Question 1:

A load is supplied by,

$V(t) = 5 + 100 \cos(314t) + 6 \cos(2(314)t+15°) + 40 \cos(3(314)t+30°)$

The current is given as,

$i(t) = 8 + 50 \cos(314t+30°) + 6 \cos(2(314)t+45°) + 10 \cos(3(314)t+65°)$

Find,

a) RMS Voltage.

b) RMS Current.

c) THD of load current.

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V(t) = 5 + 100 Cos (314t) + 6 Cos (2(314)t+15°) + 40 Cos (3(314)t+30°)
i(t) = 8 + 50 Cos (314t+15°) + 6 Cos (2(314)t+30°) + 10 Cos (3(314)t+45°)

a) RMS Voltage:

Using the formula,

$$V_{rms} = \sqrt{V_o{}^2 + V_{1,rms}{}^2 + V_{2,rms}{}^2 + \cdots}$$

$$V_{rms} = \sqrt{(5)^2 + \left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{40}{\sqrt{2}}\right)^2}$$

$$V_{rms} = 76.40\ V$$

b) RMS Current:

We know that,

$$I_{rms} = \sqrt{(8)^2 + \left(\frac{50}{\sqrt{2}}\right)^2 + \left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{10}{\sqrt{2}}\right)^2}$$

$$I_{rms} = 37.17\ A$$

V(t) = 5 + 100 Cos (314t) + 6 Cos (2(314)t+15°) + 40 Cos (3(314)t+30°)
i(t) = 8 + 50 Cos (314t+15°) + 6 Cos (2(314)t+30°) + 10 Cos (3(314)t+45°)

c) THD:

We have the formula,

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}}$$

$$THD = \sqrt{\frac{(37.17)^2 - \left(\frac{50}{\sqrt{2}}\right)^2}{\left(\frac{50}{\sqrt{2}}\right)^2}}$$

$$THD = 0.3244 = 32.44\%$$

Question 2:

A non-linear load is supplied by a voltage,

$$V(t) = 300 \cos(2\pi 50t)$$

The resulting non-linear current is given as,

$$i(t) = 10 + 70 \cos(2\pi 50t + 20°) + 40 \cos(6\pi 50t + 15°) + 30 \cos(8\pi 50t + 25°)$$

Find,

a) Power absorbed by load.

b) Power factor of load.

c) THD of load current.

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V(t) = 300 Cos (2π50t)

i(t) = 10 + 70 Cos (2π50t+20°) +
40 Cos (6π50t+15°) + 30 Cos (8π50t+25°)

a) Power Absorbed by Load:

To find the power absorbed by load, we have the formula,

$$P = V_0 I_0 + \sum V_{rms} I_{rms}\, Cos(\theta - \varphi)$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$P = (0)(10) + \left(\frac{300}{\sqrt{2}}\right)\left(\frac{70}{\sqrt{2}}\right)\cos(0 - 20) + \left(\frac{0}{\sqrt{2}}\right)\left(\frac{40}{\sqrt{2}}\right) + \left(\frac{0}{\sqrt{2}}\right)\left(\frac{30}{\sqrt{2}}\right)$$

$$P = 9.86\ kW$$

b) Power Factor:

We know that,

$$PF = \frac{P}{S} = \frac{P}{V_{in\ (rms)} I_{in\ (rms)}}$$

where,

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.13\ V$$

V(t) = 300 Cos (2π50t)

i(t) = 10 + 70 Cos (2π50t+20°) +
40 Cos (6π50t+15°) + 30 Cos (8π50t+25°)

and,

$$I_{rms} = \sqrt{(10)^2 + \left(\frac{70}{\sqrt{2}}\right)^2 + \left(\frac{40}{\sqrt{2}}\right)^2 + \left(\frac{30}{\sqrt{2}}\right)^2} = 61.64\ A$$

So,

$$PF = \frac{9866}{212.13\ x\ 61.64} = 0.75$$

c) THD of Load Current:

