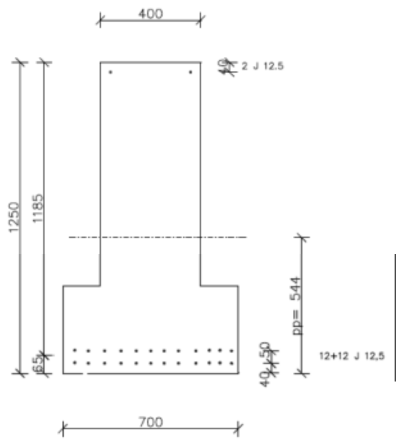
Example:
Cracked prestressed cross-section

Check if the prestresses ledge-beam satisfy the requirements of the cracked limit state



Depth of the beam	$h := 1250 \cdot mm$
Width of the compression side	$b := 400 \cdot mm$
Width of the bottom flange	$b_a := 700 \cdot mm$
Transformed cross-section values	
Area	$A_m := 604400 \cdot mm^2$
Centroid from the bottom	$p_p := 544 \cdot mm$
Section modulus about the bottom side	$W_a := 1.594 \cdot 10^8 \cdot mm^3$
Section modulus about the top side	$W_y := 1.224 \cdot 10^8 \cdot mm^3$

Concrete C30/40 Exposure class XC1

Prestress after the long-term losses
= initial prestress - losses due to shrinkage, creep and relaxation $\sigma_{p\infty 1} := 1287 \cdot MPa$

Span L=21 m

Loading at the service limit state

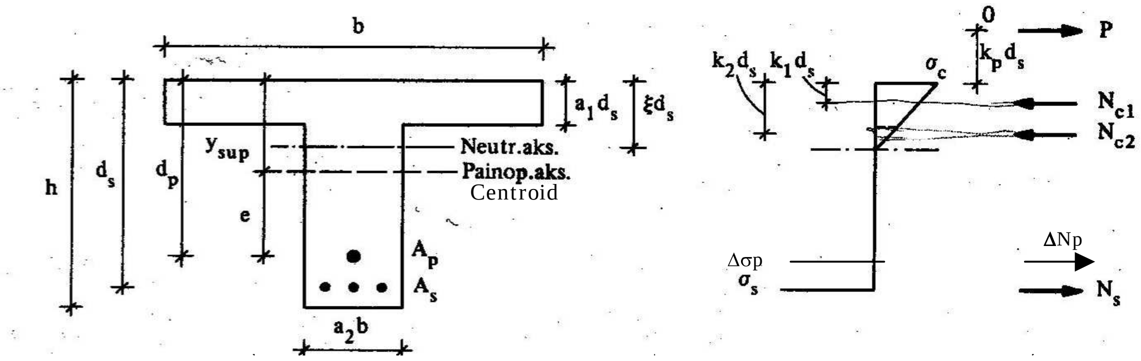
-dead load $g_k := 35 \cdot \frac{kN}{m}$ (includes the weight of the beam)

- live load $q_k := 25 \cdot \frac{kN}{m}$ combination coefficient for frequent laod combination $\psi_1 := 0.7$

combination coefficient for quasi-permanent load combination $\psi_2 := 0.3$

Calculate the cracking moment
Calculate stress in reinforcement and concrete for cracked cross-section under frequent load combination.
Hint: Use as the first approximaton of the location of the neutral axis the value for the uncracked cross-section.

ANALYSING OF CRACKED PRESTRESSED CROSS-SECTION



Cross-section   Stresses   Stress resultants

In the cross-section there are both prestressed and ordinary reinforcement steel
T-section with both prestressed and ordinary reinforcement at the cracked state.

