VIRTAUSSTATIIKKA

$\bar{E} \longrightarrow \bar{J}$ (JOS JOHTAVUUTTA) $[\bar{J}] = \frac{A}{m^2}$ $[\bar{E}] = \frac{V}{m}$

Johtavuus ja Ohmin laki

$\downarrow$

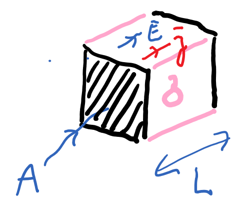
γ

$\bar{J} = \gamma \bar{E}$ OHMIN LAKI $[\gamma] = \frac{A/m^2}{V/m} = \frac{A}{Vm} = \frac{S}{m}$

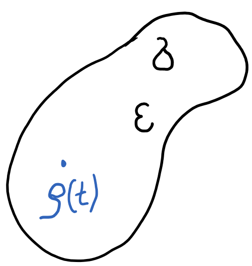
VASTUS: $E = \frac{U}{L} \Rightarrow J = \gamma E = \gamma \frac{U}{L} \Rightarrow I = JA = \gamma \frac{A}{L} U$

$\Rightarrow G = \frac{I}{U} = \gamma \frac{A}{L}$ (KONDUKTANSSI)

$R = \frac{1}{G} = \frac{U}{I} = \frac{L}{\gamma A}$ (RESISTANSSI)



Kuinka varausjakautuma käyttäytyy johtavassa aineessa?



$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \cdot (\ \checkmark\ ) \quad \Rightarrow \quad 0 = \nabla \cdot \bar{J} + \frac{\partial}{\partial t} \overbrace{\nabla \cdot \bar{D}}^{\rho}$$

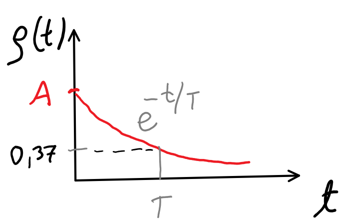
$(\nabla \cdot \nabla \times \bar{H} = 0)$

$\bar{J} = \gamma \bar{E}$

$\bar{D} = \varepsilon \bar{E}$

$$\gamma \bar{E} = \frac{\gamma}{\varepsilon} \bar{D}$$

$$-\frac{\gamma}{\varepsilon} \rho(t) = \frac{\partial}{\partial t} \rho(t) \quad \Rightarrow \quad \rho(t) = A\, e^{-\frac{\gamma}{\varepsilon} t}$$



$T = \varepsilon / \gamma$

Kupari: $T = \frac{\varepsilon_0}{\gamma} = \frac{10^{-11} \frac{As}{Vm}}{10^{8} \frac{A}{Vm}} = 10^{-19} s$!

Kuiva puu: $T = \frac{10^{-11}}{10^{-15}} s = 10^{4} s = 3h$!

MAGNETOSTATIIKKA

Magneettikentän voimakkuus $\bar{H}$ yksikkö $\frac{A}{m}$

Magneettivuon tiheys $\bar{B}$ yksikkö $\frac{Vs}{m^2} = T$ (tesla)

Magneettikentän voimakkuus $\quad H \quad$ yksikkö $\quad$ m

Magneettivuon tiheys $\quad \bar{B} \quad$ yksikkö $\frac{Vs}{m^2} = T$ (tesla)

Tyhjiön permeabiilisuus

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$$

$$\bar{B} = \mu_0 \bar{H}$$

Hans Christian Örsted (1777-1851)

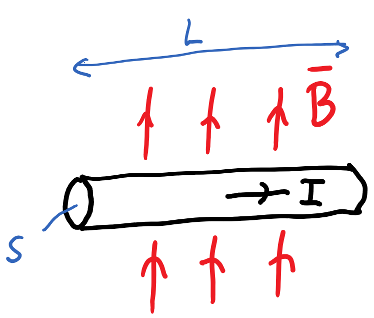
1820!