For THD, we have the formula,

$$THD = \sqrt{\frac{I_{rms}^2 - I_{1,rms}^2}{I_{1,rms}^2}} = \sqrt{\frac{(61.64)^2 - \left({}^{70}/_{\sqrt{2}}\right)^2}{\left({}^{70}/_{\sqrt{2}}\right)^2}}$$

$$THD = 0.7421 = 74.21\%$$

Question 3:

The voltage across a 10Ω resistor is

$$v(t) = 170 \sin(377t)$$

Determine,

a) Instantaneous power.
b) Average power.
c) Peak power.

v(t) = 170 Sin (377t)

a) Instantaneous Power:

We have the formulas for instantaneous power,

$$p(t) = v(t)\, i(t)$$

where,

$$i(t) = \frac{v(t)}{R}$$

$$i(t) = \frac{170\, Sin(377t)}{10} = 17\, Sin(377t)$$

So,

$$p(t) = 170\, Sin(377t)\ x\ 17\, Sin(377t)$$

$$p(t) = 2890\, Sin^2(377t)\, W$$

b) Average Power:

For average power, we have the formula,

$$P = \frac{1}{T}\int_0^T p(t)dt$$

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$$P = \frac{1}{2\pi}\int_0^{2\pi} 2890\, Sin^2(377t)\, dt$$

$$P = \frac{2890}{2\pi}\int_0^{2\pi} \frac{1 - Cos\ 2(377)t}{2}\, dt$$

$$\sin^2 t = \frac{1 - Cos\ 2t}{2}$$

$$P = \frac{2890}{4\pi}\int_0^{2\pi} 1 - Cos\ (754)t\, dt$$

$$P = \frac{2890}{4\pi}\left[t - \frac{Sin\ 754t}{754}\right]$$

$$P = \frac{2890}{4\pi}\left[(2\pi - 0) - \left(\frac{Sin\ 754 * 2\pi}{754} - \frac{Sin\ 754 * 0}{754}\right)\right]$$

$$P = \frac{2890}{4\pi}[6.27] = 1444.74\ W$$

c) Peak Power:

From the instantaneous power, we have

$$Peak\ Power = 2890\ W$$

Question 4:

The voltage and current of a circuit is given by,

$$v(t) = 3 + 5\ \mathrm{Cos}\ (2\pi 60t + 15°) + 2\ \mathrm{Cos}\ (4\pi 60t)$$

$$i(t) = 2 + 7\ \mathrm{Cos}\ (2\pi 60t + 45°) + 3\ \mathrm{Cos}\ (6\pi 60t + 25°)$$

Find,

a) RMS voltage and current.
b) Power absorbed by the circuit.
c) Power factor.

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v(t) = 3 + 5 Cos (2π60t+15°) + 2 Cos (4π60t)
i(t) = 2 + 7 Cos (2π60t+45°) + 3 Cos (6π60t+25°)

a) RMS Voltage and Current:

The RMS voltage and current is given as,

$$V_{rms} = \sqrt{(3)^2 + \left(\frac{5}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = 4.85\ V$$

$$I_{rms} = \sqrt{(2)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 5.74\ A$$

b) Power Absorbed by Load:

$$P = V_0 I_0 + \sum V_{rms} I_{rms}\ Cos(\theta - \varphi)$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$P = (3)(2) + \left(\frac{5}{\sqrt{2}}\right)\left(\frac{7}{\sqrt{2}}\right)\cos(15 - 45) + \left(\frac{2}{\sqrt{2}}\right)(0) + (0)\left(\frac{3}{\sqrt{2}}\right)$$

$$P = 21.15\ W$$

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c) Power Factor:

We know that PF is give as,

$$V_{rms} = 4.85\ V$$
$$I_{rms} = 5.74\ A$$

$$PF = \frac{P}{S} = \frac{P}{V_{in\ (rms)} I_{in\ (rms)}}$$

$$PF = \frac{21.15}{4.85\ x\ 5.74}$$

$$PF = 0.7597$$