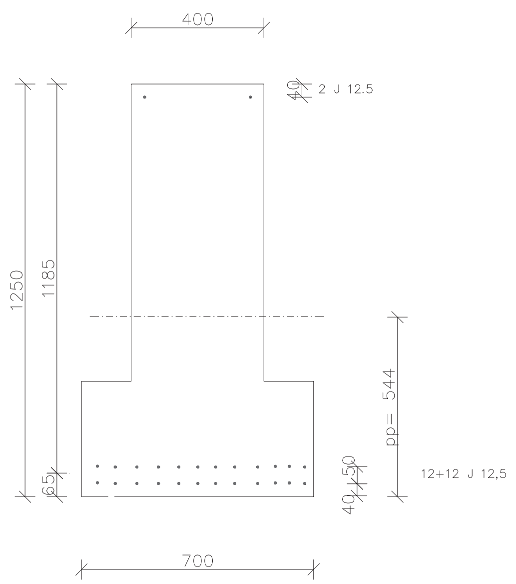
Example:
Cracked prestressed cross-section

Check if the prestresses ledge-beam satisfy the requirements of the cracked limit state



Depth of the beam	$h := 1250 \cdot mm$
Width of the compression side	$b := 400 \cdot mm$
Width of the bottom flange	$b_a := 700 \cdot mm$

Transformed cross-section values

Area	$A_m := 604400 \cdot mm^2$
Centroid from the bottom	$p_p := 544 \cdot mm$
Section modulus about the bottom side	$W_a := 1.594 \cdot 10^8 \cdot mm^3$
Section modulus about the top side	$W_y := 1.224 \cdot 10^8 \cdot mm^3$

Loading at the service limit state

-dead load $g_k := 35 \cdot \frac{kN}{m}$ (includes the weight of the beam)

- live load $q_k := 25 \cdot \frac{kN}{m}$ combination coefficient for frequent laod combination $\psi_1 := 0.7$

combination coefficient for quasi-permanent load combination $\psi_2 := 0.3$

Frequent laod conbination

$p := g_k + \psi_1 \cdot q_k$ $p = 52.5 \frac{kN}{m}$

Quasi-permanent load combination

$p_{pitkä} := g_k + \psi_2 \cdot q_k$ $p_{pitkä} = 42.5 \frac{kN}{m}$

Span $L := 21 \cdot m$

Bending moments

Quasi-permanent load combination $M_{quasi} := \frac{p_{pitkä} \cdot L^2}{8}$ $M_{quasi} = 2343\,kNm$

Frequent load combination $M_{freq} := \frac{p \cdot L^2}{8}$ $M_{freq} = 2894\,kNm$

Concrete C 30/40

characteristic cylinder strength $f_{ck} := 30 \cdot MPa$

average tensile strength $f_{ctm} := 0.3 \cdot \left(\frac{f_{ck}}{MPa}\right)^{0.667} \cdot MPa$

$f_{ctm} = 2.9\,MPa$

elastic modulus $E_{cm} := 22000 \cdot MPa \cdot \left(\frac{f_{ck} + 8 \cdot MPa}{10 \cdot MPa}\right)^{0.3}$

$E_{cm} = 3.284 \times 10^4\,MPa$

creep factor $\phi := 2$ $\chi := 0.8$ ageing coefficient

Exposure class XC1

Elastic modulus of the strands $E_p := 195000 \cdot MPa$

distancies from the bottom side

Bottom strands in two layers 12+12 ϕ_{p7} 12,5 $c_1 := 40 \cdot mm$ $c_2 := 90 \cdot mm$

Area of the bottom strands $A_p := 24 \cdot 93 \cdot mm^2$ $A_p = 2232\,mm^2$

centroid of the bottom strands from the bottom side $c_s := 65 \cdot mm$

Prestress after the long-term losses
= initial prestress - losses due to shrinkage, creep and relaxation $\sigma_{p\infty1} := 1287 \cdot MPa$

Top strands 2 ϕ_{p7} 12,5 $c_3 := 40 \cdot mm$ distance from the top side

Area of the top strands $A_{py} := 2 \cdot 93 \cdot mm^2$ $A_{py} = 186\,mm^2$

Prestress after the long-term losses $\sigma_{p\infty2} := 1072 \cdot MPa$

Prestress force after the long-term losses at the final stage

$P_\infty := A_p \cdot \sigma_{p\infty1} + A_{py} \cdot \sigma_{p\infty2}$ $P_\infty = 3.072 \times 10^3\,kN$

Distance of the resultant of the prestress force from the bottom side

$$c_p := \frac{\left[A_p \cdot \sigma_{p\infty1} \cdot c_s + A_{py} \cdot \sigma_{p\infty2} \cdot \left(h - c_3\right)\right]}{\left(A_p \cdot \sigma_{p\infty1} + A_{py} \cdot \sigma_{p\infty2}\right)}$$ $c_p = 0.139\,m$

Eccentricity of the prestress force $e_p := p_p - c_p$ $e_p = 404.682\,mm$

Cracking moment

Cracking limit state when the bending moment Mcr produce the tensile stress at the bottom fibre which is exactly the average tensile strength of concrete

$$\sigma_{ct} := \frac{N}{A} + \frac{M_{cr}}{W} \qquad \sigma_{ct} := f_{ctm}$$

$$\frac{N}{A \cdot f_{ctm}} + \frac{M_{cr}}{W \cdot f_{ctm}} \le 1 \qquad => \qquad \frac{N}{N_{cr0}} + \frac{M_r}{M_{cr0}} \le 1$$

where $N_{cr0} := A \cdot f_{ctm}$ is the cracking ressitance for tensile force, when M = 0

$M_{cr0} := W \cdot f_{ctm}$ is the cracking moment when N = 0

By solving $M_{cr} := \left(f_{ctm} - \frac{N}{A}\right) \cdot W \qquad M_{cr} := \left(1 - N \cdot \frac{W}{A}\right) \cdot W \cdot f_{ctm}$

In the case of an eccentric normal force $\sigma_{ca} := \frac{N}{A} + \frac{N \cdot e}{W} + \frac{M_{cr}}{W}$

$$M_{cr} := f_{ctm} \cdot W - N \cdot \left(\frac{W}{A} + e\right)$$

$N := -P_{\infty} \qquad M := M_{quasi}$

$$M_{cr} := f_{ctm} \cdot W_a + P_{\infty} \cdot \left(\frac{W_a}{A_m} + e_p\right) \qquad M_{cr} = 2515.6\,\text{kNm}$$

Quasi-permanent load combination

$M_{quasi} = 2342.8\,\text{kNm} \quad < \quad M_{cr} = 2515.6\,\text{kNm}$

Concrete stress at the bottom fibre $\sigma_{ca} := \frac{-P_{\infty}}{A_m} + \frac{-P_{\infty} \cdot e_p}{W_a} + \frac{M_{quasi}}{W_a} \qquad \sigma_{ca} = 1.816\,\text{MPa}$

$< \quad f_{ctm} = 2.9\,\text{MPa}$

Cross-section does not crack under the quasi-permanent laod combination

Frequent load combination

$M_{freq} = 2894.1\,\text{kNm} \quad > \quad M_{cr} = 2515.6\,\text{kNm}$

Cracking at the bottom side under the frequent load combination =>
the characteristic crack width should be calculate

We calculate first the concrete stresses and the location of the neutral axis for the uncracked cross-section
(So we get the initial value for the location of the neutral axis. The real depth of the compression zone in the cracked cross-section cannot be greater than the one in the uncracked cross-section)

Concrete stress at the bottom fibre $\sigma_{ca} := \frac{-P_\infty}{A_m} + \frac{-P_\infty \cdot e_p}{W_a} + \frac{M_{freq}}{W_a}$ $\sigma_{ca} = 5.274\,\text{MPa}$

$> \; f_{ctm} = 2.9\,\text{MPa}$

Concrete stress at teh top fibre $\sigma_{cy} := \frac{-P_\infty}{A_m} - \frac{-P_\infty \cdot e_p}{W_y} - \frac{M_{freq}}{W_y}$ $\sigma_{cy} = -18.57\,\text{MPa}$

Location of the neutral axis from the top side= depth of the compression zone $x := \frac{-\sigma_{cy}}{-\sigma_{cy} + \sigma_{ca}} \cdot h$ $x = 973.511\,\text{mm}$

Due to cracking the neutral axis moves upward when there is no creep.

Estimate the depth of the comression zone x, assuming the cross-section as a rectangle and without the top strands

Effective dept of the bottom strands $d_s := h - c_s$ $d_s = 1185\,\text{mm}$

"Effective depth" of the prestress force =distance of the prestress force resultant from the too side $d_p := h - c_p$ $d_p = 1111\,\text{mm}$

First calculation cycle is done **without creep**

Ratio of the elastic modulies $\alpha_e := \frac{E_p}{E_{cm}}$ $n_e := \alpha_e$ $\alpha_e = 5.939$

Relative amount of the tensile reinforcement $\rho := \frac{A_p}{b \cdot d_s}$ $\rho = 0.00471$

$\alpha_e \cdot \rho = 0.028$

Relative amount of the reinforcement in the compression zone $\rho_1 := \frac{A_{py}}{b \cdot d_s}$ $\rho_1 = 3.924 \times 10^{-4}$

$(\alpha_e - 1) \cdot \rho_1 = 1.938 \times 10^{-3}$

$k_p := \frac{M_{freq} - P_\infty \cdot d_p}{P_\infty \cdot d_s}$ $k_p = -0.142$

$k_2 := 6 \cdot \alpha_e \cdot \rho \cdot (1 + k_p)$ $k_2 = 0.144$

By solving the relative depth of the comression zone ξ=x/ds form the equation of 3.-degree (BY 16 appendix (Liite) S15)

$$\xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2 = 0$$

$$f(\xi) := \xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2$$

Initial value of ξ:

Given

$\xi := \dfrac{x}{d_s}$ $\qquad \xi = 0.822$

$\xi_1 := \mathrm{root}(f(\xi), \xi)$ $\qquad \xi_1 = 0.593$

Depth of the compression zone $\quad x := \xi_1 \cdot d_s \qquad x = 703.051\,\mathrm{mm}$

The neutral axis moved about 6 mm upward

Concrete stress at the top fibre

$$\sigma_{cy} := \frac{-P_\infty}{b \cdot d_s} \cdot \frac{2 \cdot \xi_1}{\xi_1^{\,2} - 2 \cdot \alpha_e \cdot \rho \cdot (1 - \xi_1) + 2 \cdot (\alpha_e - 1) \cdot \rho_1 \cdot \left(1 - \dfrac{c_3}{d_s}\right)} \qquad \sigma_{cy} = -23.094\,\mathrm{MPa}$$

The concrete stress is greater in the cracked cross-section than in the uncracked corss-section

Change of steel stress $\quad \Delta\sigma_s := \dfrac{-(1 - \xi_1)}{\xi_1} \cdot \alpha_e \cdot \sigma_{cy} \qquad \Delta\sigma_s = 94.014\,\mathrm{MPa}$

