

# EEN-1020 Heat transfer

## Week 4: Convective Heat Transfer

### External Flow

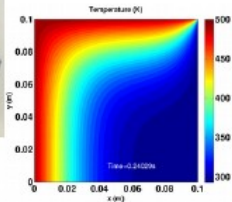
Prof. Ville Vuorinen  
November 15<sup>th</sup> - 16<sup>th</sup> 2022  
Aalto University, School of Engineering

**Week 1:** Energy conservation, heat equation, conduction  
Fourier/Newton

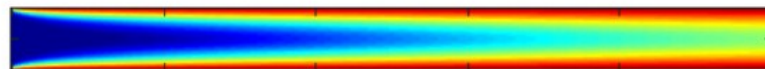


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x} \left( k \frac{\partial T(x+\Delta x/2, t)}{\partial x} - k \frac{\partial T(x-\Delta x/2, t)}{\partial x} \right)$$

**Week 2:** Fin theory, conduction, intro to convection

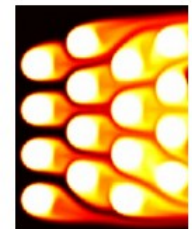


**Week 3:** convective heat transfer – internal flow (channel)



**Week 4:** convective heat transfer – external flow (fin systems)

**Week 5:** natural convection, boiling, correlations





On the heat transfer course, we have “5 friends”  
i.e. 5 main principles that are used to explain  
heat transfer phenomena

- 1) Energy conservation: “J/s thinking”
- 2) Fourier’s law
- 3) Newton’s cooling law
- 4) Energy transport equation – convection/diffusion equation
- 5) Momentum transport equation – Navier-Stokes equation



