Dilution refrigerators - concepts

Concept of Landau's Fermi liquid theory

elementary excitations of interacting Fermions are described by almost independent fermionic quasiparticles

state of Fermi liquid described simply by quasiparticle distribution

Phenomenological Theory by Landau

energy functional:

$$E = E_0 + \sum_{\vec{k},\sigma} \epsilon_\sigma(\vec{k})\delta n_\sigma(\vec{k}) + \frac{1}{2\Omega}\sum_{\vec{k},\vec{k}'}\sum_{\sigma,\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_\sigma(\vec{k})\delta n_{\sigma'}(\vec{k}')$$

deviation from ground state

$$\delta n_\sigma(\vec{k}) = n_\sigma(\vec{k}) - n_\sigma^{(0)}(\vec{k})$$

spin index $\sigma = \pm 1$

ground state distribution

$$n_\sigma^{(0)}(\vec{k}) = \Theta(k_F - |\vec{k}|)$$

filled Fermi sea

Fermi liquid theory

$$E = E_0 + \sum_{\vec{k},\sigma} \epsilon_\sigma(\vec{k})\delta n_\sigma(\vec{k}) + \frac{1}{2\Omega}\sum_{\vec{k},\vec{k}'}\sum_{\sigma,\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_\sigma(\vec{k})\delta n_{\sigma'}(\vec{k}')$$

effective quasiparticle spectrum:

$$\tilde{\epsilon}_\sigma(\vec{k}) = \frac{\delta E}{\delta n_\sigma(\vec{k})} = \epsilon_\sigma(\vec{k}) + \frac{1}{\Omega}\sum_{\vec{k}',\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_{\sigma'}(\vec{k}')$$

bare quasiparticle spectrum:

$$\epsilon_\sigma(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m^*}$$

effective mass

Fermi velocity:

$$\frac{1}{\hbar}\vec{\nabla}_{\vec{k}}\,\epsilon_\sigma(\vec{k})\Big|_{k_F} = \vec{v}_F = \frac{\hbar \vec{k}_F}{m^*}$$

density of states at ϵ_F:

$$N(\epsilon_F) = \frac{1}{\Omega}\sum_{\vec{k},\sigma}\delta(\epsilon_\sigma(\vec{k}) - \epsilon_F) = \frac{k_F^2}{\pi^2\hbar v_F} = \frac{m^* k_F}{\pi^2\hbar^2}$$

Fermi volume conserved

$$k_F = (3\pi^2 n)^{1/3}$$

Fermi liquid theory

$$E = E_0 + \sum_{\vec{k},\sigma} \epsilon_\sigma(\vec{k})\delta n_\sigma(\vec{k}) + \frac{1}{2\Omega}\sum_{\vec{k},\vec{k}'}\sum_{\sigma,\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\delta n_\sigma(\vec{k})\delta n_{\sigma'}(\vec{k}')$$

couplings:

$$f_{\sigma\sigma'}(\vec{k},\vec{k}') = f^s(\hat{k},\hat{k}') + \sigma\sigma' f^a(\hat{k},\hat{k}')$$

symmetric (charge) — antisymmetric (spin)

spherical symmetry: $$f^{s,a}(\hat{k},\hat{k}') = \sum_{l=0}^{\infty} f_l^{s,a} P_l(\cos\theta_{\hat{k},\hat{k}'})$$

Legendre Polynomials

$$\int_{-1}^{+1} dz\, P_l(z)\, P_{l'}(z) = \frac{2\delta_{ll'}}{2l+1}$$

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{\sqrt{r^2 + r'^2 - 2rr'\cos\gamma}} = \sum_{\ell=0}^{\infty} \frac{r'^\ell}{r^{\ell+1}} P_\ell(\cos\gamma),$$

$$P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = (3x^2-1)/2,\ \text{etc.}$$

Landau parameters:

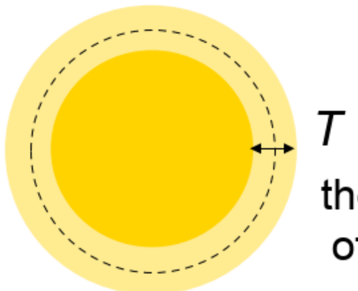
$$F_l^s = N(\epsilon_F) f_l^s \quad \text{charge}$$

$$F_l^a = N(\epsilon_F) f_l^a \quad \text{spin}$$

Fermi liquid theory

specific heat: $$\delta n_\sigma(\vec{k}) = n_\sigma^{(0)}(T, \vec{k}) - n_\sigma^{(0)}(0, \vec{k})$$

$$\frac{m^* k_F}{\pi^2 \hbar^2}$$

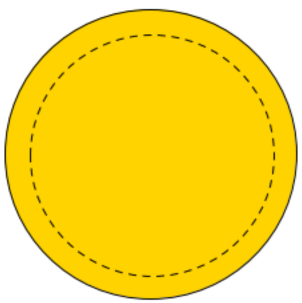


thermal softening of Fermi surface

$$C = \frac{\pi^2 k_B^2 N(\epsilon_F)}{3} T$$

compressibility: $$\kappa = -\frac{1}{\Omega} \frac{\partial \Omega}{\partial p}\bigg|_{T,N}$$

$$\delta\tilde{\epsilon}_\sigma(\vec{k}) = \frac{2}{3}\tilde{\epsilon}_\sigma(\vec{k})\kappa\delta p$$

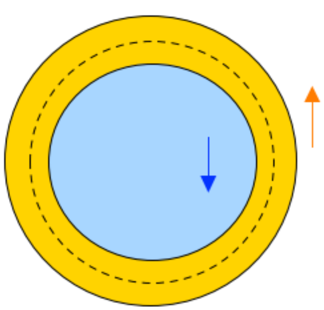


change of Fermi volume

$$\kappa = \frac{3}{2n\epsilon_F}\frac{1}{1+F_0^s} = \frac{1}{n^2}\frac{N(\epsilon_F)}{1+F_0^s}$$

spin susceptibility: $$\chi = \frac{M}{H}$$

$$\delta\tilde{\epsilon}_\sigma(\vec{k}) = -\tilde{g}\mu_B H \frac{\sigma}{2}$$



spin splitting of Fermi sea

$$\chi = \frac{\mu_B^2 N(\epsilon_F)}{1+F_0^a}$$

Let us now compute the *compressibility:*

$$\begin{cases} V = N/n \\ Vdp = dE,\ \ dE/N = d\mu \end{cases}$$

$$\kappa = -\frac{1}{V}\frac{\partial V}{\partial P} = \frac{1}{n^2}\frac{\partial n}{\partial \mu} \tag{5.38}$$

$$\delta n = \frac{1}{V}\sum_{p,\sigma}\delta n_{p,\sigma}$$

where P is the pressure. At $T = 0$,

$$\delta n_\sigma(\vec{p}) = \frac{\partial n_\sigma(\vec{p})}{\partial \varepsilon_\sigma(\vec{p})}(\delta\varepsilon_\sigma(\vec{p}) - \delta\mu) \qquad \delta\varepsilon_{p,\sigma} = \frac{1}{V}\sum_{p'\sigma'} f_{p\sigma,p'\sigma'}\delta n_{p'\sigma'}$$

The quasiparticle energy $\varepsilon_\sigma(\vec{p})$ depends on μ only through its dependence on $\delta n_{\sigma'}(\vec{p}')$ (*i.e.*, quasiparticle interactions, see Eq. 5.9). As $T \to 0$ both $\frac{\partial n}{\partial \varepsilon}$ and $\delta n_\sigma(p)$ vanish unless all momenta are *at* the Fermi-surface.

$$\delta\varepsilon_\sigma(p) = f_0^S \frac{1}{V}\sum_{\sigma',\vec{p}'}\delta n_{\sigma'}(\vec{p}') \equiv f_0^S \delta n \tag{5.40}$$

G. Baym and C. Pethick,
Landau Fermi liquid theory

where f_0^S is Landau parameter with with $l = 0$. Hence, we have

$$\delta n_\sigma(\vec{p}) = \frac{\partial n_\sigma(\vec{p})}{\partial \varepsilon_\sigma(\vec{p})}(f_0^S \delta n - \delta\mu) \tag{5.41}$$

and

$$\delta n = \frac{1}{V}\sum_{\sigma,\vec{p}} \delta n_\sigma(\vec{p}) = \frac{1}{V}\sum_{\sigma,\vec{p}} \frac{\partial n_\sigma(\vec{p})}{\partial \varepsilon_\sigma(\vec{p})}(f_0^S \delta n - \delta\mu) \tag{5.42}$$

Similarly,

$$\frac{\partial n_\sigma(\vec{p})}{\partial \varepsilon_\sigma(\vec{p})} \longrightarrow -\delta(|p| - p_F) \text{ for } T \to 0 \tag{5.43}$$