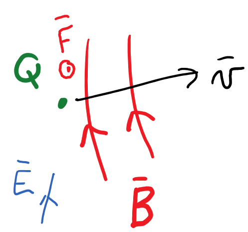
Sähköllä magneettista voimaa
https://aalto.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=42cfa837-931a-4916-9aa3-aeb60098c281

Magneetilla sähköistä voimaa
https://aalto.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=fb02c04e-fed5-416f-b695-aeba00cc6c91

Pyörrevirroista
https://aalto.cloud.panopto.eu/Panopto/Pages/Viewer.aspx?id=2b331402-074f-4b81-8d32-aeba00dafe6c

Kokeelliset lait:





$$\bar{F} = Q\,\bar{v} \times \bar{B}$$

$$\bar{F} = Q\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

(LORENTZIN VOIMA)

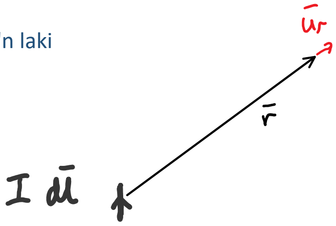
$$I = JS$$

$$J = \underbrace{\rho}_{\frac{As}{m^3}}\,\underbrace{v}_{\frac{m}{s}} \rightarrow \frac{A}{m^2}$$

$$F = \rho S L v B$$

$$= J S L B = I L B$$

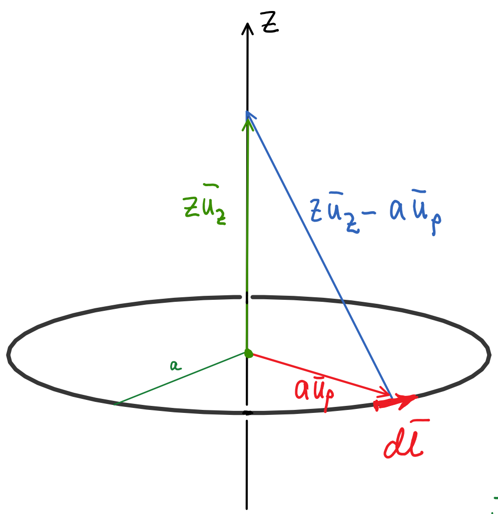
Biot-Savart'n laki



$$\bar{H}(\bar{r}) = \frac{I\, d\bar{l} \times \bar{u}_r}{4\pi r^2}$$

$$\frac{\bar{u}_r}{r^2} = \frac{\bar{r}}{r^3}$$

Virtasilmukan magneettikenttä



$$\bar{H}(z) = \int_0^{2\pi} \frac{I\, a\, d\varphi\, \bar{u}_\varphi \times (z\bar{u}_z - a\bar{u}_\rho)}{4\pi\, |z\bar{u}_z - a\bar{u}_\rho|^3}$$

$$\bar{u}_\varphi \times \bar{u}_z = \bar{u}_\rho$$

$$\bar{u}_\varphi \times \bar{u}_\rho = -\bar{u}_z$$

$$|z\bar{u}_z - a\bar{u}_\rho|^2 = (z\bar{u}_z - a\bar{u}_\rho)\cdot(z\bar{u}_z - a\bar{u}_\rho) = z^2 + a^2$$

$$\bar{H}(z) = \frac{I a}{4\pi \sqrt{z^2+a^2}^{\,3}} \underbrace{\int_0^{2\pi} (z\,\bar{u}_\rho + a\bar{u}_z)\, d\varphi}_{a\bar{u}_z\, 2\pi}$$

$$\int_0^{2\pi} \bar{u}_\rho = 0$$ (eikö niin?)

$$\bar{H}(z) = \frac{I a^2}{2\,(z^2+a^2)^{3/2}}\, \bar{u}_z$$

$$\nabla \cdot \bar{B} = 0$$

$$\bar{B} \neq \mu_0 \bar{H}$$

$z >> a \qquad (z^2+a^2)^{3/2} \simeq z^3$

$$\bar{H}(z) = \frac{I a^2}{2 z^3}\, \bar{u}_z = \frac{\overbrace{\mu_0 I \pi a^2}^{p_m}}{4\pi \mu_0 z^3}\, 2\bar{u}_z$$

= DIPOLIN KENTTÄ !

$$\bar{E}_d = \frac{p_e}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\, \bar{u}_r + \sin\theta\, \bar{u}_\theta\right) \Rightarrow \bar{E}_d(z) = \frac{p_e}{4\pi\varepsilon_0 z^3}\, 2\bar{u}_z$$