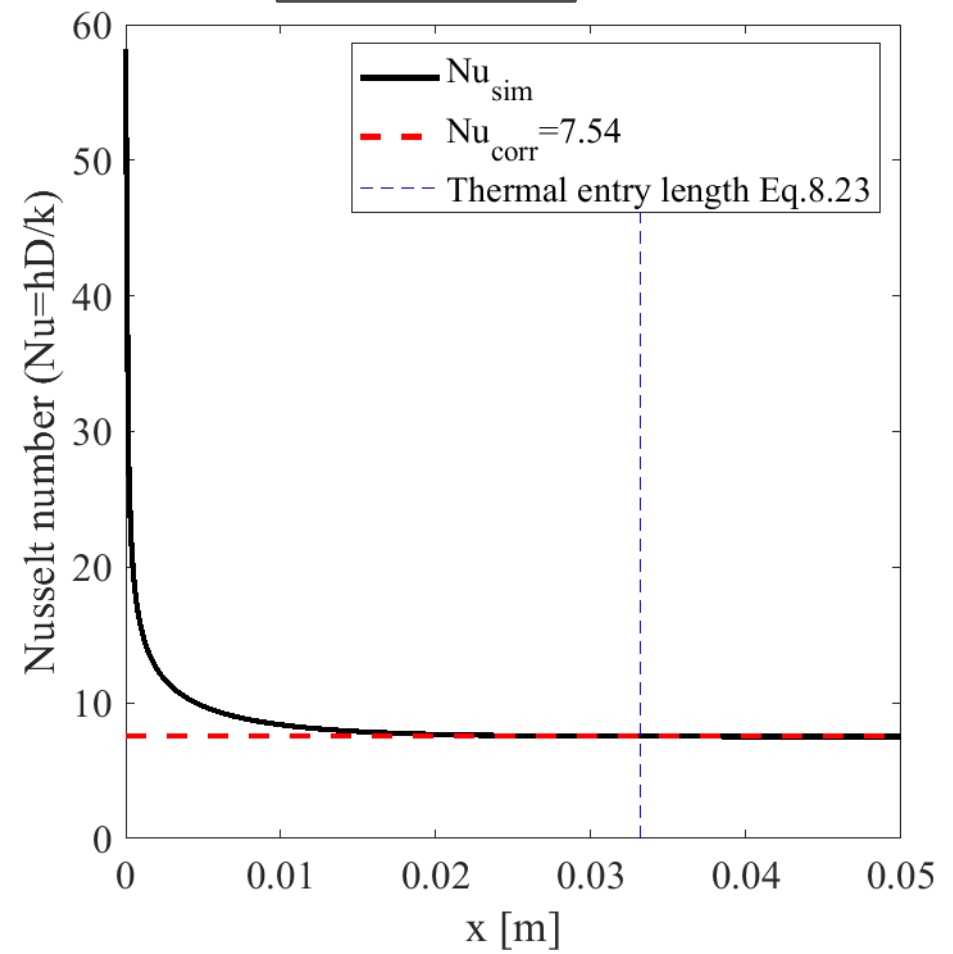
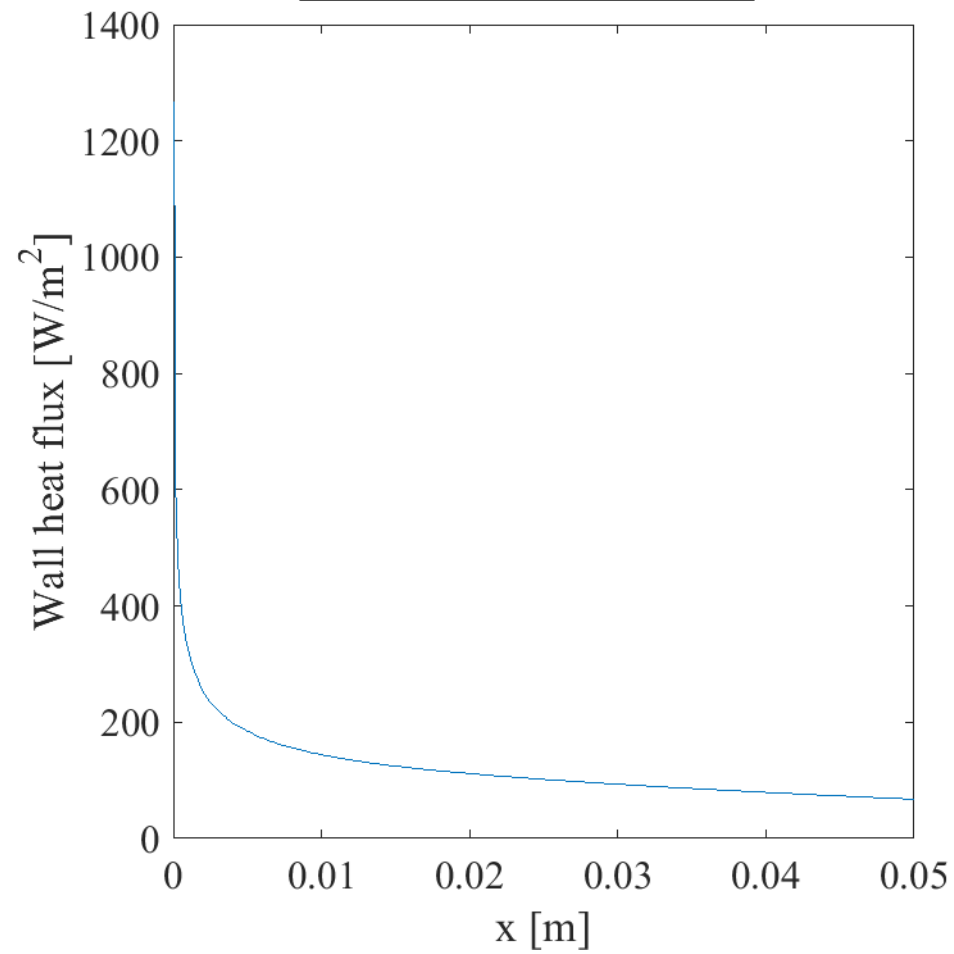
**Small recap from last week on wall heat flux  
 $q_{wall}(x)$  along channel for laminar fluid flow**