Thus,

$$\delta n = -N(0)(f_0^S \delta n - \delta\mu) \tag{5.44}$$

and

$$\delta n[1 + N(0) f_0^S] = N(0)\delta\mu \tag{5.45}$$

using the expression for the (s-wave) symmetric (singlet) Landau parameter,

$$F_0^S = N(0) f_0^S \tag{5.46}$$

we can write

$$\frac{\partial n}{\partial \mu} = \frac{N(0)}{1 + F_0^S} \tag{5.47}$$

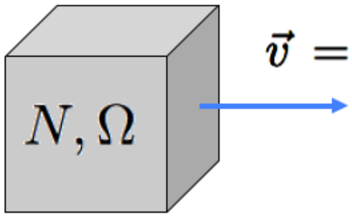
which leads to an expression for the *compressibility* κ:

$$\kappa = \frac{1}{n^2}\frac{N(0)}{1 + F_0^S} \tag{5.48}$$

$F_0^s > 10$, “less compressible”

Fermi liquid theory

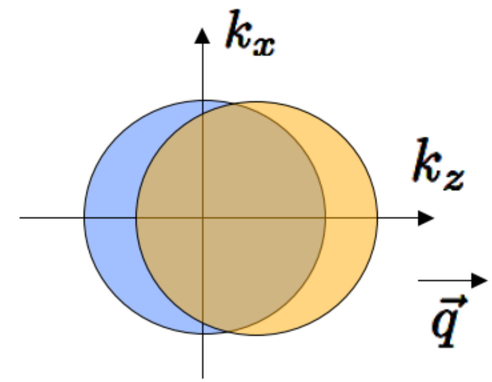
momentum shift for all particles $\vec{k} \rightarrow \vec{k} + \vec{q}$



$\vec{v} = \frac{\hbar \vec{q}}{m}$

current density

$$\vec{j}_{\vec{q}} = \frac{1}{\Omega} \sum_{\vec{k},\sigma} \vec{v}(\vec{k})\, \delta n_\sigma(\vec{k})$$



shifted Fermi sea

"bare particle" view

$$\vec{v}(\vec{k}) = \frac{\hbar \vec{k}}{m}$$

quasiparticle view

$$\vec{v}(\vec{k}) = \frac{1}{\hbar} \vec{\nabla}_{\vec{k}}\, \tilde{\epsilon}_\sigma(\vec{k})$$

$$= \frac{1}{\hbar}\left(\vec{\nabla}_{\vec{k}}\, \epsilon_\sigma(\vec{k}) + \frac{1}{\Omega} \sum_{\vec{k}',\sigma'} \vec{\nabla}_{\vec{k}}\, f_{\sigma\sigma'}(\vec{k}, \vec{k}')\delta n_\sigma(\vec{k}') \right)$$

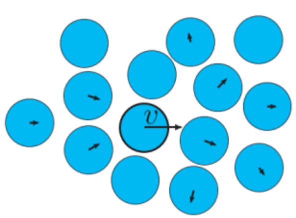
quasiparticle motion

induced motion

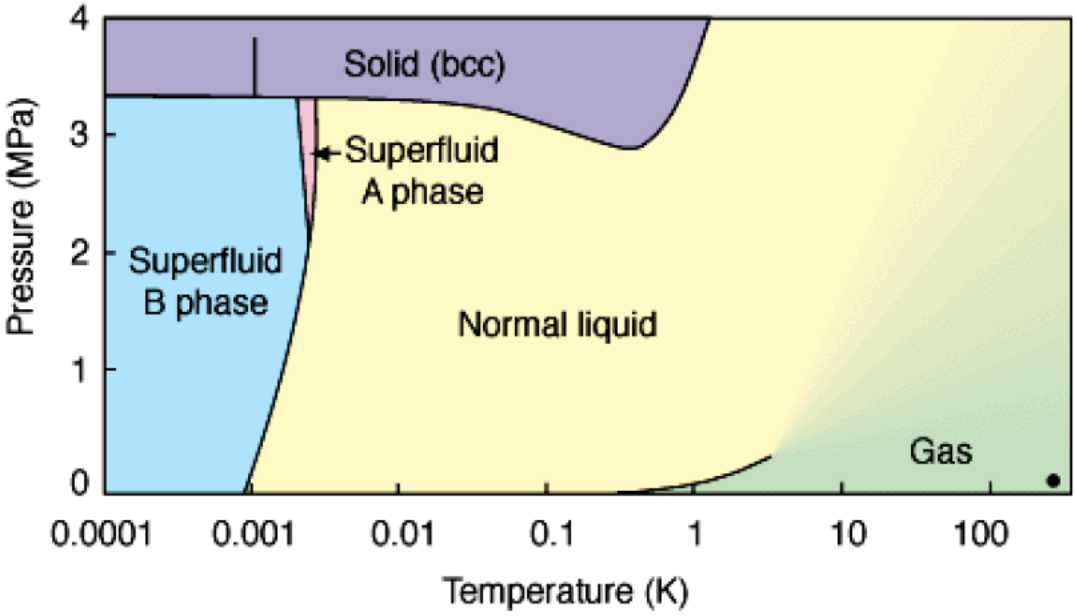
$$\frac{\hbar \vec{k}}{m} = \frac{\hbar \vec{k}}{m^*} + \frac{1}{\Omega} \sum_{\vec{k}',\sigma'} f_{\sigma\sigma'}(\vec{k}, \vec{k}')\delta(\epsilon_\sigma(\vec{k}') - \epsilon_F)\frac{\hbar \vec{k}'}{m^*}$$

$$\frac{m^*}{m} = 1 + \frac{1}{3} F_1^s$$

intrinsic consistency



^{3}He as an example Fermi liquid



strong short-range repulsion

↓

Fermi liquid

longer-ranged attraction

↓

superfluid

pressure	m^*/m	F_0^s	F_0^a	F_1^s	κ/κ_0	χ/χ_0
0	3.0	10.1	-0.52	6.0	0.27	6.3
$< p_c$	6.2	94	-0.74	15.7	0.065	24

enhanced (↑ m^*/m) diminished (↑ κ/κ_0) enhanced (↑ χ/χ_0)

$$\frac{m^*}{m} = 1 + \frac{1}{3}F_1^s$$

$$c = v_F\sqrt{\frac{1}{3}(1+F_0^s)(1+\frac{1}{3}F_1^s)}.$$

Speed of (first) sound

$$c_0 = v_F\sqrt{\frac{F_0^{(s)}}{3}}$$

Speed of zero sound

$$G(p,T) = U + pV - TS$$

$$\mu = \frac{\partial G}{\partial N}$$

Osmotic pressure

MIT MAE 545

Small water molecules can pass through a semipermeable membrane, which blocks large solute macromolecules.

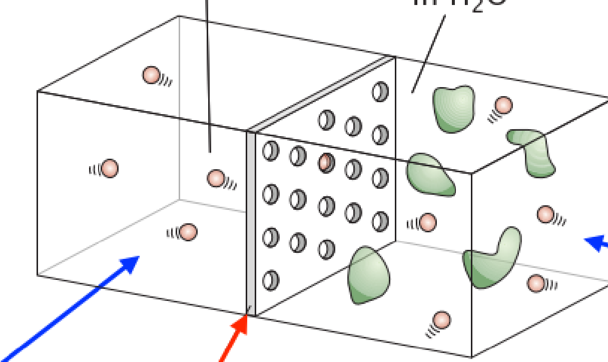


Figure from R. Phillips et al., Physical Biology of the Cell

$$G = N_1 \mu_{H_2O}(T, p_1, 0) + N_2 \mu_{H_2O}(T, p_2, c_s) + N_s \mu_s(T, p_2, c_s)$$

In thermodynamic equilibrium the Gibbs free energy *G* is minimized, which means that chemical potentials of water are the same on both sides of the semipermeable membrane!

