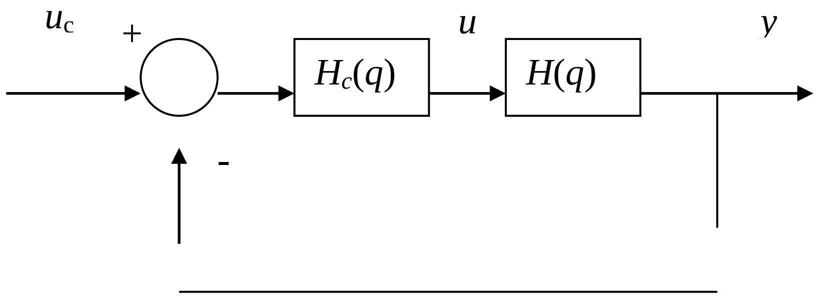
ELEC-E8101 Digital and Optimal Control
Exercise 5

1. Consider the system in the following figure.



Assume that the sampling time is h and D/A-converter holds the control signal constant during the sampling period. The control algorithm is

$$u(kh) = K\left(u_c(kh-\tau) - y(kh-\tau)\right),$$

where $K > 0$ and τ is a calculation time. The transfer function of the process is

$G(s) = \frac{1}{s}$ (this corresponds to the block $H(q)$).

a. Determine the pulse transfer function of the process.
b. For which values of gain K is the closed-loop system stable, if $\tau = 0$ or $\tau = h$?
c. Consider now the case that the continuous-time system is controlled with a continuous-time P-controller, and the process has delay τ. Calculate the values of K for which the closed loop is stable and compare to the part **b**.

2. Consider a system described by the figure in the previous problem, where input signal u_c is the unit step, and the pulse transfer function of the process is

$$H(q) = \frac{1}{q(q-0.5)}.$$

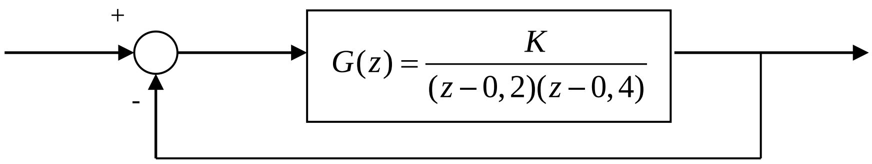
Consider two cases:

a. $H_c(q) = K$ (P-controller), $K > 0$,
b. $H_c(q) = \frac{Kq}{q-1}$ (I-controller), $K > 0$.

For which values of K is the closed loop system stable? How large is the output in the stationary state?

3.

a) Consider the system below and determine by the Jury's stability test, the Nyquist stability criterion and the "triangle rule" (see lecture slides), for which values of K the system is stable.



* **b)** Examine the situation once again, but now with an open loop pulse transfer function

$$G(z) = \frac{K}{z(z-0,2)(z-0,4)}.$$

4. (*) Consider the system

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(k).$$

a. Determine a control sequence u such that the state $\mathbf{x}$ is taken from the initial state $\mathbf{x}(0) = [1\ \ 1\ \ 1]$ to the origin.

b. Which is the minimum number of steps that solve the problem in **a.**?

c. Explain why it is not possible to find a sequence of control signals such that the state $[1\ \ 1\ \ 1]$ is reached from the origin.

5. Consider the representation

$$x(k+1) = \Phi x(k) + \Gamma u(k)$$
$$y(k) = Cx(k) + Du(k)$$

which has a stationary point (x_s, u_s, y_s). Show that it is possible to make a linear transformation by which the stationary state position is in the origin of the state space.

6. Is the following system reachable? How can a control signal be determined such that the state of the system moves from the origin to a given state $\boldsymbol{x}_f$ in three time steps?

$$\begin{cases} \mathbf{x}(k+1) = 0,2\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(k) \\ y(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\mathbf{x}(k) \end{cases}$$

7. Is the state-space representation of the previous problem observable? If the control signal is set to zero and the output of the system is observed for three time steps, how can the initial state of the system be determined?