For const. wall temperature laminar channel flow, the local wall heat flux is needed to get energy balance to find the local Nusselt number

$$q_{wall}(x) = k_f \left( \frac{\partial T}{\partial y} \right)_{wall}$$

$$Nu(x) = \frac{hD}{k_f}$$





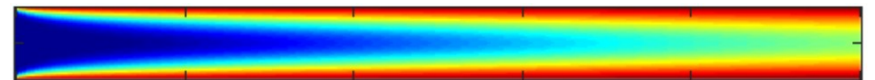
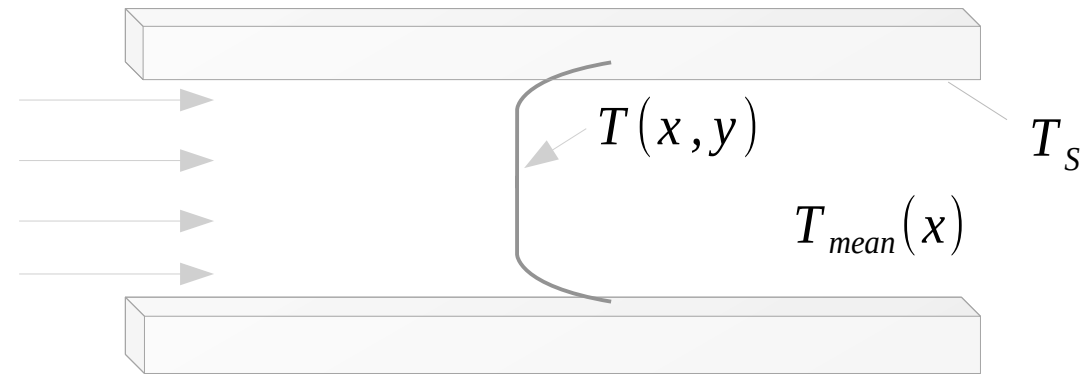
## Relevance to HW3 - Heat flux balance at the surface

Diffusive heat flux (Fourier) immediately at the wall on the fluid side =  
Heat flux from Newton's law of cooling

$$k_f \left( \frac{\partial T}{\partial y} \right)_{y=wall} = h(T_s - T_{mean})$$

We can now calculate  $h=h(x)$   
(W/m<sup>2</sup>K) at every point along  
the surface.

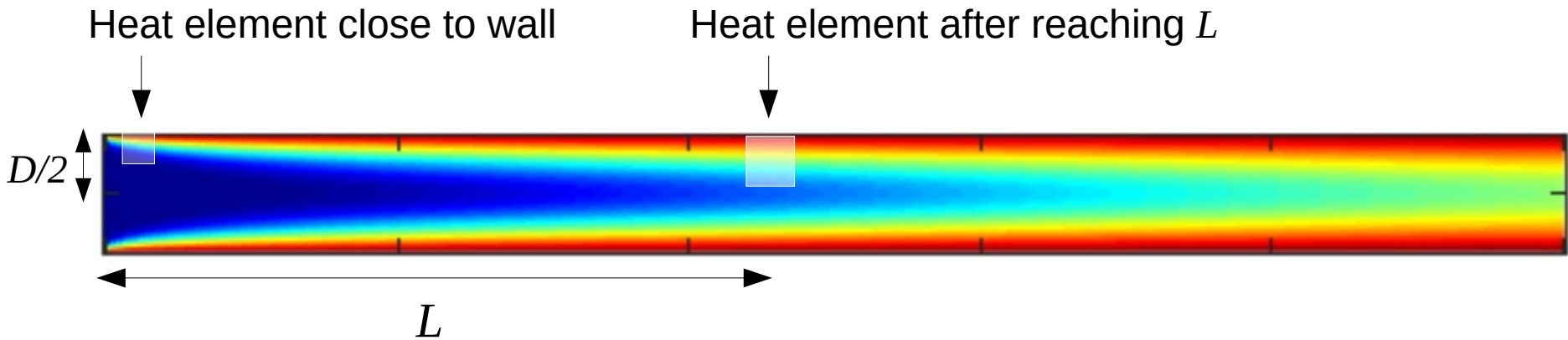
$$h = \frac{k_f \left( \frac{\partial T}{\partial y} \right)_{y=wall}}{T_s - T_{mean}}$$