Magneettidipoli

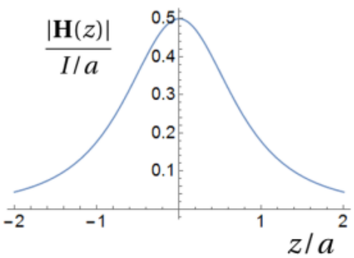
$$\bar{p}_m = \bar{u}_z\, \mu_0 \underbrace{I \pi a^2} \qquad [p_m] = \frac{Vs}{Am}\, Am^2 = Vs\,m$$

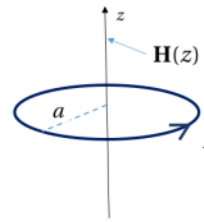
Magneettidipoli $\bar{p}_m = \bar{u}_z \mu_0 \underbrace{I\pi a^2}_{m}$

$$[p_m] = \frac{Vs}{Am} Am^2 = Vsm$$

$$[m] = Am^2$$

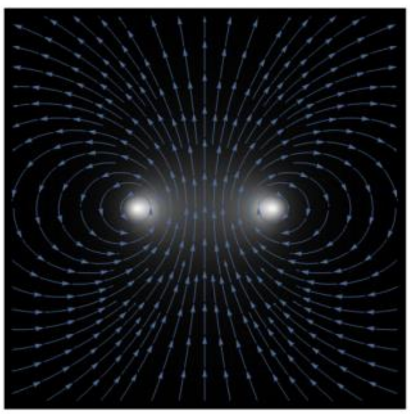
Tasavirtasilmukan magneettikenttä symmetria-akselilla

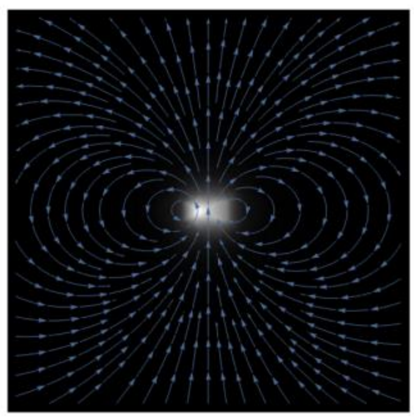




$$\mathbf{H}(z) = \mathbf{u}_z \frac{Ia^2}{2(z^2+a^2)^{3/2}}$$

Virtasilmukan magneettikenttä muuallakin kuin akselilla:
(elliptiset integraalit, oppikirja, s. 189)





$$\phi_d = \frac{p_e \cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\bar{p}_e \cdot \bar{u}_r}{4\pi\varepsilon_0 r^2}$$

$$\bar{E}_d = -\nabla\phi_d \sim \frac{2}{r^3}\cos\theta\, \bar{u}_r + \frac{1}{r^3}\sin\theta\, \bar{u}_\theta$$

$\bar{p}_e \uparrow z$ dipoli z-suuntainen: $\bar{p}_e = p_e \bar{u}_z$

Entäpä x-suuntainen dipoli?

$\rightarrow x$

$\bar{p}_e = p_e \bar{u}_x$

$$\bar{p}_e \cdot \bar{u}_r = p_e \bar{u}_x \cdot \bar{u}_r = \underline{p_e \sin\theta \cos\varphi} \quad (r^{-2})$$

$\rightarrow x$

$\bar{p}_e = p_e \bar{u}_x$

$-\nabla\phi_e \sim -\nabla\left(r^{-2}\sin\theta\cos\varphi\right)$

$= 2\bar{u}_r r^{-3}\sin\theta\cos\varphi$

$-\frac{1}{r}\bar{u}_\theta\cos\theta\cos\varphi\, r^{-2}$

$+\frac{1}{r\cancel{\sin\theta}}\bar{u}_\varphi\sin\varphi\,\cancel{\sin\theta}\, r^{-2}$

$= \left(\bar{u}_r 2\sin\theta\cos\varphi - \bar{u}_\theta\cos\theta\cos\varphi + \bar{u}_\varphi\sin\varphi\right) r^{-3}$