Change of the steel strain $\quad \Delta\varepsilon_s := \dfrac{\Delta\sigma_s}{E_p} \qquad \Delta\varepsilon_s = 0.482\,‰$

Compressive strain of concrete $\quad \varepsilon_c := \dfrac{\sigma_{cy}}{E_{cm}} \qquad \varepsilon_c = -0.703\,‰$

Check: $\quad x := \dfrac{-\varepsilon_c}{-\varepsilon_c + \Delta\varepsilon_s} \cdot d_s \qquad x = 703.051\,\mathrm{mm}$

Compressive strain of concrete at the location of the top strands $\quad \Delta\varepsilon_{sy} := \dfrac{x - c_3}{x} \cdot \varepsilon_c \qquad \Delta\varepsilon_{sy} = -0.663\,‰$

Beacuse the top strands do not taken into account in the calculation (in the 3.-degree equation) so the equilibrium is checked

Concrete stress resultant $\quad N_c := \dfrac{b \cdot x \cdot \sigma_{cy}}{2} \qquad N_c = -3.247 \times 10^3\,\mathrm{kN}$

Change of the force in the top strands $\Delta N_{py} := A_{py} \cdot \Delta\varepsilon_{sy} \cdot (E_p - E_{cm})$ $\Delta N_{py} = -20.006\,kN$

Change of the force in the bottom strands $\Delta N_p := A_p \cdot \Delta\varepsilon_s \cdot E_p$ $\Delta N_p = 209.839\,kN$

$\Sigma N := N_c + \Delta N_{py} + \Delta N_p$ $\Sigma N = -3057.4\,kN$ ~ $P_\infty = 3072\,kN$

Bending moment

$$N_c \cdot \left(k_p \cdot d_s + \frac{x}{3}\right) + \Delta N_{py} \cdot \left(k_p \cdot d_s + c_3\right) + \Delta N_p \cdot \left(d_s + k_p \cdot d_s\right) = 2.335\,kNm$$

A small inaccuracy of the equilibrium is the consequence of the fact that the top strands are not taken into account in the 3.-degree equation

2. calculation cycle: Taking account creep and making new calculation:

Effective elastic modulus of concrete with effect of creep $E_{ceff} := \dfrac{E_{cm}}{1 + \dfrac{M_{quasi}}{M_{freq}} \cdot \chi \cdot \phi}$ $E_{ceff} = 14306\,MPa$

Ratio of the elastic modulies $\alpha_e := \dfrac{E_p}{E_{ceff}}$ $\alpha_e = 13.63$

$\alpha_e \cdot \rho = 0.064$ $(\alpha_e - 1) \cdot \rho_1 = 4.956 \times 10^{-3}$

$k_p := \dfrac{M_{freq} - P_\infty \cdot d_p}{P_\infty \cdot d_s}$ $k_p = -0.142$

$k_2 := 6 \cdot \alpha_e \cdot \rho \cdot (1 + k_p)$ $k_2 = 0.33$

$f(\xi) := \xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2$

Initial value of ξ:

Given

$\xi := \dfrac{x}{d_s}$ $\xi = 0.593$

$\xi_1 := root(f(\xi), \xi)$ $\xi_1 = 0.67$

Depth of the compression zone $x := \xi_1 \cdot d_s$ $x = 793.763\,mm$ $x_1 := x$

Depth of the compression zone increases due to creep

Conrete compression stress at the top fibre

$$\sigma_{cy} := \frac{-P_\infty}{b \cdot d_s} \cdot \frac{2 \cdot \xi_1}{\xi_1^2 - 2 \cdot \alpha_e \cdot \rho \cdot (1 - \xi_1) + 2 \cdot (\alpha_e - 1) \cdot \rho_1 \cdot \left(1 - \dfrac{c_3}{d_s}\right)}$$ $\sigma_{cy} = -20.877\,MPa$

Concrete compression stress decreases du eto creep

Change of the stress in the bottom strands $\Delta\sigma_s := \dfrac{-(1 - \xi_1)}{\xi_1} \cdot \alpha_e \cdot \sigma_{cy}$ $\Delta\sigma_s = 140.256\,MPa$

Change of the strain in bottom strands $\Delta\varepsilon_s := \dfrac{\Delta\sigma_s}{E_p}$ $\Delta\varepsilon_s = 0.719\,‰$

Tension steel stress increases due to creep

Compressive strain of concrete $\varepsilon_c := \dfrac{\sigma_{cy}}{E_{ceff}}$ $\varepsilon_c = -1.459\,‰$

Check: $x := \dfrac{-\varepsilon_c}{-\varepsilon_c + \Delta\varepsilon_s} \cdot d_s$ $x = 793.763\,\text{mm}$

Compression strain of concrete at the point of the top strands $\Delta\varepsilon_{sy} := \dfrac{x - c_3}{x} \cdot \varepsilon_c$ $\Delta\varepsilon_{sy} = -1.386\,‰$

Because the top strands do not taken into account in the computing the location of the neutral axis equilibrium is checked.

Compression resultant of concrete $N_c := \dfrac{b \cdot x \cdot \sigma_{cy}}{2}$ $N_c = -3314\,\text{kN}$

Change of the force in the top strands $\Delta N_{py} := A_{py} \cdot \Delta\varepsilon_{sy} \cdot \left(E_p - E_{ceff}\right)$ $\Delta N_{py} = -46.573\,\text{kN}$

Change of the force in the bottom strands $\Delta N_p := A_p \cdot \Delta\varepsilon_s \cdot E_p$ $\Delta N_p = 313.052\,\text{kN}$

$\Sigma N := N_c + \Delta N_{py} + \Delta N_p$ $\Sigma N = -3047.8\,\text{kN}$ > $P_\infty = 3072\,\text{kN}$

error ~0,8 %

Moment about the compression line (kp):

$$N_c \cdot \left(k_p \cdot d_s + \frac{x}{3}\right) + \Delta N_{py} \cdot \left(k_p \cdot d_s + c_3\right) + \Delta N_p \cdot \left(d_s + k_p \cdot d_s\right) = 6.036\,\text{kNm}$$

the error to the bending moment Mlyhyt under the frequent loading combination is about 0,2 %

Crack width

Strain in the lowest steel layer $\Delta\varepsilon_{s1} := \frac{h - x - c_1}{d_s - x} \cdot \Delta\varepsilon_s$ $\Delta\varepsilon_{s1} = 0.765\,‰$

Change in the stress in the lowest steel layer $\Delta\sigma_{s1} := \Delta\varepsilon_{s1} \cdot E_p$ $\Delta\sigma_{s1} = 149.219\,\text{MPa}$

Amount of the effective reinforcement

Ratio of the bond strength between bonded tendons and ribbed steel
(EC2 table 6.2):

Pre-tensioned strands => $\xi := 0.6$

Adjusted ratio of the bond strength between bonded tendons and ribbed steel taking into account the different diameters of the prestressing strands and ordinary reinforcement (EC2 equation 7.5)

$$\xi_1 := \sqrt{\zeta \cdot \frac{\phi_s}{\phi_p}}$$

ϕs is the largest diameter of ordinary reinforcement with high bond ribbed steel
ϕp is the largest dameter or equivalent diameter of prestressing steel
ϕp=1,6 (Ap)^0.5 for bundled bars
ϕp=1,75*ϕwire for single 7 wire strand where ϕwire is the wire diameter
ϕp=1,20*ϕwire for single 3 wire strand

Strands used are 7 wire strand.

Area of one 7-wire strand $A_{p1} := 93 \cdot \text{mm}^2$

Area of one wire $A_{wire} := \frac{A_{p1}}{7}$ $A_{wire} = 13.286\,\text{mm}^2$

Diameter of the wire $\phi_{wire} := \sqrt{\frac{4 \cdot A_{wire}}{\pi}}$ $\phi_{wire} = 4.113\,\text{mm}$

The largest nominal diameter of the strands $\phi_p := 1.75 \cdot \phi_{wire}$ $\phi_p = 7.198\,\text{mm}$

Tension reinforcement consists only the prestressing strands, no ordinary reinforcement =>

$$\phi_s := \phi_p$$

$$\xi_1 := \sqrt{\xi \cdot \frac{\phi_s}{\phi_p}} \qquad \xi_1 = 0.775$$

Depth of the effective tension zone $h_{ef} := 2 \cdot (h - d_s)$ $h_{ef} = 130\,\text{mm}$ EC2 fig. 7.1

Area of the effective tension zone $A_{ceff} := b_a \cdot h_{ef}$ $A_{ceff} = 91000\,\text{mm}^2$

Amount of an ordinary reinforcement $A_s := 0 \cdot \text{mm}^2$

Proportional amount of the reinforcement in the effective tension zone $\rho_{peff} := \frac{A_s + \xi_1^{2} \cdot A_p}{A_{ceff}}$

$\rho_{peff} = 0.015$

Concrete cover $c := c_1 - \frac{\phi_p}{2}$ $c = 36.401\,mm$

Factor for bond characteristic $k_1 := 1.6$ prestressing strand

Bended and compressed structure =>

strain in the less tensionsioned point of the effective tension zone $\varepsilon_2 := 0$

strain in the most tensioned point of the effective tension zone $\varepsilon_1 := \frac{h - x}{d_s - x} \cdot \Delta\varepsilon_s$ $\varepsilon_1 = 0.839\,‰$

$k_2 := \frac{\varepsilon_1 + \varepsilon_2}{2 \cdot \varepsilon_1}$ $k_2 = 0.5$

$k_3 := 3.4$ $k_4 := 0.425$

Crack space $s_{rmax} := k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi_p}{\rho_{peff}}$ $s_{rmax} = 290\,mm$

Average steel strain $\varepsilon_{sm} - \varepsilon_{cm}$

Average steel strain εsm in he cracked cross-section at the point of the crack
Average concrete strain εcm between the cracks

The diffrence $\varepsilon_m := \varepsilon_{sm} - \varepsilon_{cm}$ can be obtained from the equation EC2 7.9

Factor dependent on the duration of the load $k_t := 0.6$ for short tem loading

For pre-tensioned members with bonded strands σs in the equation (7.9) may be replaced by Δσs the stress variation in the pre-tensioned strands from the state of zero strain of the concrete (decompression) at the level of the strand

As an effective tensile strength of concrete can be used the average tensile strength

$f_{cteff} := f_{ctm}$

EC2 equation (7.9)

$$\varepsilon_m := \max\left[\frac{\Delta\sigma_{s1} - k_1 \cdot \frac{f_{cteff}}{\rho_{peff}} \cdot \left(1 + \alpha_e \cdot \rho_{peff}\right)}{E_p}, 0.6 \cdot \frac{\Delta\sigma_{s1}}{E_p}\right]$$

$\varepsilon_m = 0.459\,‰$

Characteristic crack width $w_k := \varepsilon_m \cdot s_{rmax}$ $w_k = 0.13\,mm$

The characteristic crack width fulfill the requirement of exposure class XC1 $w_{ksall} := 0.2 \cdot mm$

There is no requirements in EC2 for a quasi-permanent load combination is the exposure classes XC1.
In the exposure classes XC2, XC3, XC4 the cross-section should be compressed under the quasi-permanent load combination.