**Note:** even in convective heat transfer  
the heat first purely diffuses i.e. conducts near the  
wall because  $u, v \rightarrow 0$  next to the wall.



# Thermal entry length derivation for laminar channel flow



- **Heat transport in  $y$ -direction:** Since  $V=0$  it follows that heat diffuses in  $y$ -direction.
- Diffusion time to center line can be estimated as:  $\tau_{diff} = (D/2)^2/\alpha$
- **Heat transport in  $x$ -direction:** During diffusion time heat is convected in  $x$ -direction distance  $L$
- Approximate convection time:  $\tau_{conv} = L/U$
- Equate:  $\tau_{conv} = \tau_{diff}$

*Our estimate:*  
 $(x/D) \approx 0.25 \text{Re}_D \text{Pr}$

*Text book formula for thermal entry length:*  
 $(x/D) \approx 0.05 \text{Re}_D \text{Pr}$

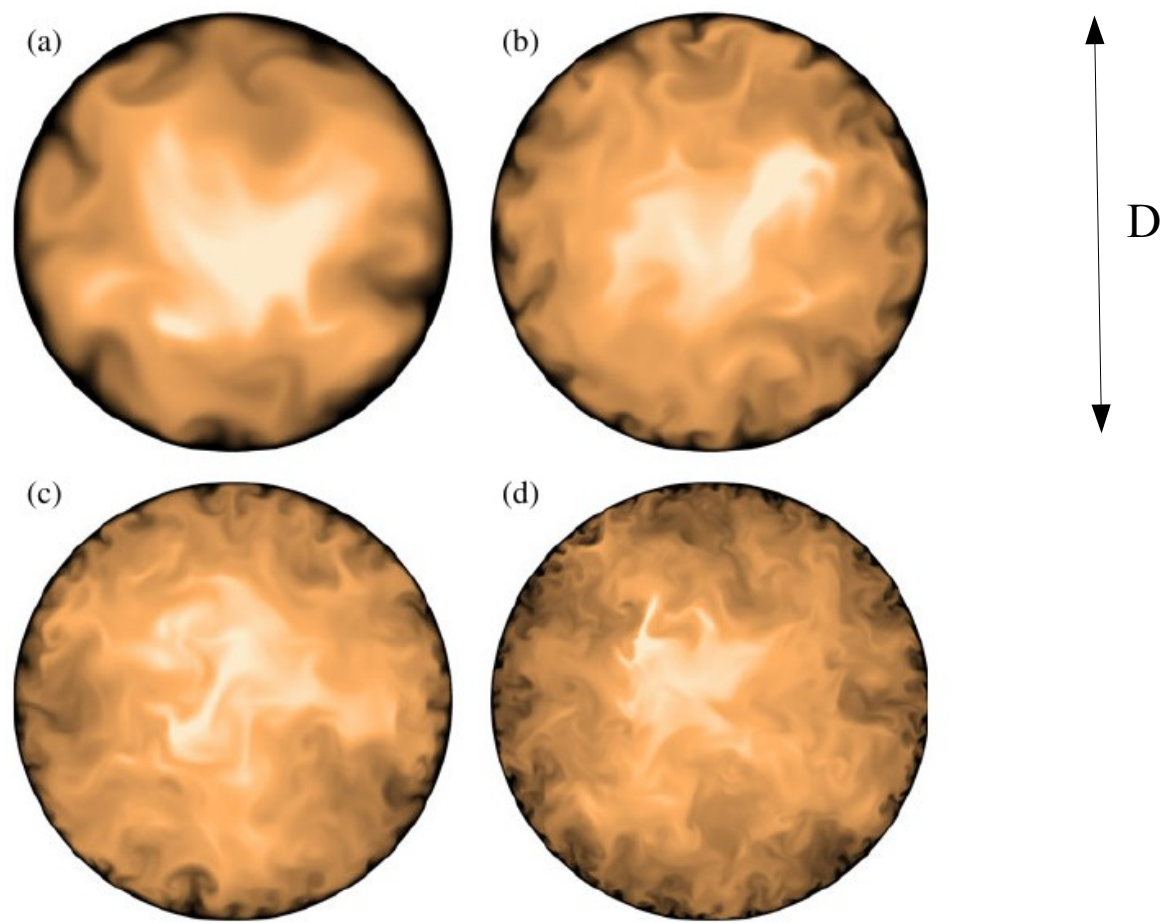
**Slide summary:** physics and way of thinking ok but the prefactor (0.25) is wrong (not 0.05) but explainable and refinable.



