Differential and Integral Calculus 1 - MS-A0111
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Practice Exam

Every problem carries an equal weight. Similarly every part of a problem carries an equal weight. Note that there is a second page! You are not allowed to use a calculator, tables or notes.

Explain the reasoning behind your solutions, do not just write the final result.

PROBLEM 1 Compute the following limits:

$$(a) \quad \lim_{x \to 0} \frac{1 - e^x}{\sin(2x)} \qquad\qquad (b) \quad \lim_{x \to 0} \frac{(e^{2x} - 1)\ln(1 + x^3)}{(1 - \cos(3x))^2}.$$

PROBLEM 2 Find the derivative of $f(x) = e^x - 1$ using the definition of derivative as a limit.

PROBLEM 3 Compute the integrals

$$(a) \quad \int_0^1 (x^2 + 1)(x^2 - 1)\, dx \qquad\qquad (b) \quad \int_0^1 \frac{1}{2 + e^x}\, dx.$$

PROBLEM 4 Find all the solutions of the equation $y' = 1 - y^2$.

PROBLEM 5 Two positive real numbers have sum equal to 7. What is the largest possible value for their product?
Hint: If you don't know a number, call it x. (The other number here can be expressed in terms of x, knowing that its sum with x is equal to 7.)

PROBLEM 6 Consider the function $f(x) = e^x + e^{-x}$

a) Compute the limits

$$\lim_{x \to -\infty} f(x), \qquad\qquad \lim_{x \to +\infty} f(x).$$

b) Compute the first derivative of f and study its sign: where is it positive, negative, zero? Where does f increase/decrease?

c) Compute the second derivative of f and study its sign: where is it positive, negative, zero? Where is f convex (happy)/concave (sad)?

d) Use the information above to draw a sketch of the graph of f.

Possibly useful formulas:

$$D \arcsin x = \frac{1}{\sqrt{1-x^2}}, \qquad D \arctan x = \frac{1}{1+x^2}$$

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \qquad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k, \qquad \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

General hint: If you have a fraction with a product in the denominator, for instance $\frac{1}{(x+1)(x-1)}$, it's a good idea to try to write it as a sum of two simpler fractions, in this example by finding two real numbers A and B such that $\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$.