In the of this example under the quasi-permanent load combination there is tension which is smaller than the tensile strength of concrete.

The cross-section is cracked under the frequent load combination. Then concrete is damaged so that at the point of cracks concrte has no tension resistance. If the load is removed cracks will close but will open again when there exist any tension stresses. The cracks wil open again with smaller bending moment than the cracking moment calculated before.

If the cross-section cracks under the characteristic or the frequent load combination the cross-section should be treated forward as a cracked cross-section also under smaller loads if this load produce any tension stress. The cross-section which is craked before can be treated as an uncracked section only if it is fully compressed.

So the crack width should be calculated also under the quasi-permanent load combination

The cracking moment (= moment of decompression) when a cross-section has cracked before under a greater load combination

$$M_{cr1} := P_\infty \cdot \left(\frac{W_a}{A_m} + e_p \right) \qquad M_{cr1} = 2053.4\,kNm \quad < \quad M_{quasi} = 2342.8\,kNm$$

Quasi-permanent load combination

When a short-term load is removed and there is only quasi-permanet loads left and this load produce tension in the uncracked section (bending moment due to quasi-permanent load is greater than the decompression moment), so cracks do not close and the cross-section must be treatet as a cracked cross-section => crack width under the quasi-permanent load combination should be checked.

$$M_{quasi} = 2342.8\,kNm \quad < \quad M_{cr} = 2515.6\,kNm$$

Stress at the bottom fibre

$$\sigma_{ca} := \frac{-P_\infty}{A_m} + \frac{-P_\infty \cdot e_p}{W_a} + \frac{M_{quasi}}{W_a} \qquad \sigma_{ca} = 1.816\,MPa$$

tension

Stress at the top fibre

$$\sigma_{cy} := \frac{-P_\infty}{A_m} - \frac{-P_\infty \cdot e_p}{W_y} - \frac{M_{quasi}}{W_y} \qquad \sigma_{cy} = -14.067\,MPa$$

Depth of the compression zone initial value

$$x := \frac{-\sigma_{cy}}{-\sigma_{cy} + \sigma_{ca}} \cdot h \qquad x = 1107.1\,mm$$

Effective elastic modulus of concrete $E_{ceff} := \dfrac{E_{cm}}{1 + \dfrac{M_{quasi}}{M_{quasi}} \cdot \chi \cdot \phi}$ $E_{ceff} = 12629.4\,MPa$

Ratio of the elastic modulies $\alpha_e := \dfrac{E_p}{E_{ceff}}$ $\alpha_e = 15.44$

Relative amount of the tension reinforcement $\rho := \dfrac{A_p}{b \cdot d_s}$ $\rho = 0.00471$

$\alpha_e \cdot \rho = 0.073$

$$k_p := \frac{M_{quasi} - P_\infty \cdot d_p}{P_\infty \cdot d_s} \qquad k_p = -0.294$$

$$k_2 := 6 \cdot \alpha_e \cdot \rho \cdot (1 + k_p) \qquad k_2 = 0.308$$

$$f(\xi) := \xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2$$

Initial value of ξ:

Given

$\xi := \dfrac{x}{d_s}$ $\xi := 0.934$

$\xi_1 := \text{root}(f(\xi), \xi)$ $\xi_1 = 0.914$

Dept of the comression zone $x := \xi_1 \cdot d_s$ $x = 1082.9\,mm$

Concrete compression stress at the top fibre

$$\sigma_{cy} := \frac{-P_\infty}{b \cdot d_s} \cdot \frac{2 \cdot \xi_1}{\xi_1^{\,2} - 2 \cdot \alpha_e \cdot \rho \cdot (1 - \xi_1) + 2 \cdot (\alpha_e - 1) \cdot \rho_1 \cdot \left(1 - \dfrac{c_3}{d_s}\right)} \qquad \sigma_{cy} = -14.21\,MPa$$

Change of the steel stress $\Delta\sigma_s := \dfrac{-(1 - \xi_1)}{\xi_1} \cdot \alpha_e \cdot \sigma_{cy}$ $\Delta\sigma_s = 20.681\,MPa$

Change of the steel strain $\Delta\varepsilon_s := \dfrac{\Delta\sigma_s}{E_p}$ $\Delta\varepsilon_s = 0.106\,‰$

Concrete compression strain $\varepsilon_c := \dfrac{\sigma_{cy}}{E_{ceff}}$ $\varepsilon_c = -1.125\,‰$

Check: $x := \frac{-\varepsilon_c}{-\varepsilon_c + \Delta\varepsilon_s} \cdot d_s$ $x = 1082.9\,mm$

Concrete strain at point of the top strands $\Delta\varepsilon_{sy} := \frac{x - c_3}{x} \cdot \varepsilon_c$ $\Delta\varepsilon_{sy} = -1.084\,‰$

Concrete stress resultant $N_c := \frac{b \cdot x \cdot \sigma_{cy}}{2}$ $N_c = -3.078 \times 10^3\,kN$

Change of the force of the top strands $\Delta N_{py} := A_{py} \cdot \Delta\varepsilon_{sy} \cdot (E_p - E_{ceff})$ $\Delta N_{py} = -36.758\,kN$

Change of the force of the botton strands $\Delta N_p := A_p \cdot \Delta\varepsilon_s \cdot E_p$ $\Delta N_p = 46.16\,kN$

Equilibrium:

$\Sigma N := N_c + \Delta N_{py} + \Delta N_p$ $\Sigma N = -3068.4\,kN$ > $P_\infty = 3072\,kN$

Moment about the compression line (kp) :

$$N_c \cdot \left(k_p \cdot d_s + \frac{x}{3}\right) + \Delta N_{py} \cdot (k_p \cdot d_s + c_3) + \Delta N_p \cdot (d_s + k_p \cdot d_s) = 10.148\,kNm$$

error about 0,4 %

Crack width under the quasi-permanent load combination

Change of strain in the lowest steel layer $\Delta\varepsilon_{s1} := \frac{h - x - c_1}{d_s - x} \cdot \Delta\varepsilon_s$ $\Delta\varepsilon_{s1} = 0.132\,‰$

Change of steel stress in the loewst layer $\Delta\sigma_{s1} := \Delta\varepsilon_{s1} \cdot E_p$ $\Delta\sigma_{s1} = 25.746\,MPa$

$$\varepsilon_m := \max\left[\frac{\Delta\sigma_{s1} - k_1 \cdot \frac{f_{cteff}}{\rho_{peff}} \cdot (1 + \alpha_e \cdot \rho_{peff})}{E_p}, 0.6 \cdot \frac{\Delta\sigma_{s1}}{E_p}\right] \qquad \varepsilon_m = 0.079\,‰$$

Characteristic crack width $w_k := \varepsilon_m \cdot s_{rmax}$ $w_k = 0.02\,mm$

The crack width fulfill the requirement for the exposure class XC1 $w_{ksall} := 0.2 \cdot mm$

Deflection

Deflection under the frequent load combination is checked

Bending moment under the frequent load combination $M_{freq} = 2894.1\,kNm$

Influence of creep on the deflection is taken into account by using the effective elastic modulus of concrete

Effective elastic modulus of concrete $E_{ceff} := \dfrac{E_{cm}}{1 + \dfrac{M_{quasi}}{M_{freq}} \cdot \chi \cdot \phi}$ $E_{ceff} = 14306\,MPa$

Ratio of the elastic modulies $\alpha_e := \dfrac{E_p}{E_{ceff}}$ $\alpha_e = 13.63$

Under the frequent load combination the cross-sections which are near the maximum bending moment where the bending moment is greater than the cracking moment are cracked but the cross-section which are near the supports where the bending moment is smaller than the cracking moment remain still uncracked.

So the flexural rigidity of the cross-section varies along the span and the deflection must be calculated by interpolating between the deflections calculated with the rigidities of an uncracked and a cracked cross-section with respect to the cracking moment.

Cracked zone of the span is calculated under the characteristic load combination because of the explanation above (it is possible that the beam is loaded by the characteristic load combination and cracks exist at a certain zone of the span and afterwards this zone remain cracked also under the smaller load combinations (frequent and quasi-permanent load combinations)

<u>Uncracked cross-section</u>

$$(\alpha_e - 1) \cdot \rho = 0.059 \qquad (\alpha_e - 1) \cdot \rho_1 = 4.956 \times 10^{-3}$$

Cross-section values of the transformed cross-sectio with the elastic modulus Eceff:

centroid from the bottom side $p_p := 533 \cdot mm$

Cross-section area $A_m := 620540 \cdot mm^2$

Second moment of area $I_m := 90.482 \cdot 10^9 \cdot mm^4$

Flexural rigidity of the uncracked cross-section $K_c := E_{ceff} \cdot I_m$ $K_c = 1.294 \times 10^3\,MN \cdot m^2$

Deflection due to the prestressing P_∞

$P_\infty = 3.072 \times 10^3\,kN$ eccentricity $e_p := p_p - c_p$ $e_p = 0.394\,m$

$a_{pI} := \frac{1}{8} \cdot \frac{\left(-P_{\infty} \cdot e_p\right) \cdot L^2}{K_c}$ $a_{pI} = -51.501\,\text{mm}$

Deflection due to imposed load $a_{qI} := \frac{5}{48} \cdot \frac{M_{freq} \cdot L^2}{K_c}$ $a_{qI} = 102.703\,\text{mm}$

Total delflection $a_I := a_{pI} + a_{qI}$ $a_I = 51.202\,\text{mm}$

Cracked cross-section

Seth of the compression zone is calculated before $x_1 = 793.763\,mm$

In the cracked cross-section the effective cross-section consists only the compression zone at the height x1 and bottom and top strands.
The cross-section values are calculated

Area of the cracked cross-section $A_{cr} := b \cdot x_1 + (\alpha_e - 1) \cdot A_{py} + \alpha_e \cdot A_p$ $A_{cr} = 3.503 \times 10^5\,mm^2$

Centroid from **the top side**

$$p_{cr} := \frac{b \cdot \frac{x_1^2}{2} + (\alpha_e - 1) \cdot A_{py} \cdot c_3 + \alpha_e \cdot A_p \cdot d_s}{A_{cr}} \qquad p_{cr} = 462.939\,mm$$

Second moment of area

$$I_{cr} := \frac{b \cdot x_1^3}{12} + b \cdot x_1 \cdot \left(p_{cr} - \frac{x_1}{2}\right)^2 + (\alpha_e - 1) \cdot A_{py} \cdot (p_{cr} - c_3)^2 + \alpha_e \cdot A_p \cdot (p_{cr} - d_s)^2$$

$$I_{cr} = 0.034\,m^4$$

Flexural rigidity of the cracked cross-section $K_r := E_{ceff} \cdot I_{cr}$ $K_r = 491.251\,MN \cdot m^2$

Deflection due to prestress

The eccentricity of the prerstress force is different in the cracked cross-section than in the uncracked cross-section

$$e_{pcr} := h - c_p - p_{cr} \qquad e_{pcr} = 647.743\,mm$$

$$a_{pII} := \frac{1}{8} \cdot \frac{(-P_{\infty} \cdot e_{pcr}) \cdot L^2}{K_r} \qquad a_{pII} = -223.288\,mm$$

Deflection due to the imposed load

$$a_{qII} := \frac{5}{48} \cdot \frac{M_{freq} \cdot L^2}{K_r} \qquad a_{qII} = 270.628\,mm$$

For a cracked cross-section without a normal force the flexural rigidity obtained for the equation

$$K_{r1} := A_p \cdot E_p \cdot \left(d_s - \frac{x_1}{3}\right) \cdot (d_s - x_1) \qquad K_{r1} = 156.73\,MN \cdot m^2$$

is valid. But this equation is not valid for a prestressed cross-section because the neutral axis is not located at the centroid of the cross-section due to a normal force or a prestress force. In the above equation is taken into account the fact that the neutral axis lies at the level of tyhe centroid if there is pure bending no normal force.