**Internal flow:** When Reynolds number increases large enough (below  $>5300$ ), pipe and channel flows become **turbulent**. Velocity field below indicates that also wall-normal velocity emerges along with chaotic swirling due to turbulence  $\rightarrow$  much higher heat transfer via enhanced convection and mixing

<https://link.springer.com/article/10.1007/s10494-013-9482-8>

$$Re = \frac{U D}{\nu}$$



**Fig. 4** Pseudo-colour visualisation of the instantaneous axial velocity  $u_z$  normalised by the bulk velocity  $U_b$ . **a**  $Re_b = 5,300$ ; **b**  $Re_b = 11,700$ ; **c**  $Re_b = 19,000$ ; **d**  $Re_b = 37,700$ . Here, the colours vary from 0 (black) to 1.3 (white)

El-Khoury et al. (2013)



# Lecture 4.1 Theory and experiment: Heat transfer of flow over a cylinder

**ILO 4:** Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.





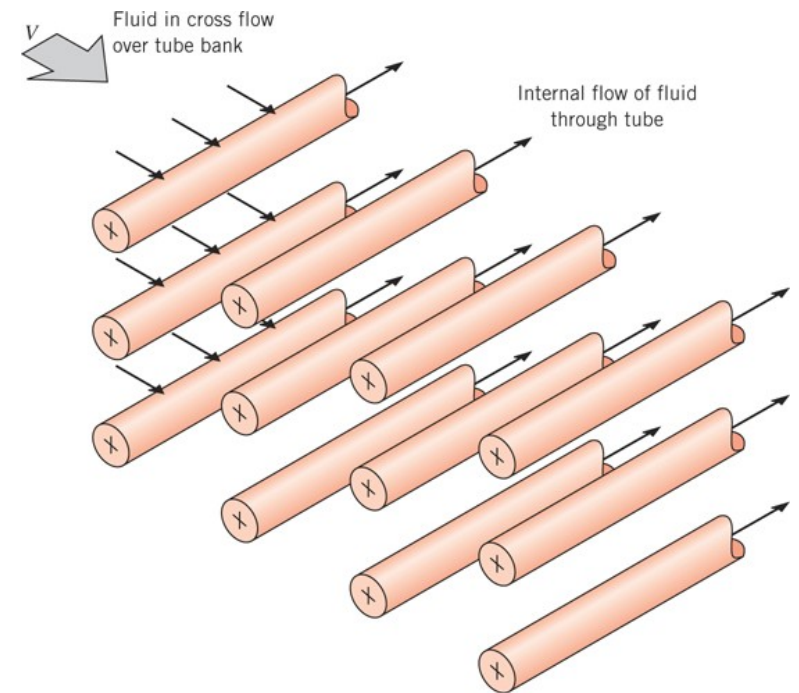
**Example:** Heat transfer increases when fluid velocity relative to the object increases → higher Reynolds number → higher heat transfer coefficient





# Cylindrical fins are common in energy applications

- Cylinder or pin fin beds very commonly used in heating and cooling applications
- E.g. cooling system in a ship where cool sea water is pumped through pipes and air is blown by fan over the array for AC cooling of cabins
- Understanding single cylinder heat transfer is really essential.