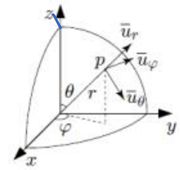
Pallokoordinaatisto

$$\nabla f(\bar{r}) = \bar{u}_r\frac{\partial}{\partial r}f + \bar{u}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}f + \bar{u}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f$$

$$\nabla\times\bar{f} = \frac{1}{r^2\sin\theta}\begin{vmatrix}\bar{u}_r & r\bar{u}_\theta & r\sin\theta\,\bar{u}_\varphi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ f_r & r f_\theta & r\sin\theta\, f_\varphi\end{vmatrix}$$

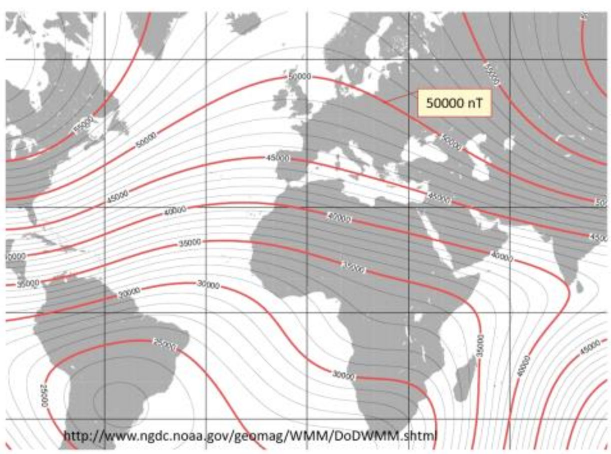
$$\nabla\cdot\bar{f} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f_\varphi$$

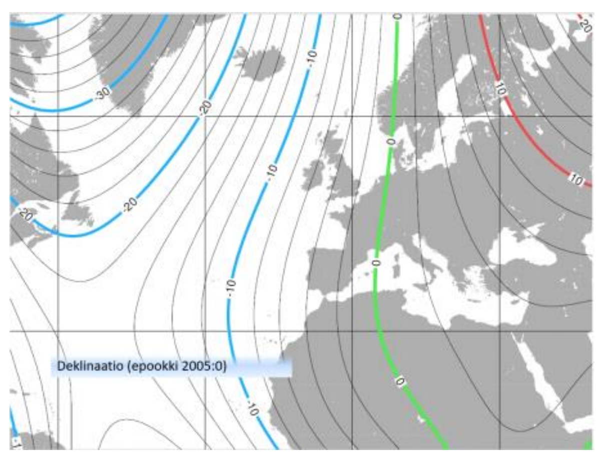
$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$



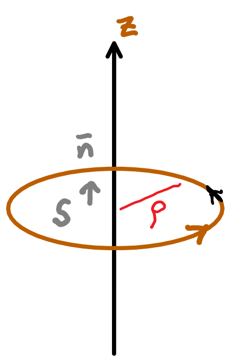
Mathematica-animaatio

Maamagnetismi





Suoran virtalangan magneettikenttäkenttä



$d\bar{l} = \bar{u}_\varphi \rho\, d\varphi$

$\bar{H} = \bar{u}_\varphi \frac{I}{2\pi\rho}$

$$\int_S \underbrace{\nabla\times\bar{H}}_{\bar{J}}\cdot d\bar{S} = \oint_C \bar{H}\cdot \underbrace{d\bar{l}}_{\bar{u}_\varphi \rho\, d\varphi} = I$$

$\nabla\times\bar{H} = \bar{J}$

$\frac{I}{2\pi\rho}\bar{u}_\varphi$

$\int \bar{J}\cdot d\bar{S} = I$

$\bar{u}_\varphi\cdot\bar{u}_\varphi = 1$

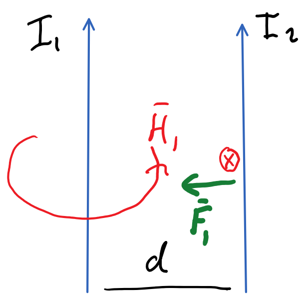
$2\pi\rho$

$\oint$ $\qquad \int_S \bar{J}\cdot d\bar{S} = I$ $\qquad \bar{u}_\varphi \cdot \bar{u}_\varphi = 1$