Total deflection for fully cracked beam $a_{II} := a_{pII} + a_{qII}$ $a_{II} = 47.339\,\text{mm}$

Steel stress σsr for the cracking moment used in the interpolation equation of the deflection is in this case the change of steel stress due to the cracking moment $M_{cr} = 2515.6\,\text{kNm}$

The change of the steel stress and the depth of the compression zone are calculated when the bending moment is **the cracking moment**

$$k_p := \frac{M_{cr} - P_\infty \cdot d_p}{P_\infty \cdot d_s} \qquad k_p = -0.246$$

$$k_2 := 6 \cdot \alpha_e \cdot \rho \cdot \left(1 + k_p\right) \qquad k_2 = 0.29$$

$$f(\xi) := \xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2$$

The initial value of ξ:

Given

$$\xi := \frac{x}{d_s} \qquad \xi := 0.934$$

$$\xi_1 := \text{root}\left(f(\xi), \xi\right) \qquad \xi_1 = 0.818$$

Depth of the compression zone $x_{cr} := \xi_1 \cdot d_s$ $x_{cr} = 0.969\,\text{m}$

Concrete compression stress at the top fibre

$$\sigma_{cy} := \frac{-P_\infty}{b \cdot d_s} \cdot \frac{2 \cdot \xi_1}{\xi_1^{\,2} - 2 \cdot \alpha_e \cdot \rho \cdot \left(1 - \xi_1\right) + 2 \cdot \left(\alpha_e - 1\right) \cdot \rho_1 \cdot \left(1 - \frac{c_3}{d_s}\right)} \qquad \sigma_{cy} = -16.182\,\text{MPa}$$

Change of the steel stress $\Delta\sigma_{sr} := \dfrac{-\left(1 - \xi_1\right)}{\xi_1} \cdot \alpha_e \cdot \sigma_{cy}$ $\Delta\sigma_{sr} = 49.111\,\text{MPa}$

In the interpolation equation the steel stress σs is the change of the steel stress under **the characteristic load combination**

Bending moment due to the characteristic load combination

$$M_k := \frac{\left(g_k + q_k\right) \cdot L^2}{8} \qquad M_k = 3307.5\,\text{kNm}$$

Steel stress due to this moment

$$k_p := \frac{M_k - P_\infty \cdot d_p}{P_\infty \cdot d_s} \qquad k_p = -0.029$$

$$k_2 := 6 \cdot \alpha_e \cdot \rho \cdot (1 + k_p) \qquad k_2 = 0.374$$

$$f(\xi) := \xi^3 + 3 \cdot k_p \cdot \xi^2 + k_2 \cdot \xi - k_2$$

The initial value of ξ:

Given

$$\xi := \frac{x}{d_s} \qquad \xi := 0.934$$

$$\xi_1 := \mathrm{root}(f(\xi), \xi) \qquad \xi_1 = 0.573$$

Depth of the compression zone $x_{cr} := \xi_1 \cdot d_s$ $x_{cr} = 0.679\,\mathrm{m}$

Concrete compression stress at the top finbre under the characteristic load combination

$$\sigma_{cy} := \frac{-P_\infty}{b \cdot d_s} \cdot \frac{2 \cdot \xi_1}{\xi_1^{\,2} - 2 \cdot \alpha_e \cdot \rho \cdot (1 - \xi_1) + 2 \cdot (\alpha_e - 1) \cdot \rho_1 \cdot \left(1 - \frac{c_3}{d_s}\right)} \qquad \sigma_{cy} = -26.228\,\mathrm{MPa}$$

Change of the steel stess under the characteristic load combination

$$\Delta\sigma_s := \frac{-(1 - \xi_1)}{\xi_1} \cdot \alpha_e \cdot \sigma_{cy} \qquad \Delta\sigma_s = 266.282\,\mathrm{MPa}$$

Remark! $\frac{\Delta\sigma_{sr}}{\Delta\sigma_s} = 0.184$ < $\frac{M_{cr}}{M_k} = 0.761$ for a prestressed structure

For **prestressed structure**s it is **not** allowed to use $\frac{M_{cr}}{M_k}$ instead of $\frac{\Delta\sigma_{sr}}{\Delta\sigma_s}$

For normally reinforced sructures without normal force $\frac{M_{cr}}{M_k}$ can be used instead of $\frac{\Delta\sigma_{sr}}{\Delta\sigma_s}$

$\beta := 0.5$ for long-term load combination (the load combination considered includes long-term load)

Interpolation factor $\xi_a := 1 - \beta \cdot \left(\frac{\Delta\sigma_{sr}}{\Delta\sigma_s}\right)^2$ $\xi_a = 0.983$

Deflection $a := \xi_a \cdot a_{II} + (1 - \xi_a) \cdot a_I$ $a = 47.405\,\mathrm{mm}$

Spalling reinforcement of the end block (anchor block) of post-tensioned members

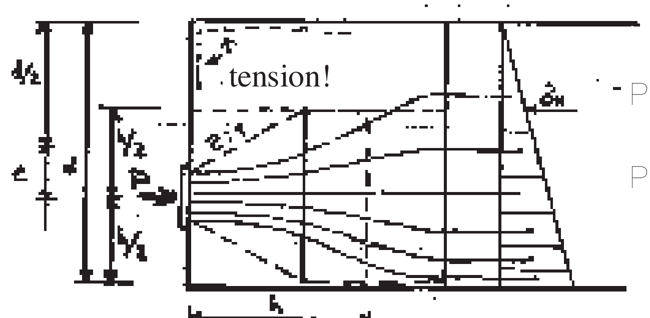
The prestressing force affect to the member by an anchor. The size of the anchor is small compared to the dimensions of the member; so the prestressing force is a concentrated load which spread at a certain distance over the whole cross-section. The prestress force is assumed to spread at the inclination of 1:2. The spreading causes transversal tensile stresses which are taken by spalling reinforcement.

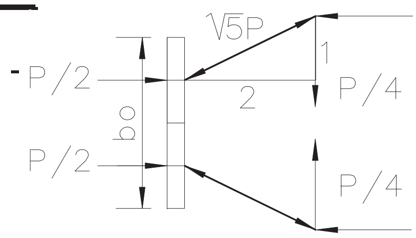
Nearby the anchor about at the distance which corresponds the dept of the member is a local disturbace region, so called D-region where the stress distibution is not linear (so the Bernoulli's principle is not valid). The stress distribution in this region can be calculated by plate theory. This phenomena is called St. Venant's principle.

Forces in te disturbance region can be examined with so call the strut-and tie.method (a modified truss theory). Compressed strut members and tension tie members can be made according to elastic stress distribution (stress trajectories). Compression is taken to concrete by the compressed struts and reinforcement acts as tensile ties. The disturbance region is assumed to be separated from the rest of the structure (B-region; Bernoulli region).
Then at the separation line there is a linear stress distribution. At the other end of the D-region there is the concentrated laod which cause the disrturbation. The D-region can be treated as a deep beam which is supported by the concetrated loads (prestressing forces) and loaded by the linear stress distribution at the other end of the region.

The concentrated load (point load, prestressing force) is divided by two parts P/2.
The force P/2 is assumed spread to one direction at the inclination 1:2. From the resultant of P/2 is drawn the force vector in the inclination 1:2.
The stress distribution at the other end of the D-region is also divided into two parts which resultants are P/2. From the point of the resultant of P/2 is drawn the force vector parallel to the stresses to the intersection of the inclined force vector of P/2. In this intersection point (so called node point) the components of force resultants parallel to the stresses reverse each other but in the perpendicular direction the equilibrium requires tension force
(spalling force) which may cause cracking in concrete. The intersection points of two halves is connected by tensile tie member. The tensile force of this tie corresponds the resultant of the spalling stresses caused by the concentrated load. The value of this tensile force is 0.5*P/2.
The required spalling reinforcement is calculated for this force.
The part of the stress distribution which is at the area of the anchor is assumed to go straight from the anchor without any spreading and it does not cause any spalling stresses. This is taken into account by the reduction factor bo/b, where bo is the dimension of the anchor and b is the dimension of the distrubution afre of the stresses.

According the above the spalling force due to concentrated load can be obtained form from the equation





$$F_t := 0.25 \cdot \gamma_{p.unfav} \cdot P \cdot \left(1 - \frac{b_0}{b}\right)$$

where b_0 is the side of the loading area (anchor) in the direction of dimension b of the structure

P is the prestress force at the anchor at the time of tensioning

$\gamma_{p.unfav}$ =1.15 partial safety factor for prestress force, when the prestress force is unfavourable

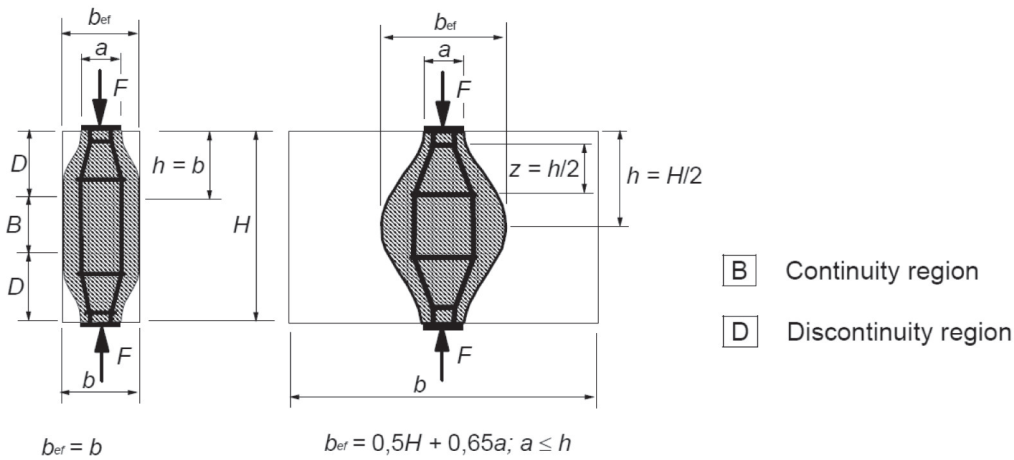
F_t is the spalling force in the direction of b

The part $\gamma_{p.unfav} \cdot P \cdot \left(\frac{b_0}{b}\right)$ of the total prestress force goes through and does not cause spalling stresses.