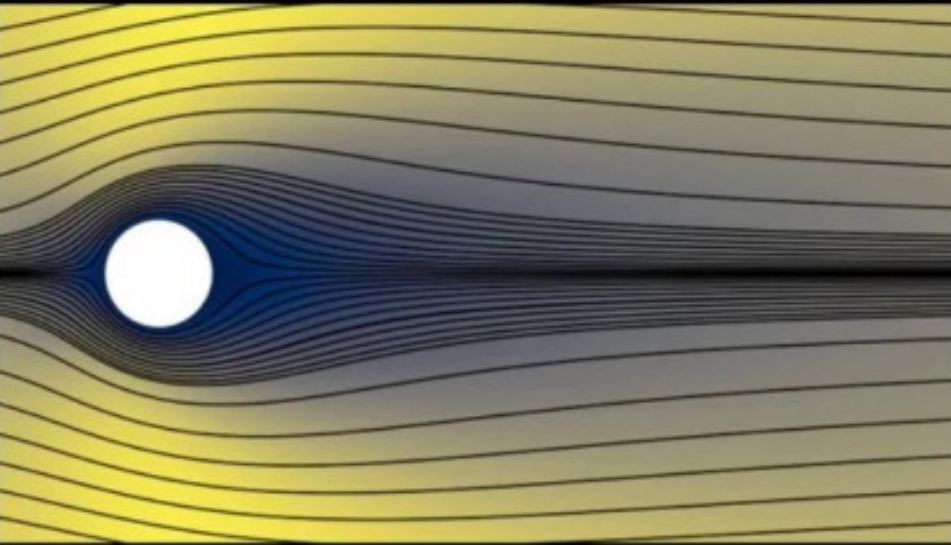




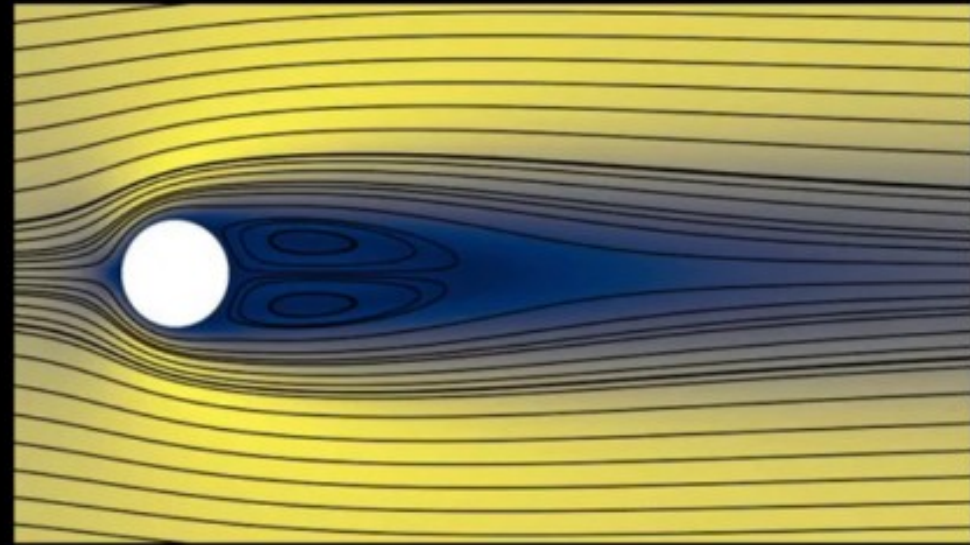
**External flows:** The fluid dynamical flow pattern over a cylinder depends on the Reynolds number ( $Re=UD/\nu$ ).