$$\mu_{H_2O}(T, p_1, 0) = \mu_{H_2O}(T, p_2, c_s)$$

H_2O => ^4He
membrane => superleak

Osmotic pressure

MIT MAE 545

$$G(p,T,c) - G(p,T,0) = -TS$$

$$S_{mix} = -R(c_1 \ln c_1 + c_2 \ln c_2)$$

$$\approx -Rc_1 \ln c_1 \approx -Rc_2$$

Chemical potential of water

$$\mu_{H_2O} = \frac{\partial G}{\partial N_{H_2O}} = \mu^0_{H_2O} - k_B T \frac{N_s}{N_{H_2O}}$$

$$\mu_{H_2O}(T,p,c_s) = \mu^0_{H_2O}(T,p) - k_B T c_s v$$

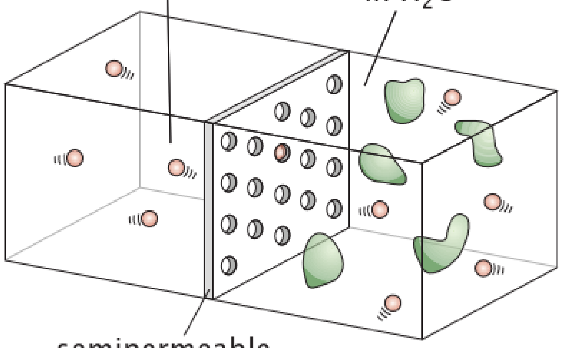


Figure from R. Phillips et al., Physical Biology of the Cell

$$\mu_{H_2O}(T,p_1,0) = \mu_{H_2O}(T,p_2,c_s)$$

$$\mu_{H_2O}(T,p_2,c_s) = \mu^0_{H_2O}(T,p_2) - k_B T c_s v$$

$$\mu_{H_2O}(T,p_2,c_s) \approx \mu^0_{H_2O}(T,p_1) + \left(\frac{\partial \mu^0_{H_2O}}{\partial p}\right)(p_2 - p_1) - k_B T c_s v$$

v

$$\Pi = p_2 - p_1 = k_B T \Delta c_s$$

Osmotic pressure depends only on temperature and concentration difference across the membrane!

Dilution refrigerator

Only available CONTINUOUS cooling method below 0.3 K

- 1951 H. London proposed the operating principle at LT meeting in Oxford
- 1962 First practical concept about it (London, Clarke & Mendoza)
- 1965 First realization (Das, De Bruyn Ouboter & Taconis, Leiden, 0.22 K)
- 1966 Dubna, 25 mK
- 1999 Lancaster, 1.7 mK

Physical grounds: QUANTUM-effects make it possible !

- **^{3}He atoms take more space than ^{4}He** (zp motion) => distinguishable
- ^{3}He dissolves into ^{4}He (Fermi/Bose systems) even at absolute zero temperature; $x_{3s}(T = 0) = 6.6$ %
- molar entropy of ^{3}He is higher in dilute mixture than in pure phase; heat of mixing: $L_m = 84\ T^2$ J/(mol K^2) $L = \Delta S T$; $\Delta S \sim T$
- ^{4}He is **superfluid and has no entropy** ($T < 0.5$ K)
- osmotic pressure of ^{3}He keeps balance against thermal gradient in mixture
- ^{3}He vapor pressure higher than of ^{4}He (possible to distill ^{3}He out of ^{4}He)

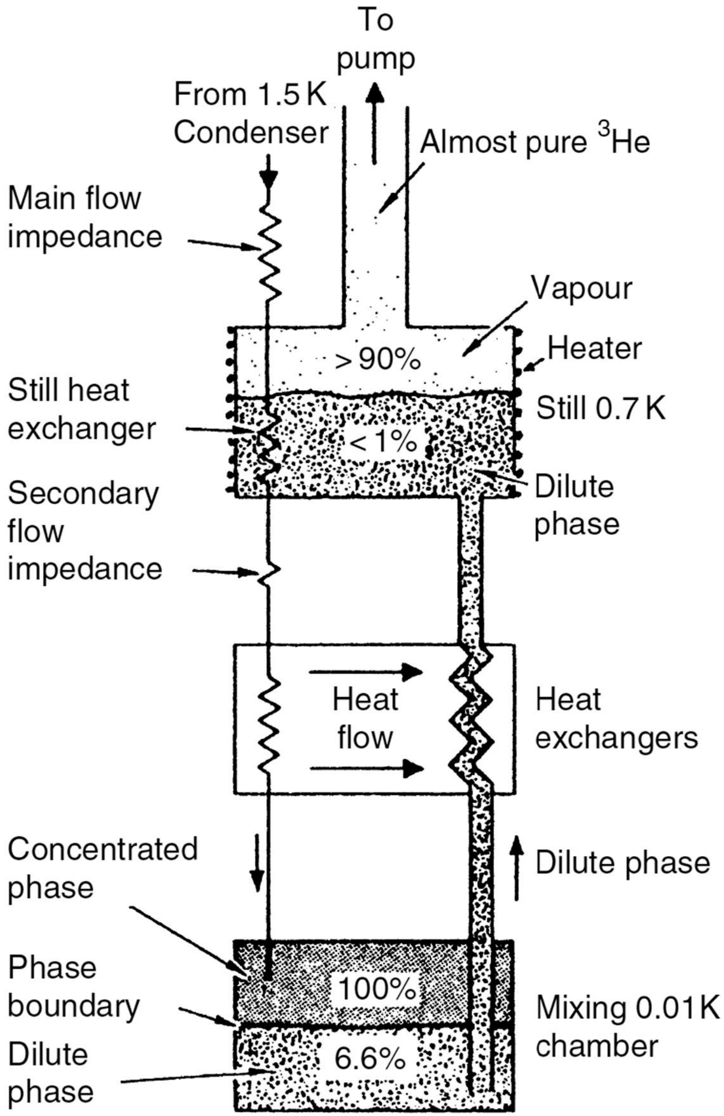


Fig. 7.9. Schematic ^{3}He–^{4}He dilution refrigerator. This part will sit in a vacuum chamber that is immersed in a ^{4}He bath at 4.2 K. The incoming ^{3}He gas is condensed on a continuously operating ^{4}He pot at 1.5 K (Sect. 5.2.4)

Operation of a dilution refrigerator requires pretty elaborate pumping and gas handling systems

– efficient booster-, roots-, or turbo pumps + backing pumps

=> 10^4 m^3/h (~ 1 mmol/s, p ~ 1 Pa)

– still pumping line ϕ ~ 0.3 m
– LN2-charcoal trap
– safety measures, etc

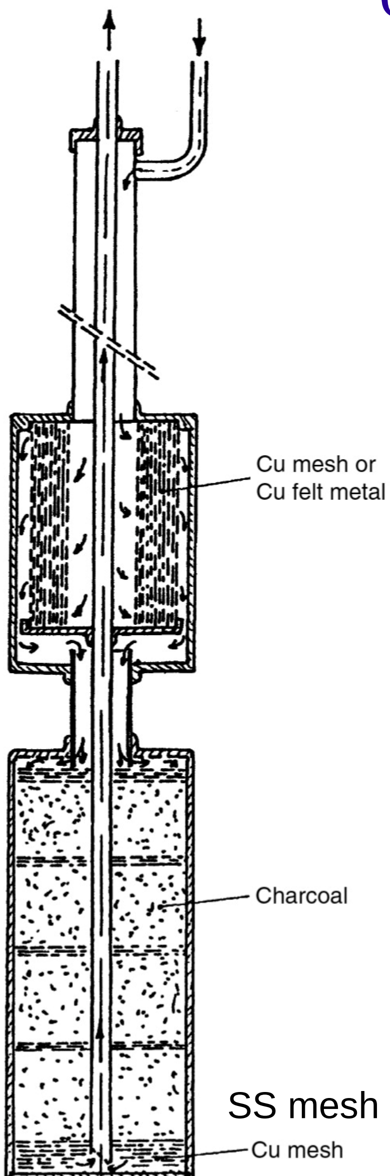
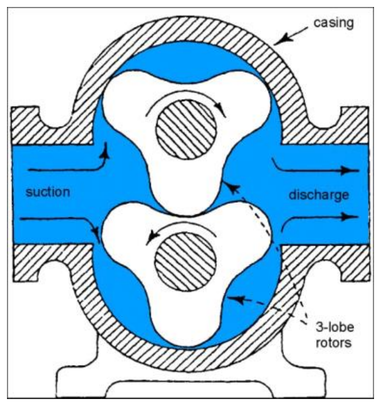
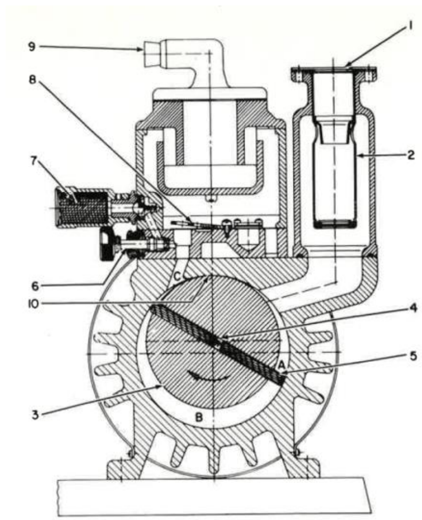


Fig. 7.25. LN_2 cooled activated charcoal/Cu mesh trap (courtesy of R.M. Mueller and P. Sekowski)