The side dimension of the distribution area is not more than 2*the smallest dimension a of the loading area

It is supposed that the resultant of the stress distribution is at the same level than the concentrate load. So in the case of an eccentric load the stresses are supposed to distributed equally for the distance 2*a.

According to EC2 (EN-1992-1-1)



a) Partial discontinuity b) Full discontinuity

Figure 6.25: Parameters for the determination of transverse tensile forces in a compression field with smeared reinforcement

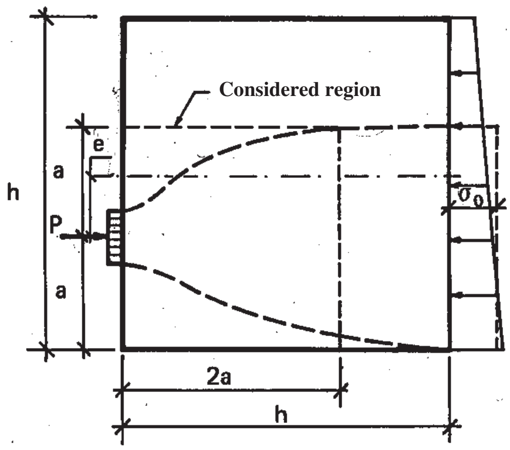
(3) Reinforcement required to resist the forces at the concentrated nodes may be smeared over a length (see Figure 6.25 a) and b)). When the reinforcement in the node area extends over a considerable length of an element, the reinforcement should be distributed over the length where the compression trajectories are curved (ties and struts). The tensile force T may be obtained by:

a) for partial discontinuity regions $\left(b \leq \frac{H}{2}\right)$, see Figure 6.25 a:

$$T = \frac{1}{4}\frac{b-a}{b}F \qquad (6.58)$$

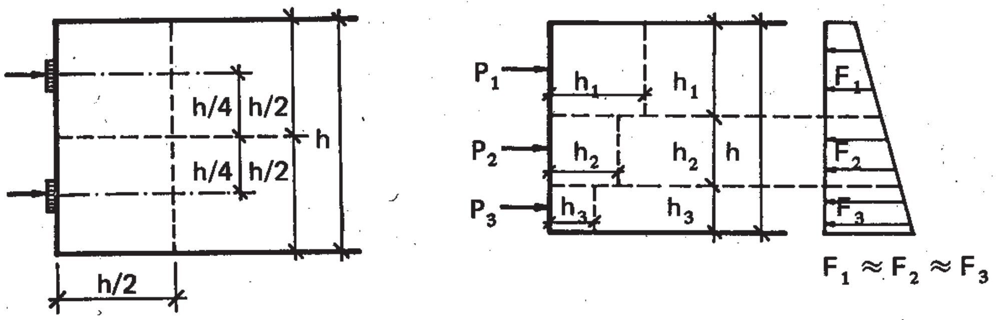
b) for full discontinuity regions $\left(b > \frac{H}{2}\right)$, see Figure 6.25 b:

$$T = \frac{1}{4}\left(1 - 0{,}7\frac{a}{h}\right)F \qquad (6.59)$$

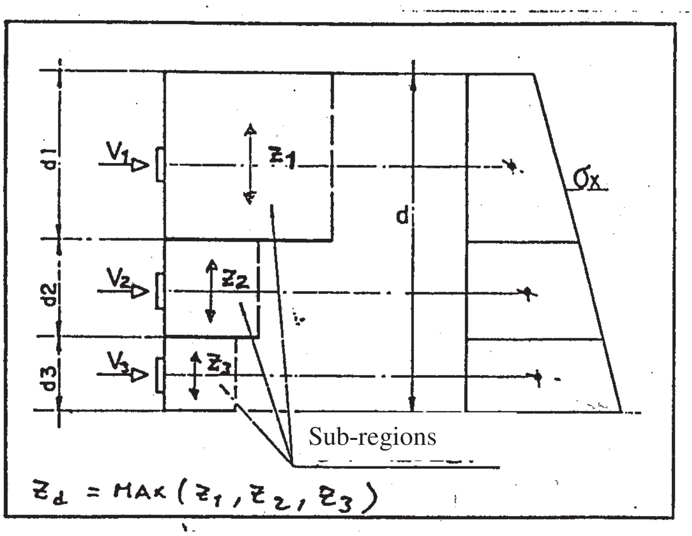


Eccentric point load at the end of a member, distribution width 2*a

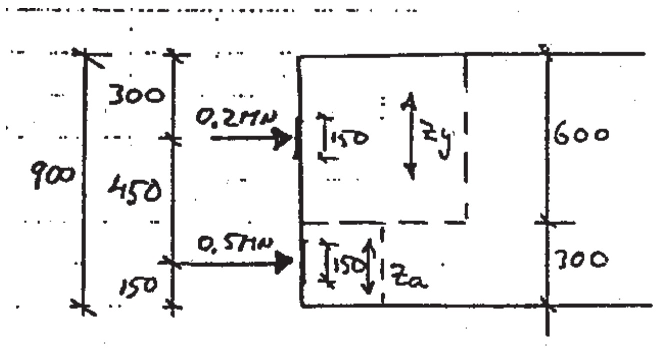
If a structure is loaded by several concentrated loads, the structure may be divided to the regions which centroid is at the same level than the load. If these regions do not cross each others the dimensiong spalling force is the greatest spalling force in these regions. The spalling reinforcement is distributed to the distance which represents the depth of the member.



Several point loads (anchors) at the end of a member



Example:



Two loads $P_y := 200 \cdot kN$ $a_1 := 300 \cdot mm$ from the top side and

$P_a := 500 \cdot kN$ $a_2 := 150 \cdot mm$ from the bottom side

Partial safety factor of prestress force (unfavourable) $\gamma_{p.unfav} := 1.2$

Dimension of the anchor $b_0 := 150 \cdot mm$

Depth of the cross-section $d := 900 \cdot mm$

Spalling force due to Py: Dimension of the distribution area $b_1 := 2 \cdot a_1$ $b_1 = 600\,mm$

$$Z_y := 0.25 \cdot \gamma_{p.unfav} P_y \cdot \left(1 - \frac{b_0}{b_1}\right) \qquad Z_y = 45\,kN$$

Spalling force due to Pa: Dimension of the distribution area $b_2 := 2 \cdot a_2$ $b_2 = 300\,mm$

$$Z_a := 0.25 \cdot \gamma_{p.unfav} \cdot P_a \cdot \left(1 - \frac{b_0}{b_2}\right) \qquad Z_a = 75\,kN$$

Dimensiong spalling force $Z_d := \max(Z_y, Z_a)$ $Z_d = 75\,kN$

Partial safety factor for reinforcement $\gamma_s := 1.1$

Spalling reinforcemnent A500 HW $f_{yd} := \frac{500 \cdot MPa}{\gamma_s}$ $f_{yd} = 454.545\,MPa$

Spalling reinforcement $A_{s.spall} := \frac{Z_d}{f_{yd}}$ $A_{s.spall} = 165\,mm^2$

Spalling reinforcement is distributed to the distance 600 mm

stirrups 3 T 6 k 200 $A_{s.spall} := 3 \cdot 2 \cdot 28.3 \cdot mm^2$ $A_{s.spall} = 170\,mm^2$

Splitting force due to eccentricity of the concentrated load at the very end of the structure

top tendons $e_1 := 150 \cdot mm$

bottom tendons $e_2 := 300 \cdot mm$

$$F_{t2Ed} := 0.015 \cdot \gamma_{p.unfav} \cdot \left(\frac{P_y}{1 - \sqrt{2 \cdot \frac{e_1}{d}}} + \frac{P_a}{1 - \sqrt{2 \cdot \frac{e_2}{d}}} \right) \qquad F_{t2Ed} = 57.563\,kN$$

$$A_{s.split} := \frac{F_{t2Ed}}{f_{yd}} \qquad A_{s.split} = 126.639\,mm^2$$

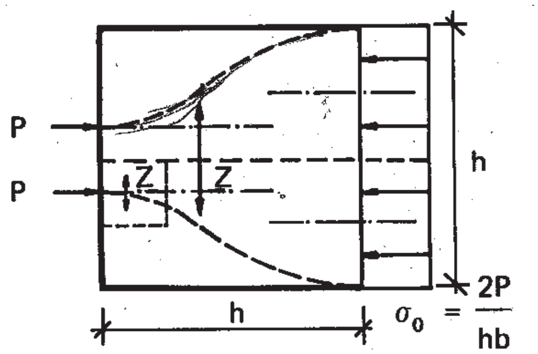
Splitting reinforcement can be included to the amount of the spalling reinforcement so that a part of the spalling reinforcement which represtents the amount of the located at the very end of the structure and the rest of the spalling reinforcement is distributed to the distance of h.

In this case 3 T 6 strirrups are located at the very end of the structure and then 2 T 6 k 200

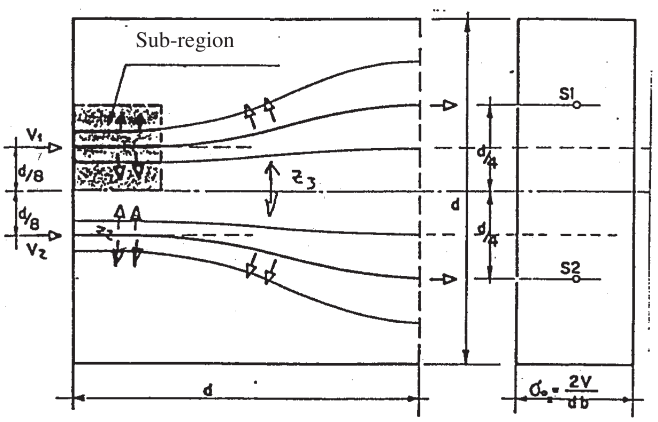
When concentrated loads are located so that the stress distribution cannot be divided to the regions which centroid coinside to the loads, so the whole depth cannot be used as the distribution area.

If the concentrated loads are located inside of the resultant if the distribution area, the local spalling force is calculated for each concentrated load and then another spalling force Z3 due to the interaction of the concentrated loads far from the end of the structure.

If the concentrated laods are located so that the distribution areas cross each other the whole depth can be use as a distiribution area.