Video below courtesy of: J.Ramsay, M.Sellier, W.Ho

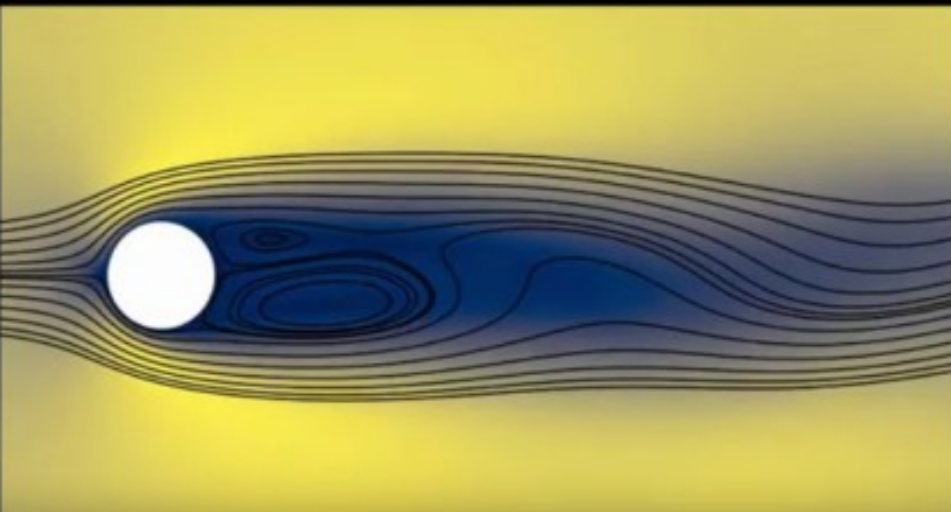
Attached Steady Flow,  $Re=4$



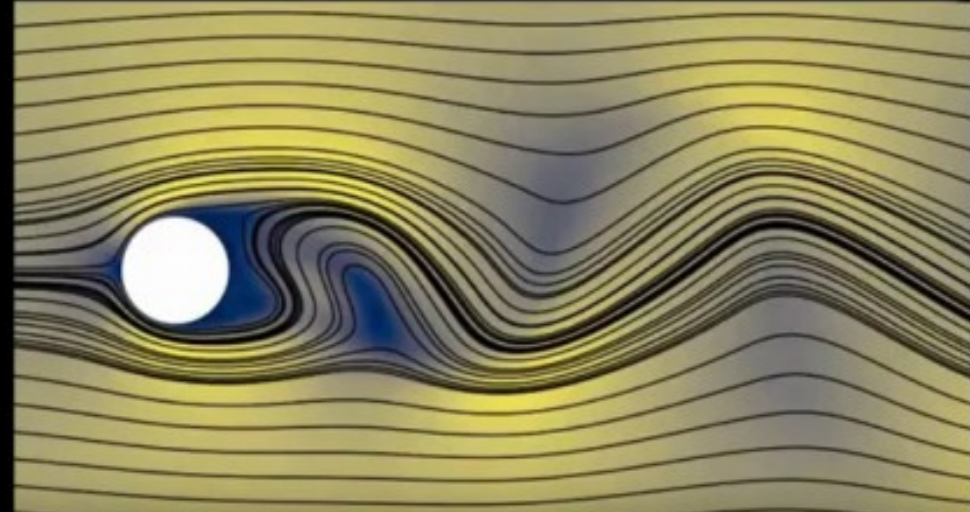