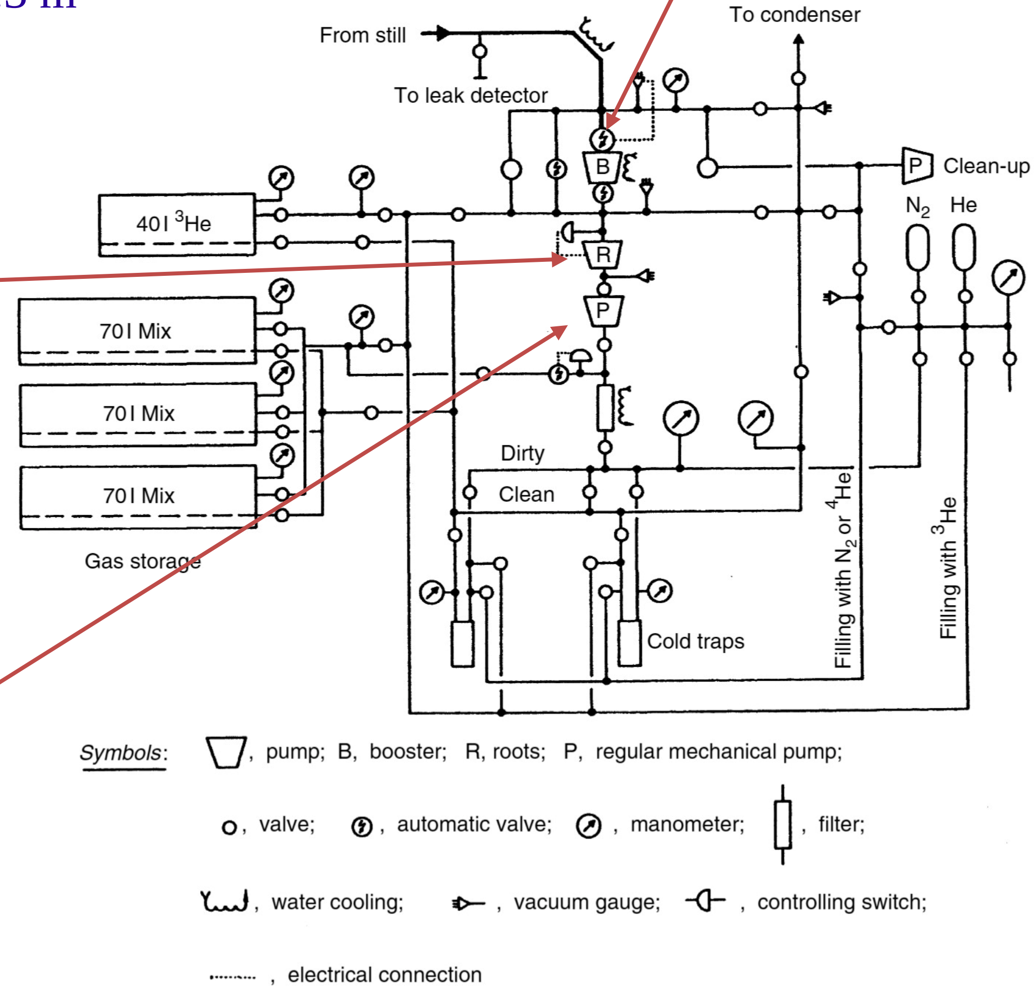


Fig. 7.24. Schematic diagram of the gas handling and pumping system of a ^3He–^4He dilution refrigerator (courtesy of P. Sekowski, Universität Bayreuth)

Dilution units are commercially available (price around 200-300 k€…700k€)

- Oxford Instruments, UK
- Leiden Cryogenics, The Netherlands
- Bluefors Cryogenics, Finland
- etc.

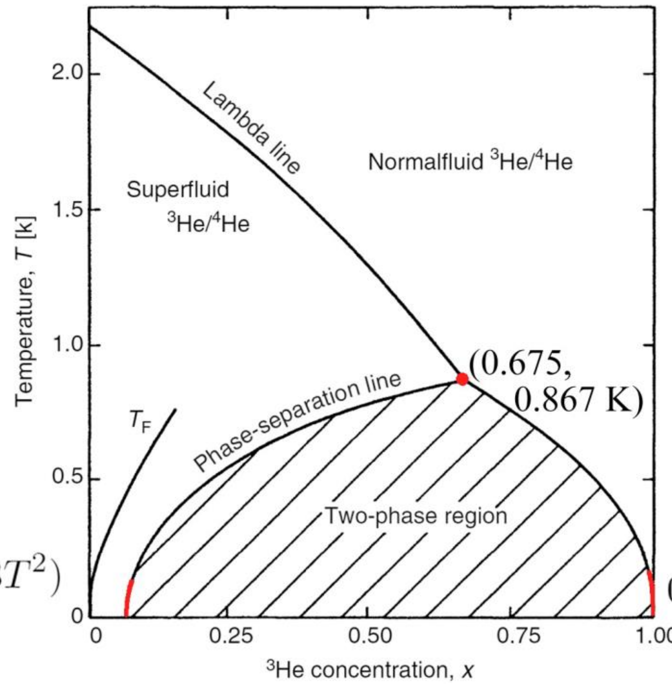
Special types may be adapted to specific conditions:

- **Dry cryostats**
  - pulse tube cooler provides the ~ 2 (3.5) K base
  - have become the mainstream of mK-refrigerators
  - just one vacuum space for the pulse tube and the dilution fridge
- **Miniature cryostats**
  - can be dipped into a storage dewar (often max diam ~ 5 cm)
  - quick operation, $T_{\mathrm{min}} \sim 10 \ldots 20$ mK in couple hours, limited power
- **Fully plastic dilution fridges**
  - can be operated in very high magnetic fields
  - Kapitza resistance between helium and plastics is smaller than He–metal
- **Monster machines**
  - $dn/dt \sim 10$ mmol/s with $A_{\mathrm{hex}} > 2000\ \mathrm{m}^2$

CF-CS110-1500 Maglev-2PT
Tmin < 7 mK
Q > 1500 microW @ 100 mK

$x_3 = 0.066(1 + 8.3T^2)$

0.064 - 0.068



$x_4 = 0.85T^{3/2}\ e^{-0.56/T}$

Fig. 7.1. Phase diagram of liquid ^{3}He–^{4}He mixtures at saturated vapour pressure. The diagram shows the lambda line for the superfluid transition of ^{4}He, the phase separation line of the mixtures below which they separate into a ^{4}He-rich and a ^{3}He-rich phase, and the line of the Fermi temperatures T_F of the ^{3}He component

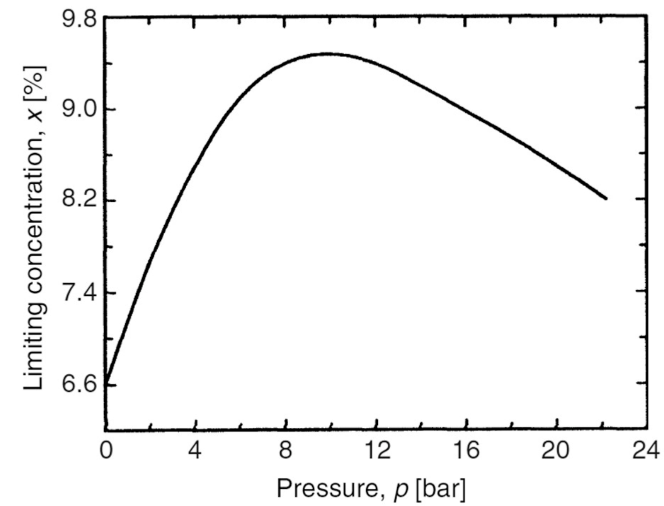


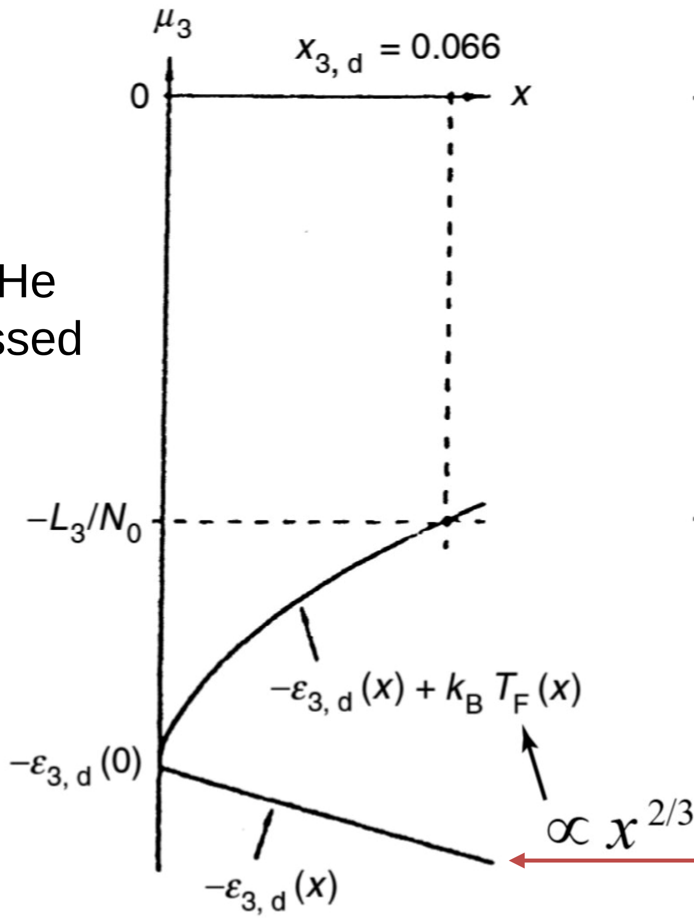
Fig. 7.2. Limiting low-temperature concentration of ^{3}He in ^{4}He at $T = 50$ mK as a function of pressure