Two point loads close to each others

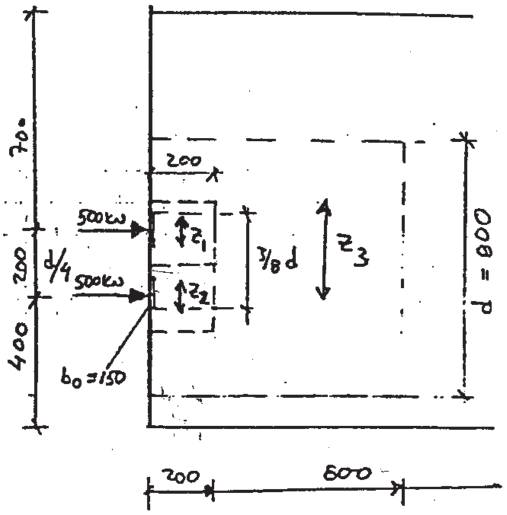


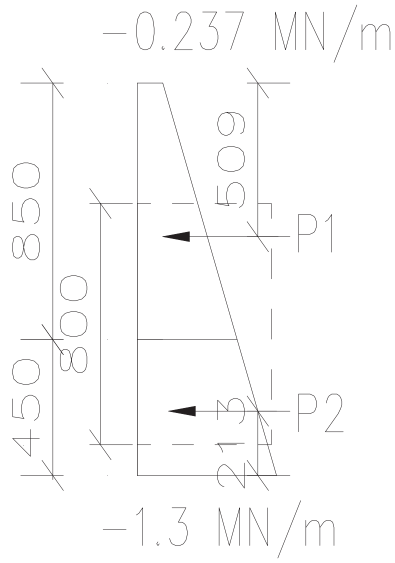
Z1 and Z2 are the spalling froces in the sub-regions

The local spalling forces Z1 and Z2 due to concentrated loads V1 and V2 are calculated as before.
In addition of these forces the interaction of the both loads causes spalling force Z3 far from the anchors. Spalling force Z3 is calcultated for the total load V1+V2 with the side of the loading area 3/8 d.

=> $Z_3 := 0.25\cdot(V_1 + V_2)\cdot\left(1 - \frac{\frac{3}{8}\cdot d}{d}\right)$ $\quad Z_3 := \frac{5}{32}\cdot(V_1 + V_2)$

Example.





Two prestressing forces $P_1 := 500 \cdot kN$ distance $a_1 := 700 \cdot mm$ from the top side

$P_2 := 500 \cdot kN$ distance $a_2 := 400 \cdot mm$ from the bottom side

Dimension of the anchor $b_0 := 150 \cdot mm$

Depth of the cross-section $h := 1300 \cdot mm$

The location of the resultant of the loads $a := 500 \cdot mm$ from the bottom side

=> eccentricity $e := \frac{h}{2} - a$ $\quad e = 150 mm$

Stress at the bottom fibre: $b\sigma_{ca} := \frac{-(P_1 + P_2)}{h} + \frac{-(P_1 + P_2) \cdot e \cdot 6}{h^2}$ $\quad b\sigma_{ca} = -1.302 \frac{MN}{m}$

Stress at the top fibre: $b\sigma_{cy} := \frac{-(P_1 + P_2)}{h} - \frac{-(P_1 + P_2) \cdot e \cdot 6}{h^2}$ $\quad b\sigma_{cy} = -0.237 \frac{MN}{m}$

h is the depth of the member and b is the width of the member

The stress distribution can be divided into parts so that the stress resultant corresponding P1 is located at the distance 850 mm from the top side and the stress resultant corresponding P2 is located at the distance 450 mm from the bottom side.
The stress resultant of the upper part is located at the distance 509 mm from the top side and the stress resultantt of the lower part is located at the distance 213 mm from
the bottom side. The concentrated loads P1 and P2 are located between these resultants so. indside. In this case the locations of the stress resultants corresponding the loads P1 and P2 do not coincide the forces P1 and P2.

The distance between the concentrated loads is 200 mm. The distance of the point loads from the bottom side is 500 mm and the distance of the individual load from the stress resultant is 100 mm.

Suppose that the distance of the individual force is d/8=100 mm .
So the distiribution width is d = 800 mm.
Stresses are supposed to distributed over this distance uniformly.

$d := 800 \cdot mm$

Spalling force due to P1:

Width of the distribution area $b_1 := 2 \cdot \left(\frac{d}{2} - 100 \cdot mm\right)$ $b_1 = 600\,mm$

$h - a_1 = 600\,mm$

$Z_1 := 0.25 \cdot \gamma_{p.unfav} \cdot P_1 \cdot \left(1 - \frac{b_0}{b_1}\right)$ $Z_1 = 112.5\,kN$

Spalling force due to P2:

Width of the distribution area $b_2 := 2 \cdot \left(\frac{d}{2} - 100 \cdot mm\right)$ $b_2 = 600\,mm$

$Z_2 := 0.25 \cdot \gamma_{p.unfav} \cdot P_2 \cdot \left(1 - \frac{b_0}{b_2}\right)$ $Z_2 = 112.5\,kN$

Local spalling force near the anchors $Z := \max(Z_1, Z_2)$ $Z = 112.5\,kN$

Spalling reinforcement A500HW $f_{yd} := \frac{500 \cdot MPa}{\gamma_s}$ $f_{yd} = 454.545\,MPa$

$A_{s1} := \frac{Z}{f_{yd}}$ $A_{s1} = 247.5\,mm^2$

3 T 8 $A_{s1} := 3 \cdot 2 \cdot 50.3 \cdot mm^2$ $A_{s1} = 301.8\,mm^2$

Reinforcement s distributed to the distance 200 mm => 3 T 8 k 100

Spalling force due to the total force $P_1 + P_2 = 1000\,kN$

$Z_3 := \frac{5}{32} \cdot \gamma_{p.unfav} \cdot (P_1 + P_2)$ $Z_3 = 187.5\,kN$

$A_{s3} := \frac{Z_3}{f_{yd}}$ $A_{s3} = 412.5\,mm^2$

5 T 8 $A_{s3} := 5 \cdot 2 \cdot 50.3 \cdot mm^2$ $A_{s3} = 503\,mm^2$

Reinforcement is divided to the distance 800 mm-200 mm= 600 mm 5 T 8 k 100

Total amount of reinforcement is 8 T 8 at the distance 0 ... 800 mm

Splitting force due to the eccentricity in the very end of the structure

top tendons $e_1 := 50 \cdot mm$

bottom tendons $e_2 := 250 \cdot mm$

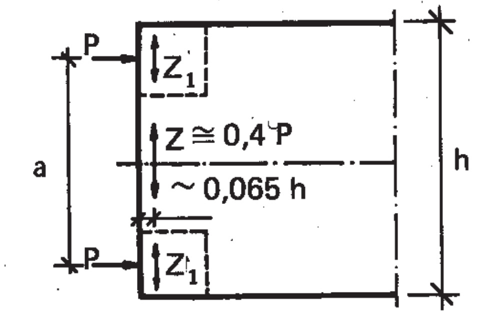
$$F_{t2Ed} := 0.015 \cdot \gamma_{p.unfav} \cdot \left(\frac{P_1}{1 - \sqrt{2 \cdot \frac{e_1}{d}}} + \frac{P_2}{1 - \sqrt{2 \cdot \frac{e_2}{d}}} \right) \qquad F_{t2Ed} = 56.896\,kN$$

$$A_{s.end} := \frac{F_{t2Ed}}{f_{yd}} \qquad A_{s.end} = 125\,mm^2$$

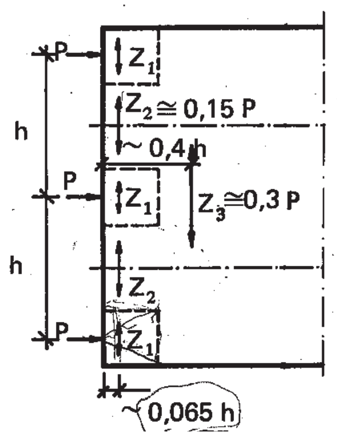
From the spalling reinforcement As3 2 T 10 (157 mm2) vertical links is located in the very end of the structure and stirrups 3 T 10 k100 and then stirrups 5 T 8 k100.

If the point loads are far from each other and near the edge, the point loads are outside the the cenroid of the region of the stress diagram corresponding these point loads. Point loads arise both local spalling forces Z1 near the point loads and also near the end of the structure tensile forces (Z) between the point loads.
Tensile force Zp can be calculated by treating the end of the structure as a deep beam which span is the distance between the point loads and the depth is the depth of the structure.
The support reactions of this deep beam are the point loads (prestress force P) and the distributed loading is the stress distribution $b\sigma_{cox}$ due to the point loads.

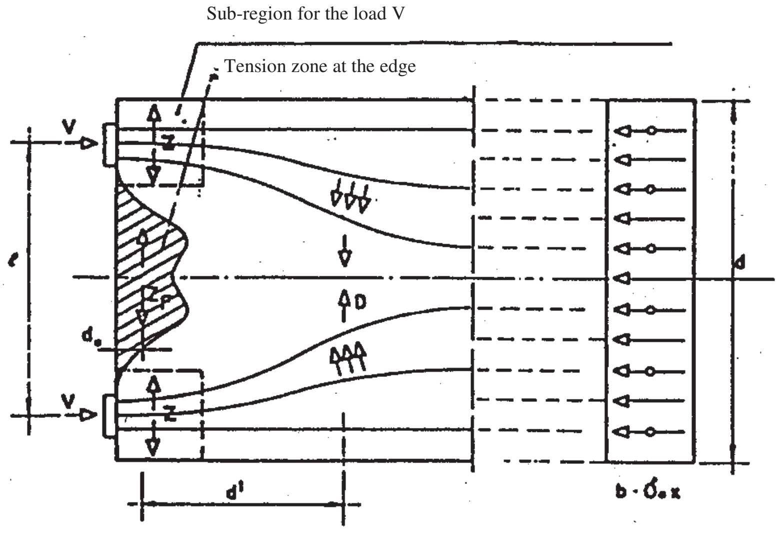
The breadth of the support is the dimension of the loading area (anchor size).



