PHYS-E055102, Low Temperature Physics, Exercise 6 DL 19.10.2022

1. Derive an expression for the coefficient of performance (COP) of an ideal pulse tube refrigerator. The schematic of the setup is shown in Fig. 1. You can assume the working fluid to be ideal gas.

2. ^{3}He circulating in a dilution refrigerator is transported from the heat exchangers to the mixing chamber in a tube, which is $l = 30\,\mathrm{cm}$ long and has a diameter of $d = 1\,\mathrm{mm}$. The circulation is $0.2\,\mathrm{mmol/s}$. What is the lowest attainable temperature if the mixing chamber heat load is only due to the viscous flow of ^{3}He in the tube? You can use $\eta_3 = 0.33 \cdot 10^{-6}\ (\mathrm{K}/T)^2\,\mathrm{Pa\,s}$ for ^{3}He viscosity.

3. Dilute ^{3}He - ^{4}He mixture leaving the mixing chamber is lead to the heat exchangers in a tube, which is $l = 3\,\mathrm{cm}$ long and has a diameter of $d = 1\,\mathrm{mm}$. Osmotic pressure drives the ^{3}He in this tube. The flow causes a resisting pressure gradient. What is the ^{3}He concentration difference between the upper and lower ends of the tube in an equilibrium state? The ^{3}He circulation is $0.2\,\mathrm{mmol/s}$ and the temperature of the mixing chamber is $6\,\mathrm{mK}$. The viscosity of the dilute phase at $6.6\,\%$ concentration is $\eta_d = 0.05 \cdot 10^{-6}\ (\mathrm{K}/T)^2\,\mathrm{Pa\,s}$. Neglect all heating effects.

4. Analogous to the lecture's derivation of the compressibility derive the formula for the spin susceptibility χ in the Landau theory of the Fermi liquid. As a starting point calculate the caused change to the energy distribution function. Assume the field is applied only in z direction and the quasiparticle energies change by a fixed amount $-\frac{1}{2}\hbar\gamma\sigma_z H$ (Zeeman coupling, γ is the gyromagnetic ratio, σ_z is the Pauli matrix and H the external uniform magnetic field).

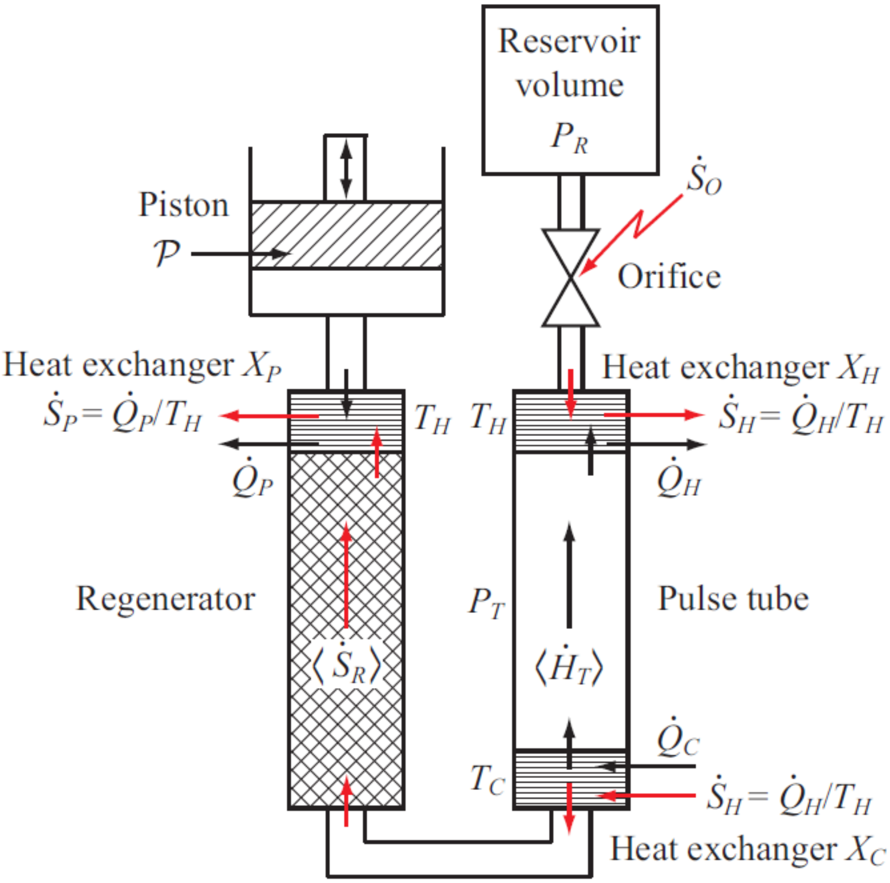

Figure 1: Single orifice pulse tube refrigerator. Energy flows are indicated with black arrows and entropy flows with red arrows.