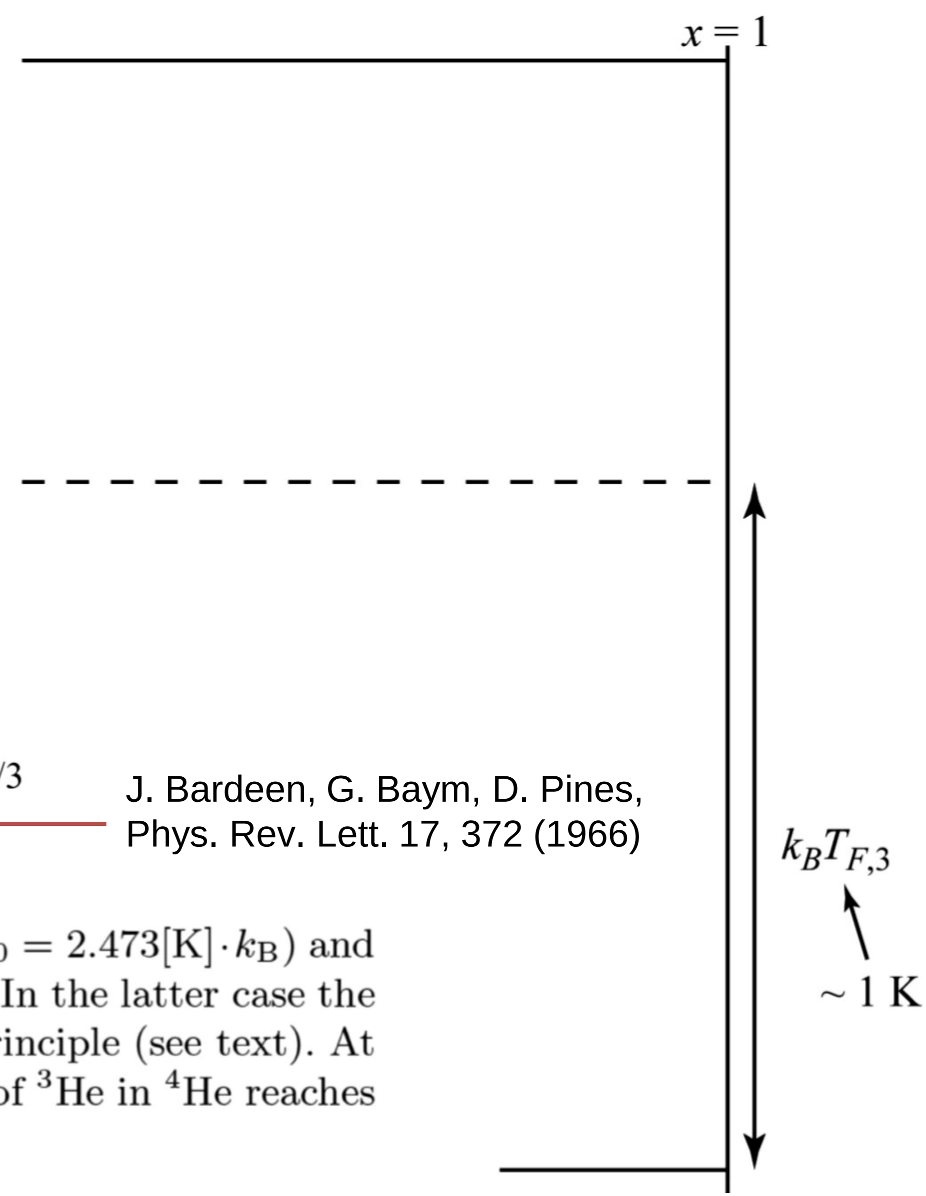
Helium isotopes	^{4}He	^{3}He
$T < 0.5$ K	– Bose condensate	– Fermi fluid
	– no thermal excitations	– $T_F \sim 1$ K
	– $\eta = 0$, $S \sim 0$, $C \sim 0$	– $C = \pi^2 R/2\ T/T_F$
		– $m^* \sim 2.8\ (3.0)\ m_3$

$$T_F = \frac{\hbar^2}{2m_3^* k_B}\left(3\pi^2 n_3\right)^{2/3}$$

Finite solubility of ^{3}He in ^{4}He at $T = 0$

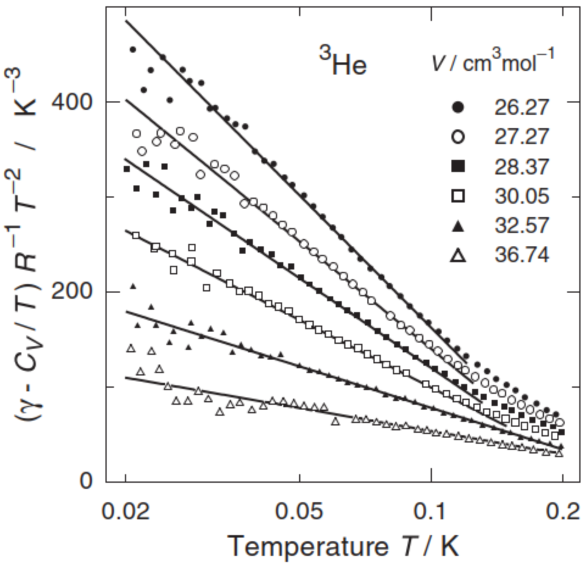
Stronger binding of ^{3}He atoms in liquid ^{4}He than in ^{3}He is expressed by $\varepsilon_{3,d}(0, 0) > L_3/N_0$



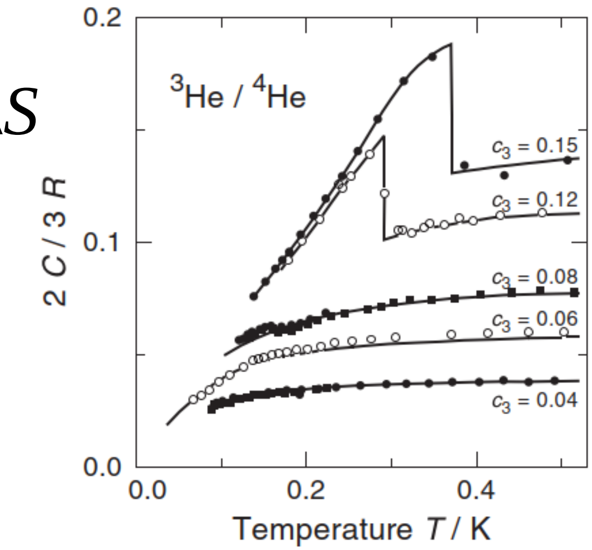


J. Bardeen, G. Baym, D. Pines, Phys. Rev. Lett. 17, 372 (1966)

Fig. 7.4. Chemical potential of a ^{3}He atom in pure ^{3}He ($L_3/N_0 = 2.473[\mathrm{K}]\cdot k_\mathrm{B}$) and in ^{3}He–^{4}He mixtures as a function of the ^{3}He concentration x. In the latter case the chemical potential is enhanced by $k_\mathrm{B}T_\mathrm{F}(x)$ due to the Pauli principle (see text). At the limiting concentration $x_{3,\mathrm{d}} = 6.6\%$ the chemical potential of ^{3}He in ^{4}He reaches the chemical potential of pure ^{3}He



