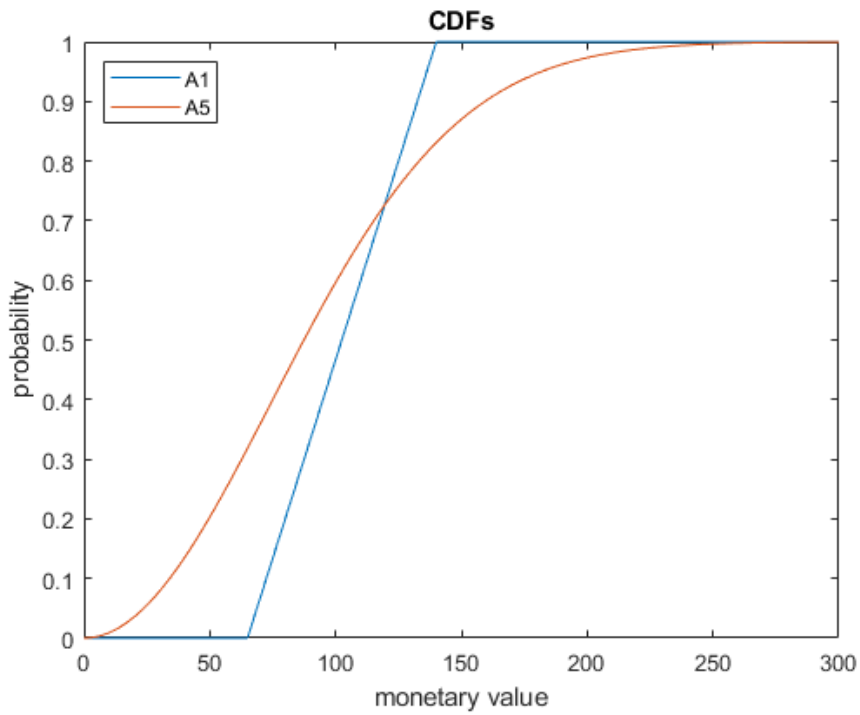


Figure 2.3: CDFs of A1 and A5

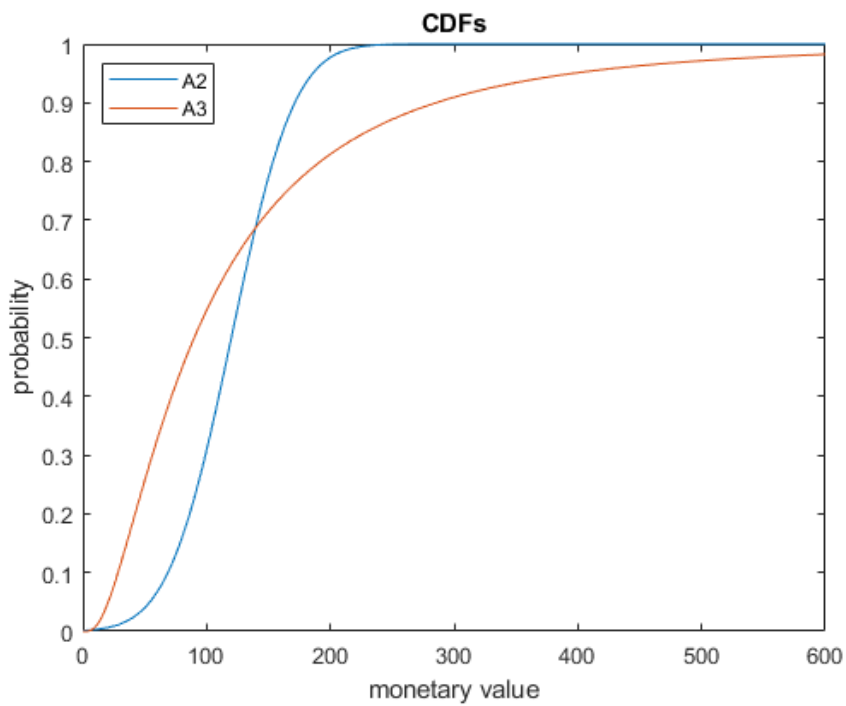


Figure 2.4: CDFs of A2 and A3