The cross-section loaded only by the prestressing force the compression line lies at the point of the prestressing force resultant. When the cross-section is under the bending moment M at the service limit state, the compression line rises the distance e + y_{sup} + kp·d_s to the point O.
So the bending moment is

$$M = P \cdot (e + y_{sup} + k_p \cdot d)$$

where

P — is the resultant of the prestress force

e — is the eccentricity of the prestressing force from the centroid of the cross-section

y_{sup} — is the distance of the centroid from the top (compression) face

$$d = d_s \cdot \frac{A_s + \frac{E_p}{E_s} \cdot A_p \cdot \frac{d_p}{d_s}}{A_s + \frac{E_p}{E_s} \cdot A_p}$$ is the effective depth of the tension reinforcement

d_s — is the effective depth of the ordinary reinforcement

d_p — is the effective depth of the prestress strands

$k_p \cdot d$ — is the location of the compression line (point 0) from the top (compression face)
positive, if the point O is outside the cross-section
negative, if the point O is inside the cross-section

So the parameter $k_p = \dfrac{M - P \cdot (e + y_{sup})}{P \cdot d} = \dfrac{M - P \cdot d_p}{P \cdot d}$

Further

The resultant of concrete compression stress in the flange $N_{c1} = \frac{\sigma_c \cdot b \cdot d_s}{2} \cdot \left(\frac{2 \cdot \xi - a_1}{\xi} \right) \cdot a_1$

The resultant of the compression stress in the web $N_2 = \frac{\sigma_c \cdot b \cdot d_s}{2} \cdot \left(\frac{(\xi - a_1)^\wedge 2}{\xi} \right) \cdot a_2$

The resultant force of the ordinary reinforcement

$$N_s = \sigma_s \cdot A_s = \varepsilon_s \cdot E_s \cdot A_s = \frac{\frac{d_s}{d} - \xi}{\xi} \varepsilon_c \cdot E_s \cdot A_s = \frac{\frac{d_s}{d} - \xi}{\xi} \cdot \alpha_e \cdot \sigma_c \cdot \rho_s \cdot b \cdot d_s$$

The addition of the force in the prestress reinforcement

$$\Delta N_p = \Delta\sigma_s \cdot A_p = \Delta\varepsilon_p \cdot E_p \cdot A_p = \frac{\frac{d_p}{d} - \xi}{\xi} \Delta\varepsilon_p \cdot E_p \cdot A_p = \frac{\frac{d_p}{d} - \xi}{\xi} \cdot \alpha_p \cdot \sigma_c \cdot \rho_p \cdot b \cdot d_s$$

The total resultant of the tension force $N_s + \Delta N_p = \frac{1 - \xi}{\xi} \cdot \alpha_e \cdot \sigma_c \rho \cdot b \cdot d$

where

$\rho = \frac{A_s + \frac{E_p}{E_s} \cdot A_p}{b \cdot d}$ is the relative amount of the tension reinforcement

$\alpha_e = \frac{E_s}{E_{c,eff}}$ is the ratio of the elastic modulies of ordinary reinforcement and concrete

E_s is the elastic modulus of the ordinary reinforcement
E_p is the elastic modulus of prestress reinforcement

$E_{c,ee}$ is the effective elastic modulus of concrete $E_{c,eff} = \frac{E_{cm}}{1 + \phi \cdot \frac{M_{Ed,long-term}}{M}}$

E_{cm} is the elastic modulus of concrete
ϕ is the creep factor
$M_{Ed,lon-term}$ is the bending moment due to quasi-permanent load combination
M is the bending moment at the service limit state under the load combination which is considered (frequent or characteristic)

$\xi = \frac{x}{d}$ is the relative depth of the compression zone

If the tension reinforcement consist only prestress strands and no ordinary reinforcement:
$A_s = 0$ and $d=d_p$ and $d_s=d_p$ and $E_s=E_p$

The equilibrium of the axial forces: $N_{c1} + N_{c2} - N_s - P = 0$

The moment equilibrium about the point O: $N_{c1} \cdot (k_1 + k_p) + N_{c2} \cdot (k_2 + k_p) - N_s(1 + k_p) = 0$