Heat of mixing $L_m = T\Delta S$



$$C = \frac{dQ}{dT} = \frac{TdS}{dT}$$

$$dS = \frac{C}{T}dT \qquad S(T) = \int_0^T \frac{C}{T}dT$$

$$C_3 = 2.7\,RT = 22\;T\;\mathrm{J/(mol\;K^2)}$$

$$S_3 = 22\;T\;\mathrm{J/(mol\;K^2)}$$

– empirical fact

– no first-principles theory

$$C_V = \gamma T + \Gamma T^3 \ln\left(\frac{T}{\Theta_c}\right)$$

Fermi-liquid theory works fine: $C_d =$

$$N_A k_B \frac{\pi^2}{2}\frac{T}{T_F} = \frac{0.745\;\mathrm{J}}{\mathrm{mol\;K}}\;\frac{T}{\mathrm{K}}\;\frac{m^*}{m_3}\left(\frac{V_m\;\mathrm{mol}}{x\;\mathrm{cm}^3}\right)^{2/3}$$

$x = 0.066, \quad m^* = 2.5\,m_3$

$x \rightarrow 0, \quad m^* = 2.34\,m_3$

=> $C_d\,(0.066) = 106\;T\;\mathrm{J/(mol_3\;K^2)}$

=> $S_d\,(0.066) = 106\;T\;\mathrm{J/(mol_3\;K^2)}$

THUS: $L_m = T\Delta S = T^2\,(106 - 22)\;\mathrm{J/(mol\;K^2)} = 84\,(T/\mathrm{K})^2\;\mathrm{J/mol}$

Cooling power

$$\dot{Q} = \dot{n} L_m = 84\, \dot{n}\, (T/\mathrm{K})^2 \ \mathrm{J/mol}$$

Compare **evaporation** vs. **dilution** $(\dot{Q}_i = \dot{n} L_i)$

evaporation	dilution
$\dot{n} \propto e^{-L_e/RT}$	$\dot{n}$ indep. of T
L_e indep. of T	$L_m \propto T^2$

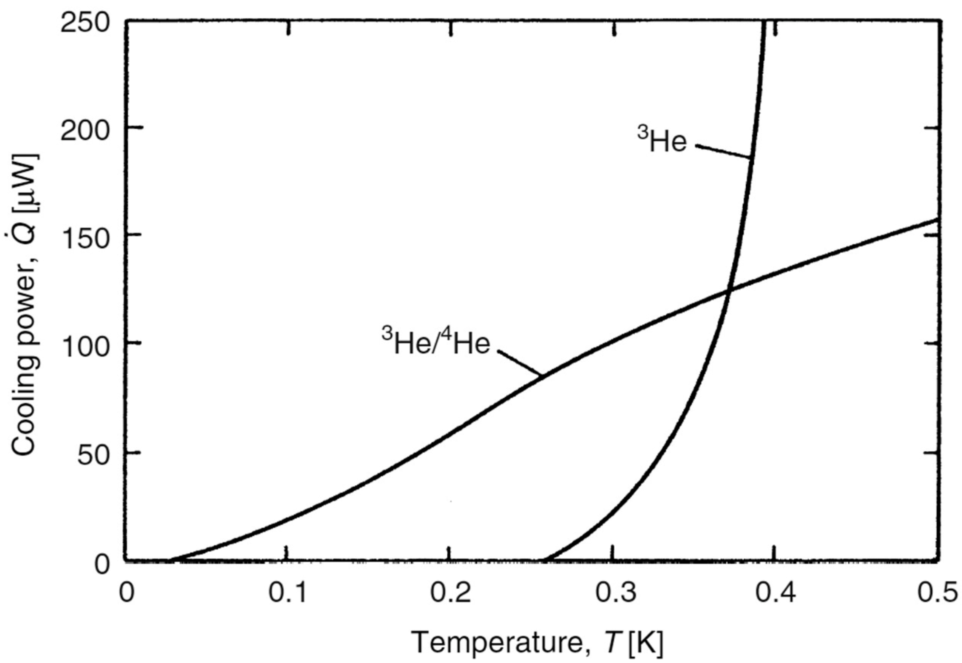
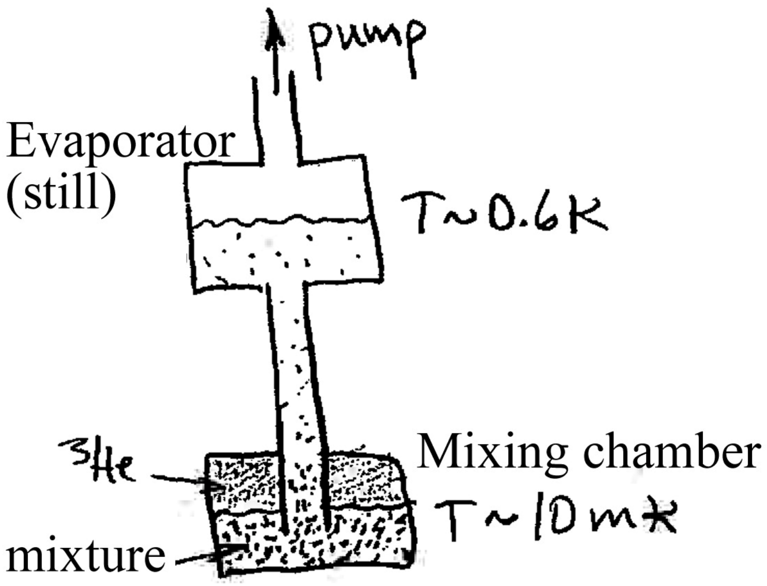


Fig. 7.3. Cooling power of a ^{3}He evaporation cryostat and of a ^{3}He–^{4}He dilution refrigerator, assuming that the same pump with a helium gas circulation rate of $5\,l\,\mathrm{s}^{-1}$ is used [7.9]

Osmotic pressure

Large thermal gradient in DR causes concentration gradients, $x(T)$



Ideal mixture model:

$T > T_F \qquad \pi V_{m,4} = x R T$

$T < T_F \qquad \pi V_{m,4} = 0.4\, R\, T_F$

π is osmotic pressure

Equilibrium:
$\pi(z)$ is constant
=> concentration gradient is opposite to
dT/dz

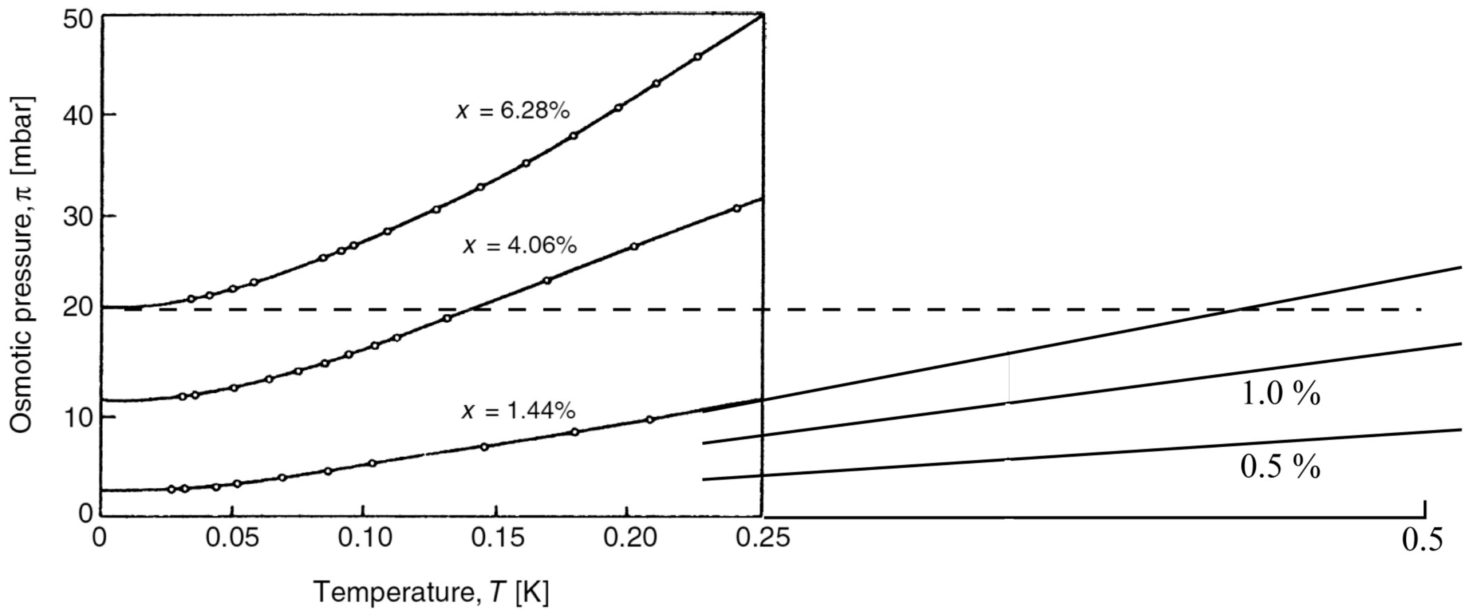


Fig. 7.7. Osmotic pressures of some dilute ^3He–^4He mixtures at a pressure of 0.26 bar (from [7.11] who used the data of [7.31])