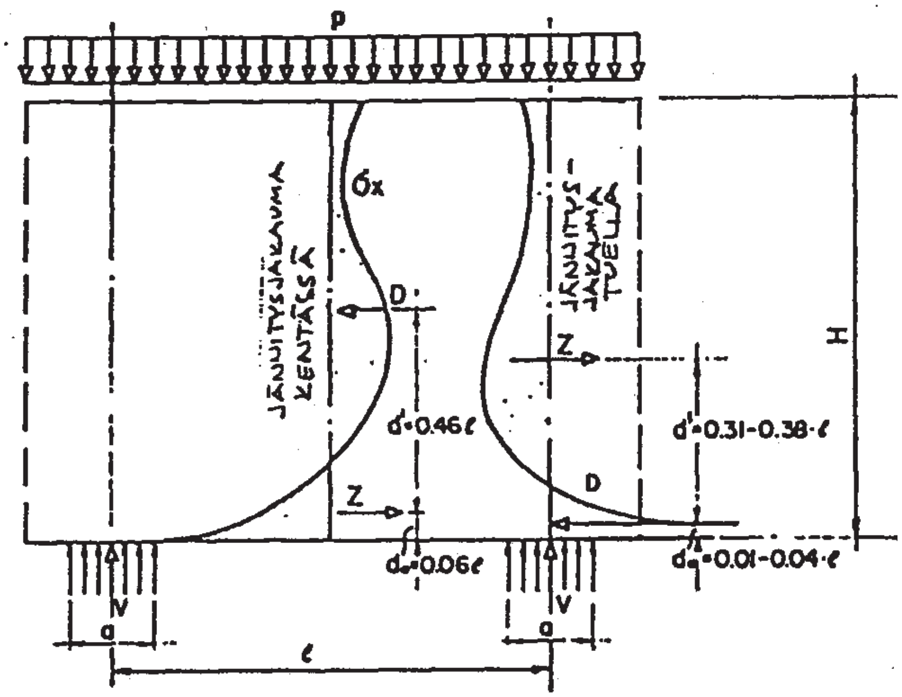
Two point forces (anchors) which are close to the corners far from each others



Three point loads which affect outside there compression stress zones



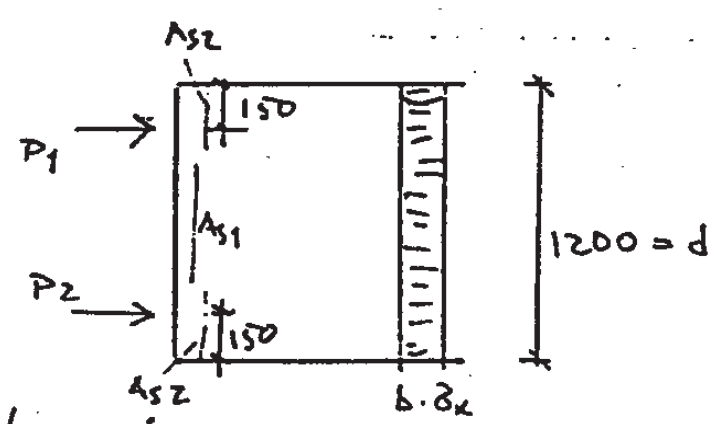
End block of the structure as a deep beam



Deep beam

$\frac{\text{Depth H}}{\text{Span l}} = 1{,}0$	Inner lever arm d'		Location of the tension resultant d_o	
$\frac{a}{\ell} = 0{,}05...0{,}25$	$\frac{a}{\ell} = 0{,}05$	$\frac{a}{\ell} = 0{,}25$	$\frac{a}{\ell} = 0{,}05$	$\frac{a}{\ell} = 0{,}25$
Continuous beam, n spans				
- support moment	$0{,}31 \cdot l$	$0{,}38 \cdot l$	$0{,}01 \cdot l$	$0{,}04 \cdot l$
- field moment	$0{,}46 \cdot l$	$0{,}46 \cdot l$	$0{,}06 \cdot l$	$0{,}06 \cdot l$
2-span continuous beam				
- support moment at the middle support	$0{,}42 \cdot l$	$0{,}48 \cdot l$	$0{,}02 \cdot l$	$0{,}07 \cdot l$
- field moment	$0{,}63 \cdot l$	$0{,}63 \cdot l$	$0{,}08 \cdot l$	$0{,}08 \cdot l$
Simply supported beam	$0{,}67 \cdot l$	$0{,}67 \cdot l$	$0{,}11 \cdot l$	$0{,}11 \cdot l$

Example.



2 tendons $P_1 := 800 \cdot kN$ distance from the top side $a_1 := 150 \cdot mm$

$P_2 := 800 \cdot kN$ distance from the bottom side $a_2 := 150 \cdot mm$

Anchor dimension $a_0 := 200 \cdot mm$ $b_0 := 200 \cdot mm$

Depth of the beam $d := 1200 \cdot mm$

The point loads are symmetrical about the centroid line of the beam

Stress*b at the distance d from the anchor $b\sigma_x := \frac{P_1 + P_2}{d}$ $b\sigma_x = 1333.3 \frac{kN}{m}$

Deep beamm the span $L := d - a_1 - a_2$ $L = 900 mm$

Depth of the deep beam $d_1 := d$ $d_1 = 1200 mm$

Bending moment considering the stress distribution as the load of the deep beam

$$M_{Ed} := \frac{\gamma_{p.unfav} \cdot b\sigma_x \cdot L^2}{8} \qquad M_{Ed} = 162 kNm$$

The end block is treated as a simply supported deep beam

Breadth of the support is the dimension of the anchor $a_0 = 200 mm$

Lever arm of the deep beam $z := 0.67 \cdot L$ $z = 603 mm$

Tensile force $Z_{Ed} := \frac{M_{Ed}}{z}$ $Z_{Ed} = 268.657 kN$

Splitting reinforcement A500 HW $f_{yd} := \frac{500 \cdot MPa}{\gamma_s}$ $f_{yd} = 454.545 MPa$

$$A_s := \frac{Z_{Ed}}{f_{yd}} \qquad A_s = 591 mm^2$$

4 T 12 vertical links + 1 T 10 stirrup $A_s := 4 \cdot 113 \cdot mm^2 + 2 \cdot 78.5 \cdot mm^2$ $A_s = 609 mm^2$

Comparing:

Eccentricity of the both prestress force $e := 450 \cdot mm$

From the equation: $F_{t2.Ed} := 0.015 \cdot \gamma_{p.unfav} \cdot \left(\frac{P_1}{1 - \sqrt{2 \cdot \frac{e}{h}}} + \frac{P_2}{1 - \sqrt{2 \cdot \frac{e}{h}}} \right)$ $F_{t2.Ed} = 171.48\,kN$

At the corners the spalling reinforcement for the spalling forces Z1:

Size of the distribution area $b := 2 \cdot a_1$ $b = 300\,mm$

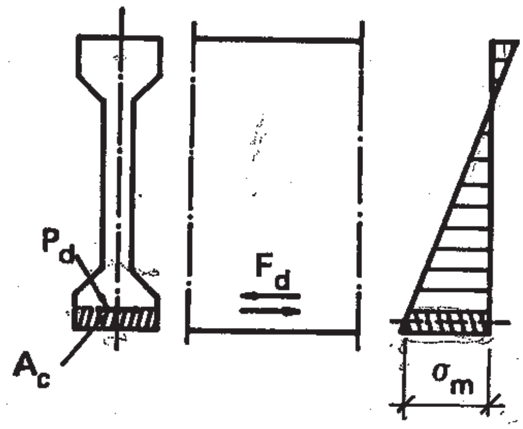
$Z_{1Ed} := 0.25 \cdot \gamma_{p.unfav} \cdot P_1 \cdot \left(1 - \frac{a_0}{b} \right)$ $Z_{1Ed} = 80\,kN$

Spalling reinfrocement $A_{s1} := \frac{Z_{1Ed}}{f_{yd}}$ $A_{s1} = 176\,mm^2$

The same vertical links between the point loads is adequite also for the spalling reinforcement in the corners.

Spalling reinforcement in pre-tensioned members with bonded strands

Reinforcement for spalling forces due to a group of pre-tensioned strands is dimensioned for a shear force Fd, which is difference between the total prestressing force and the stress resultant ir the part Ac of the concrete section below the centroid of the prestessing strands



Spalling force due to a group of the pre-tensioned strands

Shear force Fd is obtained from the equation

$$F_d := P_d - |\sigma_{cm}| \cdot A_c$$

where $P_d := \gamma_{p.unfav} \cdot P_0$ prestressing force at the considered time

σcm is the average stress in Ac

Ac is the part of the concrete section below the centroid of the group of the predstressir strands

The term σcmAc corresponds in the equation of the spalling force for post-tension members the term b0/b. So the part of the prestressing force which corresponds the stress in the area of the anchor and does not induce spallimng stresses.

Total amount of reinforcement As for the spalling force is obtained from the equation

$$A_s := 0.3 \cdot \frac{F_d}{f_{yd}}$$

where the design strength of spalling reinforcement fyd is not greater than 300 MPa.

Spalling reinforcement is divided equally at the distance of the transmission length;
one or two stirrups is located at the very end of the member for taking the splitting force due to eccentricity.

Splitting force due to te eccentricity at the end of the member is calculated in the same way than for post-tensioned structures for the equation

$$F_{t2.Ed} := \frac{0.015 \cdot P_d}{1 - \sqrt{2 \cdot \frac{e}{h}}}$$

Example of the spalling force due to pre-tensioned strands

Number of the strands $n := 10$

Area of one strand $A_{p1} := 93 \cdot mm^2$

Prestressing just after the transfer $\sigma_{p0} := 1230 \cdot MPa$

Prestress force $P_0 := n \cdot A_{p1} \cdot \sigma_{p0}$ $P_0 = 1143.9\,kN$

Partial safety factor for prestressing $\gamma_{p.unfav} = 1.2$

Design value of the restressing force $P_d := \gamma_{p.unfav} \cdot P_0$ $P_d = 1372.7\,kN$

Eccentricity of the prestressing force $e := 612 \cdot mm$

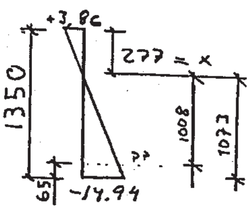
I-profile depth $h := 1350 \cdot mm$

width of the bottom flange $b_{bot} := 400 \cdot mm$

Area of the cross-section of the I-beam $A := 0.2498 \cdot m^2$

Section modulus about bottom side $W_{bot} := 0.0890 \cdot m^3$

Section modulus about top side $W_{top} := 0.08984 \cdot m^3$



Sresses due to prestressing:

Stress at the top fibre $\sigma_{ctop} := \frac{-P_d}{A} + \frac{P_d \cdot e}{W_{top}}$ $\sigma_{ctop} = 3.856\,M$

Stress at the bottom fibre $\sigma_{cbot} := \frac{-P_d}{A} - \frac{P_d \cdot e}{W_{bot}}$ $\sigma_{cbot} = -14.934\,MPa$

Cenroid of the strands from the bottom side $c_p := 65 \cdot mm$

Concrete stress at the centroid of the strands

$$\sigma_{cp} := \sigma_{cbot} + \left(\sigma_{ctop} - \sigma_{cbot}\right) \cdot \frac{c_p}{h} \qquad \sigma_{cp} = -14.03\,MPa$$

Average concrete stress below the centroid of the strands (so at the height of 65 mm)

$$\sigma_{cm} := \frac{\left(\sigma_{cbot} + \sigma_{cp}\right)}{2} \qquad \sigma_{cm} = -14.482\,MPa$$