Ampèren laki

$$\nabla \times \bar{H} = \bar{J}$$

Virtalankojen välinen voima



$$F_1 = I_2 L \underset{\mu_0 \bar{H}_1}{B_1} = \mu_0 \frac{I_1}{2\pi d} I_2 L$$

Ampeerin määritelmä

$I_1 = I_2 = 1A$

$d = 1m$

$$\frac{F_1}{L} = \mu_0 \frac{1A \cdot 1A}{2\pi \cdot 1m}$$

$$= \underbrace{\cancel{4\pi}^{2} \cdot 10^{-7} \frac{1 \cdot 1}{\cancel{2\pi} \cdot 1}}_{0{,}2\,\mu} \underbrace{\frac{Vs}{\cancel{A}m} \cdot \frac{\cancel{A} \cdot A}{m}}_{N/m}$$

Permeabiilisuus

$$\mu = \mu_r \mu_0 \quad \mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$$

$$\bar{B} = \mu \bar{H}$$

DIAMAGNEETTINEN $\quad \mu_r \approx 1 \quad < 1 \quad 0{,}9998 \quad$ Au, Ag, Cu, VESI

PARAMAGNEETTINEN $\quad \mu_r \approx 1 \quad > 1 \quad 1{,}00002 \quad$ Mg, Al

FERROMAGNEETTINEN $\quad$ Fe, Co, Ni

Magneettiset materiaalit

Vektoripotentiaali $\bar{A}$

$\nabla \times \bar{H} = \bar{J}$ & $\nabla \cdot \bar{B} = 0$ $\qquad \bar{B} = \nabla \times \bar{A} \Rightarrow \nabla \cdot (\nabla \times \bar{A}) = 0$

$$\nabla \times \underbrace{\mu_0 \bar{H}}_{\bar{B} = \nabla \times \bar{A}} = \mu_0 \bar{J}$$

$$\Rightarrow \nabla \times (\nabla \times \bar{A}) = \mu_0 \bar{J}$$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$B = \nabla \times A$

$$\nabla(\underbrace{\nabla \cdot \bar{A}}_{=0}) - \nabla^2 \bar{A} = \mu_0 \bar{J}$$

$$\nabla^2 \bar{A}(\bar{r}) = -\mu_0 \bar{J}(\bar{r})$$

$$\bar{A}(\bar{r}) = \int \frac{\mu_0 \bar{J}(\bar{r}') \, dV'}{4\pi |\bar{r} - \bar{r}'|}$$

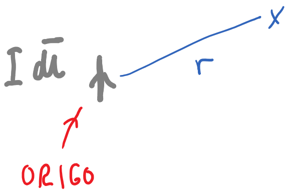
SÄHKÖSTATIIKKA

$$\nabla^2 \phi(\bar{r}) = -\frac{\rho(\bar{r})}{\varepsilon_0}$$

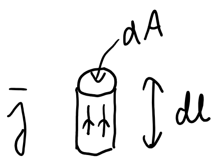
$$Q \cdot \xrightarrow{r} \times \quad \phi = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\times \quad \phi(\bar{r}) = \int \frac{\rho(\bar{r}') \, dV'}{4\pi\varepsilon_0 |\bar{r} - \bar{r}'|}$$

Pienen virta-alkion magneettikenttä



$$\bar{A}(\bar{r}) = \int \frac{\mu_0 \bar{J}(\bar{r}') \, dV'}{4\pi |\bar{r} - \cancel{\bar{r}'}|} = \frac{\mu_0}{4\pi r} \underbrace{\int \bar{J} \, dV'}_{\int \underbrace{\bar{J} \, dA}_{I} \, dl} = \frac{\mu_0 I \bar{dl}}{4\pi r}$$



$$\bar{B} = \nabla \times \bar{A} = \frac{\mu_0 I}{4\pi} \nabla \times \left(\frac{\bar{dl}}{r} \right) = -\frac{\mu_0 I}{4\pi} \frac{\bar{u}_r}{r^2} \times \bar{dl}$$

$$= \mu_0 \frac{I \bar{dl} \times \bar{u}_r}{4\pi r^2}$$

BIOT-SAVART'in laki !!!