Mixing chamber

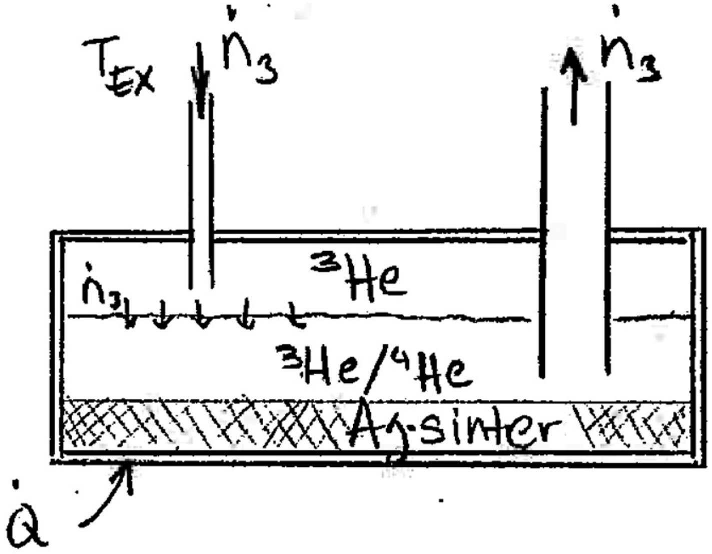
Power balance:

Enthalphy $\Delta H = c_p \Delta T$

$$\dot{Q} = \dot{n}_3 L_m - 11(T_{EX}^2 - T_{MC}^2)\ \mathrm{J/(mol\,K^2)}$$

$$L_m = 84 T_{MC}^2\ \mathrm{J/(mol\,K^2)}$$

$$\Rightarrow \quad \dot{Q} = \dot{n}_3 (95 T_{MC}^2 - 11 T_{EX}^2)\ \mathrm{J/(mol\,K^2)}$$



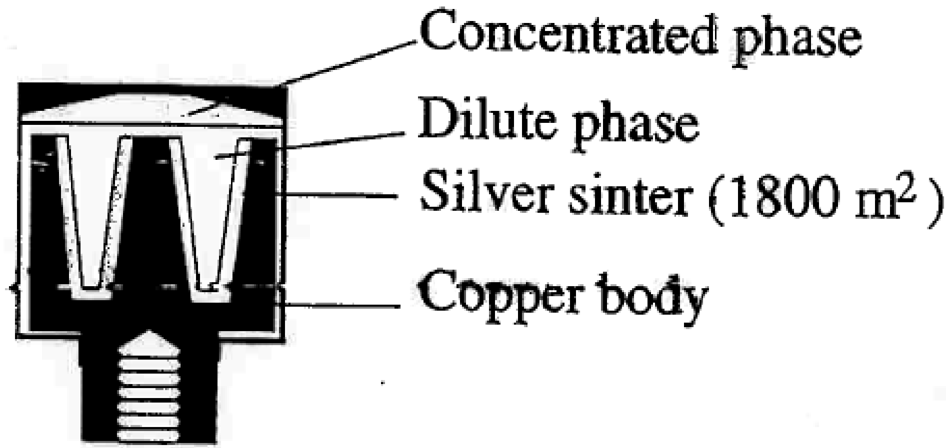
If $T_{EX} = T_{MC}$ (or operation in single cycle)

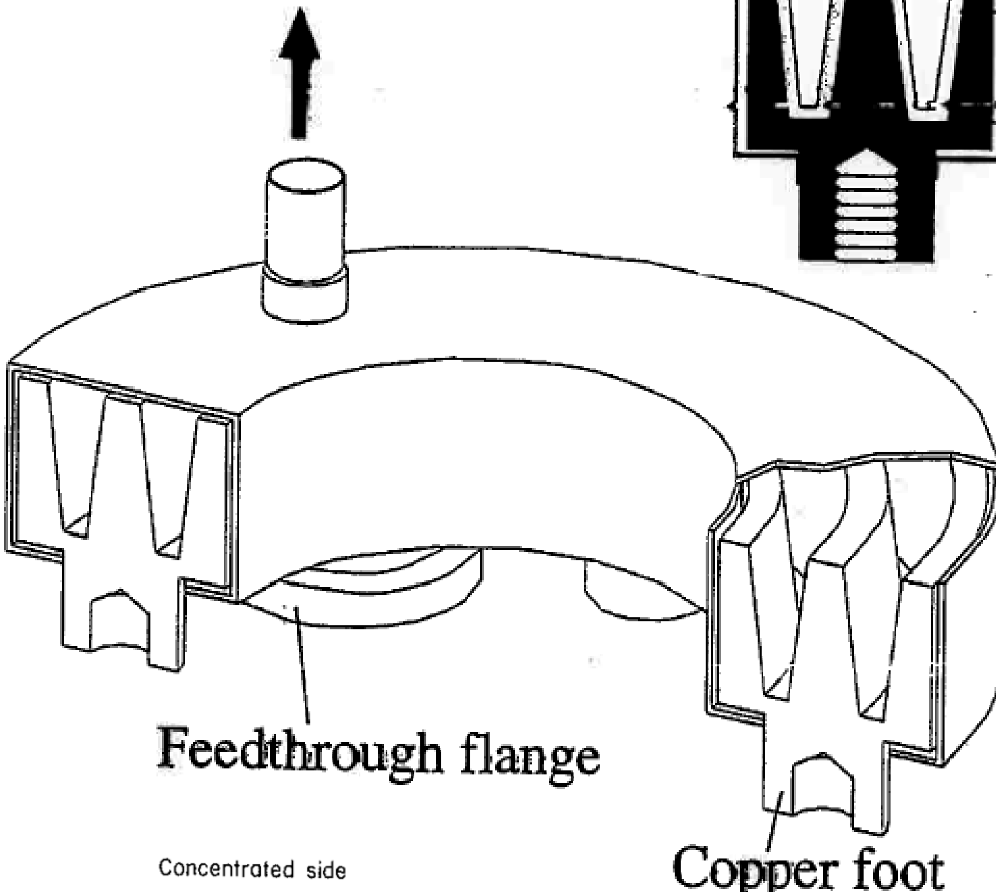
$$\dot{Q} = \dot{n}_3 84 T_{MC}^2\ \mathrm{J/(mol\,K^2)}$$

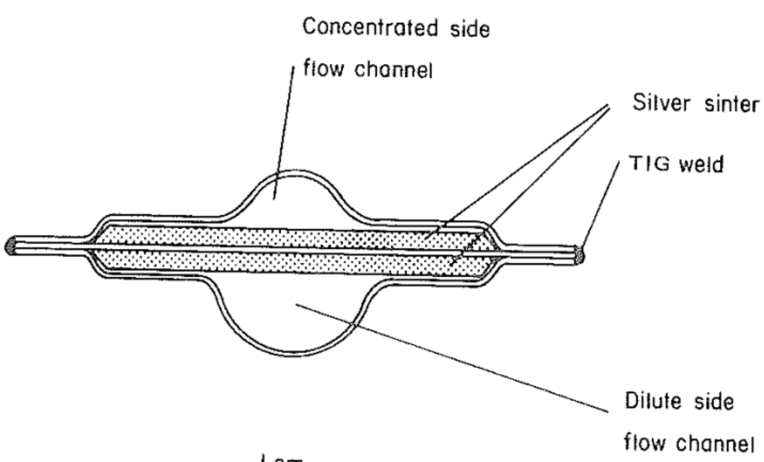
if $\quad \dot{Q} \to 0 \quad T_{MC} = T_{EX} / 2.9$

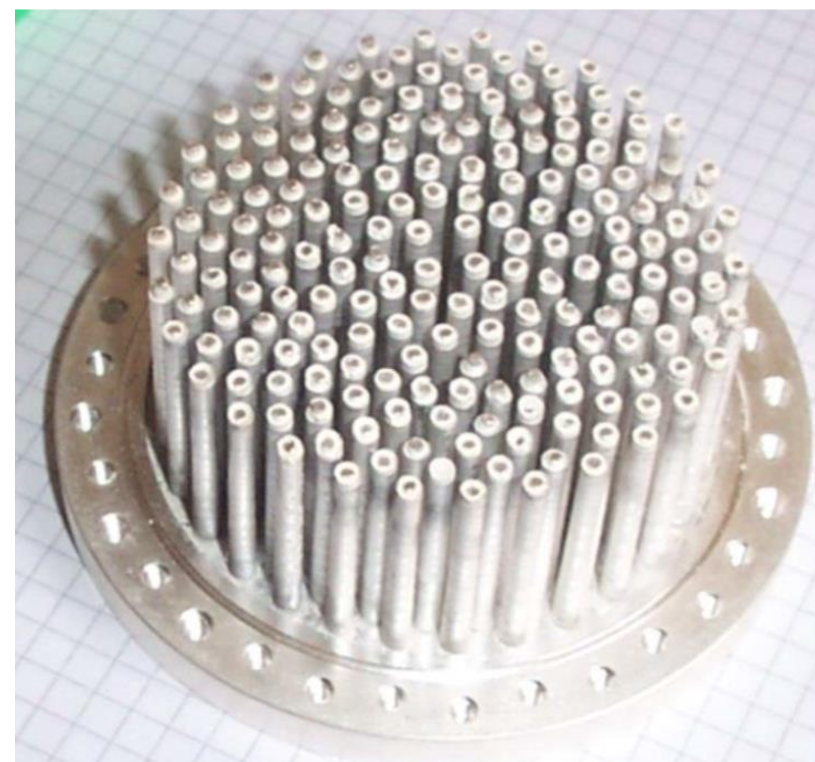
=> best possible heat exchangers to obtain minimum T_{EX}