Area of the cross-section below the centroid of the strands $A_c := b_{bot} \cdot c_p$ $A_c = 0.026\,m^2$

Spalling force $F_d := P_d - \left|\sigma_{cm}\right| \cdot A_c$ $F_d = 996.151\,kN$

Spalling reinforcement A500 HW $f_{yd} := \min\left(300 \cdot MPa, \frac{500 \cdot MPa}{\gamma_s}\right)$ $f_{yd} = 300\,MPa$

Spalling reinforcement $A_{s.spall} := 0.3 \cdot \frac{F_d}{f_{yd}}$ $A_{s.spall} = 996\,mm^2$

Spalling reinforcement is divided equally at the distance of the transission length $l_{pt2} := 700 \cdot mm$

7 T 10 k 100 stirrups $A_{s.spall} := 7 \cdot 2 \cdot 78.5 \cdot mm^2$ $A_{s.spall} = 1099\,mm^2$

Splitting force in the web at the end of the member due to the eccentricity of the strands

$$F_{t2.Ed} := \frac{0.015 \cdot P_d}{1 - \sqrt{2 \cdot \frac{e}{h}}} \qquad F_{t2.Ed} = 430.671\,kN$$

Splitting reinforcement $f_{yd} := \frac{500 \cdot MPa}{\gamma_s}$ $f_{yd} = 454.545\,MPa$

$$A_{s.split} := \frac{F_{t2.Ed}}{f_{yd}} \qquad A_{s.split} = 947\,mm^2$$

At the of the member vertical links 2 T 16 + stirrups 4 T 10 k50

$$A_{s.split} := 2 \cdot 201 \cdot mm^2 + 4 \cdot 2 \cdot 78.5 \cdot mm^2 \qquad A_{s.split} = 1030\,mm^2$$

The splitting reinforcement As.split can be included to the spalling reinforcement As.spall, so there is at the end the vertical links 2 T 16 and the stirrups 4 T 10 k 50 and then the stirrups 5 T 10 k 100.

Spalling stress at the end of the HC500-slab (no spalling reinforcement)

(according to the standard EN-1168 Precast concrete products, Hollow core slabs)

Depth of the slab $h := 500 \cdot mm$

Cross-section values without the strands:

Area of concrete $A_c := 0.2760323 \cdot m^2$

Second moment of area $I_c := 0.0084639 \cdot m^4$

Centroid from the bottom fibre $pp_c := 250.5 \cdot mm$

Total width of the webs $b_w := 343.8 \cdot mm$

Self-weight of the slab $g := A_c \cdot 25 \cdot \frac{kN}{m^3}$ $g = 6.901 \frac{kN}{m}$

Length of the slab $L := 17 \cdot m$

Bending moemnt due to self-weight $M_g := \frac{g \cdot L^2}{8}$ $M_g = 249.292\,kNm$

Diameter of the strand $\phi := 12.5 \cdot mm$

Area of one strand $A_{p1} := 93 \cdot mm^2$

number of the strands in the layers

Distance of the 1. layer from the bottom. $c_{p1} := 35 \cdot mm$ $n_{p1} := 13$

Distance of the 2. layer from the bottom. $c_{p2} := 72 \cdot mm$ $n_{p2} := 8$

Total number of the strands $n_p := n_{p1} + n_{p2}$ $n_p = 21$

Total area of the strands $A_p := n_p \cdot A_{p1}$ $A_p = 1953\,mm^2$

Centroid of the strands from the botttom $c_p := \frac{n_{p1} \cdot c_{p1} + n_{p2} \cdot c_{p2}}{n_p}$ $c_p = 0.049\,m$

Elastic modulus of concrete $E_p := 195000 \cdot MPa$

Initial prestress $\sigma_{p0} := 1000 \cdot MPa$

Concrete grade C60/75

Characteristic compressive strength $f_{ck} := 60 \cdot MPa$

Average compressive strength $f_{cm} := f_{ck} + 8 \cdot MPa$ $f_{cm} = 68\,MPa$

Concrete strength at transfer 70 % of the average nominal strength (strength at 28 days)

Average comresssive strength at the time of transfer	$f_{cm.ti} := 0.7 \cdot f_{cm}$	$f_{cm.ti} = 47.6\,MPa$
Develoment coefficient for concrete compressive strength	$\beta_{cc} := \frac{f_{cm.ti}}{f_{cm}}$	$\beta_{cc} = 0.7$
Elastic modulus of concrete	$E_{cm} := 22000 \cdot \left(\frac{f_{cm}}{10 \cdot MPa}\right)^{0.3} \cdot MPa$	$E_{cm} = 3.91 \times 10^{4}\,MPa$
Elastic modulus of concrete at transfer	$E_{cm.ti} := \left(\frac{f_{cm.ti}}{f_{cm}}\right)^{0.3} \cdot E_{cm}$	$E_{cm.ti} = 3.513 \times 10^{4}\,MPa$
Average tensile strength of concrete	$f_{ctm} := 2.12 \cdot \ln\left(1 + \frac{f_{cm}}{10 \cdot MPa}\right) \cdot MPa$	$f_{ctm} = 4.355\,MPa$
Average tensile strengt at transfer	$f_{ctm.ti} := \beta_{cc} \cdot f_{ctm}$	$f_{ctm.ti} = 3.048\,MPa$
Characteristic tensile strength at transfer	$f_{ctk.ti} := 0.7 \cdot f_{ctm.ti}$	$f_{ctk.ti} = 2.134\,MPa$
Partial safety factor for concrete	$\gamma_c := 1.5$	
Design value of tensile strength	$f_{ctd.ti} := \frac{f_{ctk.ti}}{\gamma_c}$	$f_{ctd.ti} = 1.423\,MPa$

Transformed cross-section values

Ratio of the elastic modulies	$\alpha_p := \frac{E_p}{E_{cm.ti}}$	$\alpha_p = 5.55$
Area of the transformed cross-sectoon	$A := A_c + (\alpha_p - 1) \cdot A_p$	$A = 0.285\,m^2$
Centroid from the bottom	$pp := \frac{A_c \cdot pp_c + A_p \cdot c_p}{A}$	$pp = 0.243\,m$

Second moment of area

$$I := I_c + A_c \cdot (pp - pp_c)^2 + (\alpha_p - 1) \cdot n_{p1} \cdot A_{p1} \cdot (pp - c_{p1})^2 + (\alpha_p - 1) \cdot n_{p2} \cdot A_{p1} \cdot (pp - c_{p2})^2$$

$$I = 8.816 \times 10^{-3}\,m^4$$

Section modulus about the top fibre $W_y := \frac{I}{h - pp}$ $W_y = 0.034\,m^3$

Section modulus about the bottom fibre $W_a := \frac{I}{pp}$ $W_a = 0.036\,m^3$

Section modulus about the centroidal axis $W_p := \frac{I}{pp - c_p}$ $W_p = 0.045\,m^3$

Size of the core $k := \frac{W_y}{A}$ $k = 120.414\,mm$

Initial prestress $\sigma_{p0} := 1000 \cdot MPa$

Prestress losses before the transfer about 3 %

Prestress just before the transfer $\sigma_{p1} := 0.97 \cdot \sigma_{p0}$ $\sigma_{p1} = 970\,MPa$

Prestress force $P_1 := \sigma_{p1} \cdot A_p$ $P_1 = 1.894 \times 10^3\,kN$

Eccentricity of the prestress force $e_0 := pp - c_p$ $e_0 = 193.928\,mm$

Concrete stress at the level of the centroid of the strands

$$\sigma_{cp} := \frac{-P_1}{A} + \frac{-P_1 \cdot e_0}{W_p} + \frac{M_g}{W_p} \qquad \sigma_{cp} = -9.246\,MPa$$

Sress in the strands just after the transfer

$$\sigma_{pm0} := \sigma_{p1} + \alpha_p \cdot \sigma_{cp} \qquad \sigma_{pm0} = 918.678\,MPa$$

Transmission length of prestress

Bond coefficient at transfer $\eta_{p1} := 3.2$

Coefficient for good bond condition $\eta_1 := 1$

Bond strength $f_{bpt} := \eta_{p1} \cdot \eta_1 \cdot f_{ctd.ti}$ $f_{bpt} = 4.552\,MPa$

Coefficient for rapid transfer $\alpha_1 := 1.25$

Coefficient for 7-wire strands $\alpha_2 := \frac{A_{p1}}{\pi \cdot \phi^2}$ $\alpha_2 = 0.189$

Basic value of the transmission length $l_{pt} := \alpha_1 \cdot \alpha_2 \cdot \phi \cdot \frac{\sigma_{pm0}}{f_{bpt}}$ $l_{pt} = 597.419\,mm$

Lower design value of the transmission length $l_{ptd} := 0.8 \cdot l_{pt}$ $l_{ptd} = 477.935\,mm$

Average spalling stress

Factor due to the eccentricity $\alpha_e := \frac{e_0 - k}{h}$ $\alpha_e = 0.147$

Prestress force just after the transfer $P_{m0} := \sigma_{pm0} \cdot A_p$ $P_{m0} = 1.794 \times 10^3\,kN$

$$\frac{P_{m0}}{b_w \cdot e_0} = 26.91\,MPa$$

Spalling stress $\sigma_{sp} := \frac{P_{m0}}{b_w \cdot e_0} \cdot \frac{15 \cdot \alpha_e^{2.3} + 0.07}{1 + \left(\frac{l_{ptd}}{e_0}\right)^{1.5} \cdot (1.3 \cdot \alpha_e + 0.1)}$ $\sigma_{sp} = 3.195\,MPa$

$f_{ctk.ti} = 2.134\,MPa$

$f_{ctm.ti} = 3.048\,MPa$

Isolated web, where is 5 strands

Width of the web $b_{w1} := 70 \cdot mm$

Centroid of the strand in one web from the bottom $c_{p11} := \frac{3 \cdot c_{p1} + 2c_{p2}}{5}$ $c_{p11} = 49.8\,mm$

Prestress force in one web $P_{m01} := \sigma_{pm0} \cdot 5 \cdot A_{p1}$ $P_{m01} = 427.185\,kN$

Eccentricity $e_{01} := pp - c_{p11}$ $e_{01} = 193.223\,mm$

Parameter due to the eccentricity $\alpha_{e1} := \frac{e_{01} - k}{h}$ $\alpha_{e1} = 0.146$

$$\frac{P_{m01}}{b_{w1} \cdot e_{01}} = 31.583\,MPa$$

Max. spalling stress in one web $\sigma_{sp1} := \frac{P_{m01}}{b_{w1} \cdot e_{01}} \cdot \frac{15 \cdot \alpha_{e1}^{2.3} + 0.07}{1 + \left(\frac{l_{ptd}}{e_{01}}\right)^{1.5} \cdot (1.3 \cdot \alpha_{e1} + 0.1)}$ $\sigma_{sp1} = 3.692\,MPa$

$f_{ctk.ti} = 2.134\,MPa$

$f_{ctm.ti} = 3.048\,MPa$