Parameters $k_1 = \left(\frac{3 \cdot \xi - 2 \cdot a_1}{2 \cdot \xi - a_1} \right) \cdot \frac{a_1}{3}$

$$k_2 = \frac{2 \cdot a_1 + \xi}{3}$$

Putting the equations of N_{c1}, N_{c2} , k_1, k_2 and N_s to the equation of the moment equilibrium we get first the 4. degree equation for solving the parameter ζ . A one of it's root is $\zeta = 0{,}5\ a_1$. Then the equation returns to the 3.-degree equation

$$\xi^3 + 3 \cdot k_p \cdot \xi^2 + \left(\frac{K_1 + K_2}{a_2} - K_1 \right) \cdot \xi + \left(K_3 - \frac{K_3 + K_2}{a_2} \right) = 0$$

where

$$K_1 = 3 \cdot a_1 \cdot (a_1 + 2 \cdot k_p)$$
$$K_2 = 6 \cdot \alpha_e \rho_s \cdot (1 + k_p)$$
$$K_3 = a_1^{\ 2} \cdot (2 \cdot a_1 + 3 \cdot k_p)$$
$$k_p = \frac{M - P \cdot (e + y_{sup})}{P \cdot d_s}$$

Putting N_{c1}, N_{c2} and N_s to the equilibrium equation for the axial forces we can solve the concrete stress at the top fibre

$$\sigma_c = \frac{P}{b \cdot d_s} \cdot \left[\frac{2 \cdot \xi}{a_1(2 \cdot \xi - a_1) + a_2 \cdot (\xi - a_1)^2 - 2 \cdot \alpha_e \cdot \rho_s \cdot (1 - \xi)} \right]$$

Stress in the ordinary tension reinforcement is $\sigma_s = \frac{\frac{d_s}{d} - \xi}{\xi} \cdot \alpha_e \cdot \sigma_c$

Addition of the stress in the prestress reinforcenemt is $\Delta\sigma_p = \frac{\frac{d_p}{d} - \xi}{\xi} \alpha_p \cdot \sigma_c$

For the retangular cross-section the 3. degree equation is $\xi^3 + 3 \cdot k_p \cdot \xi^2 + K_2 \cdot \cdot \xi - K_2 = 0$

when $a_1 = 0$ and $a_2 = 1$

Concrete compression stress is $\sigma_c = \frac{P}{b \cdot d_s} \cdot \left[\frac{2\xi}{\xi^2 - 2\alpha_e \cdot \rho_s \cdot (1-\xi)} \right]$

If the cross-section consists also the top strands the prestress force resultant and it's eccentricity are calculated by taking into account the prestress force in the top strands.
In the equilibrium equation the compressive change of the force in the top strands due to concrete compression is taked into account

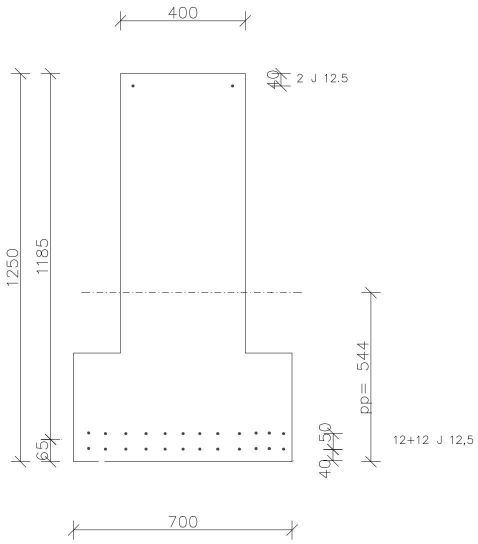
$$N_s^{'} = A_{py} \cdot \Delta\sigma_{sy} = A_{py} \cdot (E_p - E_c) \cdot \Delta\varepsilon_{py} = A_{py} \cdot (\alpha_e - 1) \cdot E_c \cdot \frac{x - c_y}{x} \cdot \varepsilon_c$$

$$= (\alpha_e - 1) \cdot \rho^{'} \cdot \sigma_c \cdot \frac{(\xi - \frac{c_y}{d_s})}{\xi} \cdot b \cdot d_s$$

In the above equation term $(\alpha_e\text{-}1)$ means that there is no concrete at the point of the top strands

Putting N_s' to the equilibrium equation the concrete stress can be solved

$$\sigma_c = \frac{2 \cdot P}{b \cdot d_s} \cdot \frac{\xi}{\xi^2 - 2 \cdot (1-\xi) \cdot \alpha_e \cdot \rho + 2 \cdot (\xi - \frac{c_y}{d_s}) \cdot (\alpha_e - 1) \cdot \rho^{'}}$$



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