Examples on real geometry



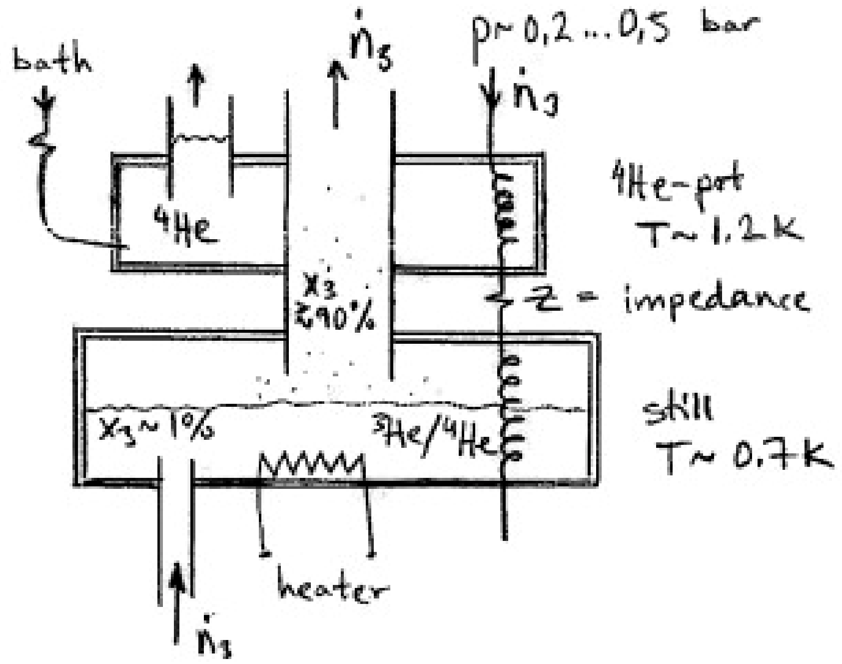






Kurt Uhlig, arXiv 1412.3597

Distiller (still):



Optimal still temperature is 0.6 – 0.7 K

T/K	x_3 /%	$p_3 + p_4$ /Pa	$p_3/(p_3 + p_4)$ /%
0.6	1.2	4.6	99
0.7	1.0	8.8	97

– ^{3}He coming from RT is condensed in ^{4}He evaporator (pot), T ~ 1.2 K
– further cooling of returning ^{3}He occurs in still, T ~ 0.7 K
– simple spiral tube heat exchangers
– impedance $Z \sim 10^{12}$ cm^{-3}

Still must be heated up:

– latent heat of ^{3}He evaporation is L_3 ~ 20 J/mol

– **incoming ^{3}He flow gives a load ΔH_3 ~ 2 J/mol**

– additional heating dQ/dt ~ 18 dn/dt J/mol (~ few mW typically) must be provided to keep up circulation

– still is a good thermal-anchor point (**additional load up to ~ 1 mW** is OK)

Important to prevent ^{4}He from evaporating in the still. **Superfluid film may creep** up to a point where T ~ 2 K => higher vapor pressure

– may become **as much as 10 ... 20 %** unless attention is paid

– pumps are loaded by additional mass flow

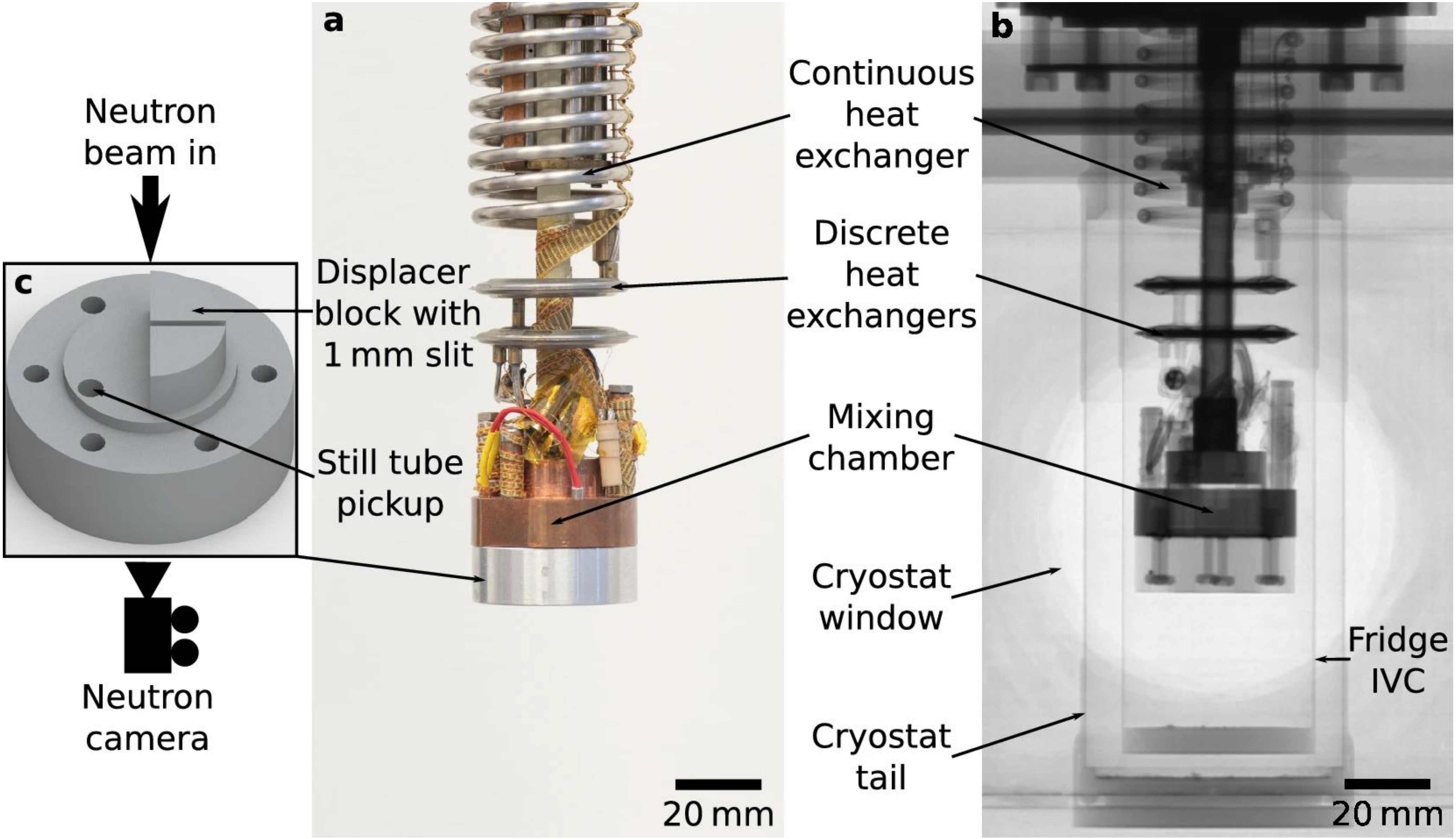
– heat exchangers are loaded as **mixture has higher heat capacity than pure ^{3}He**

– before MC ^{3}He separates from ^{4}He and produces heating (reverse to mixing)

– ^{4}He may accumulate to heat exchangers if geometry is not right (**gravity**)

– down at the mixing chamber extra ^{4}He does no harm, except that some ^{3}He is away from taking part in the mixing process

– **check ^{3}He/^{4}He ratio in circulation by a leak detector** (mass spectrometer)



https://www.nature.com/articles/s41598-022-05025-0#Sec13

Lawson, C.R., Jones, A.T., Kockelmann, W. *et al.* Neutron imaging of an operational dilution refrigerator. *Sci Rep* **12**, 1130 (2022). https://doi.org/10.1038/s41598-022-05025-0