Separated Vortex Pair,  $Re=40$



Unsteady Vortex Shedding,  $Re=48$

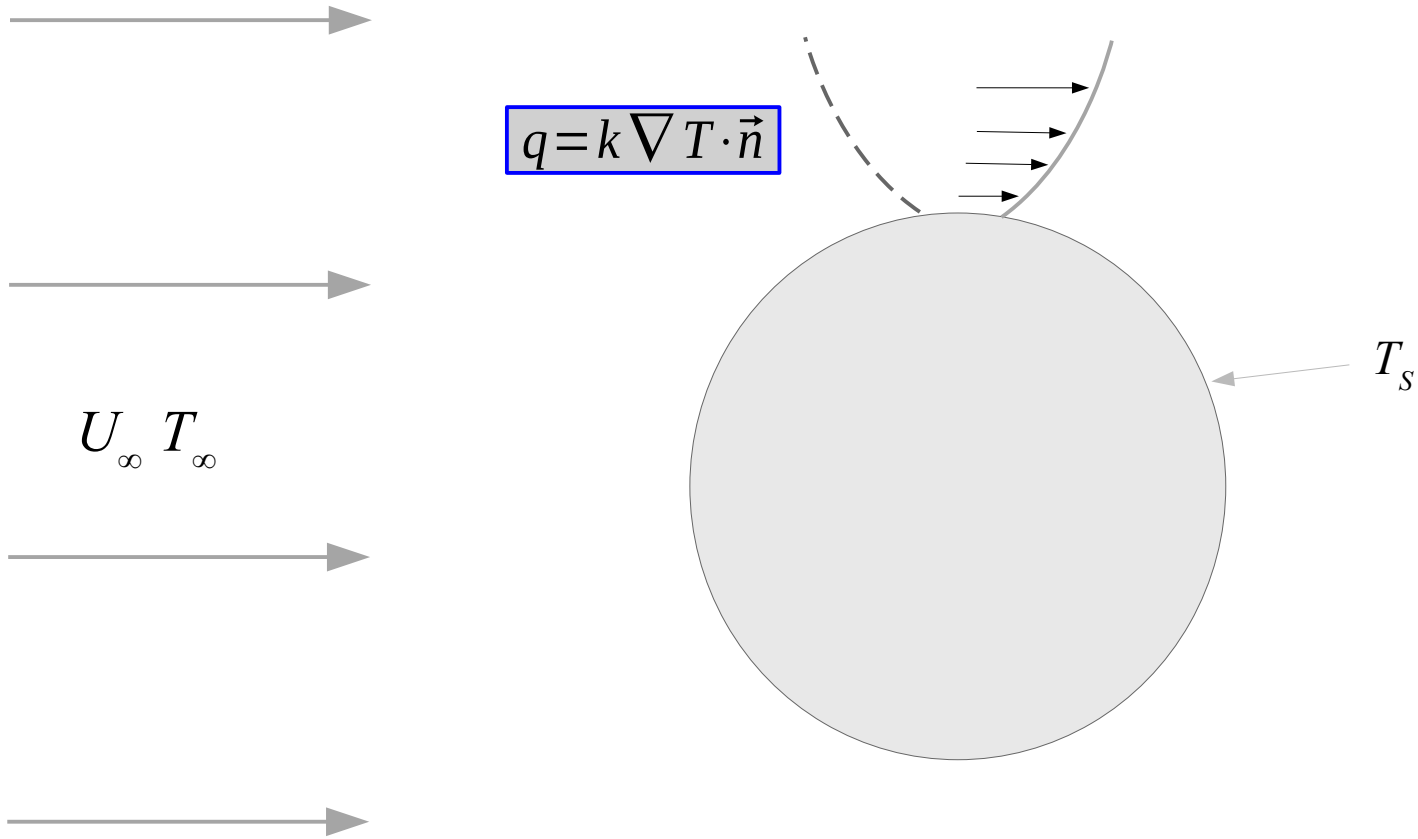


Fully Developed Vortex Shedding,  $Re=180$





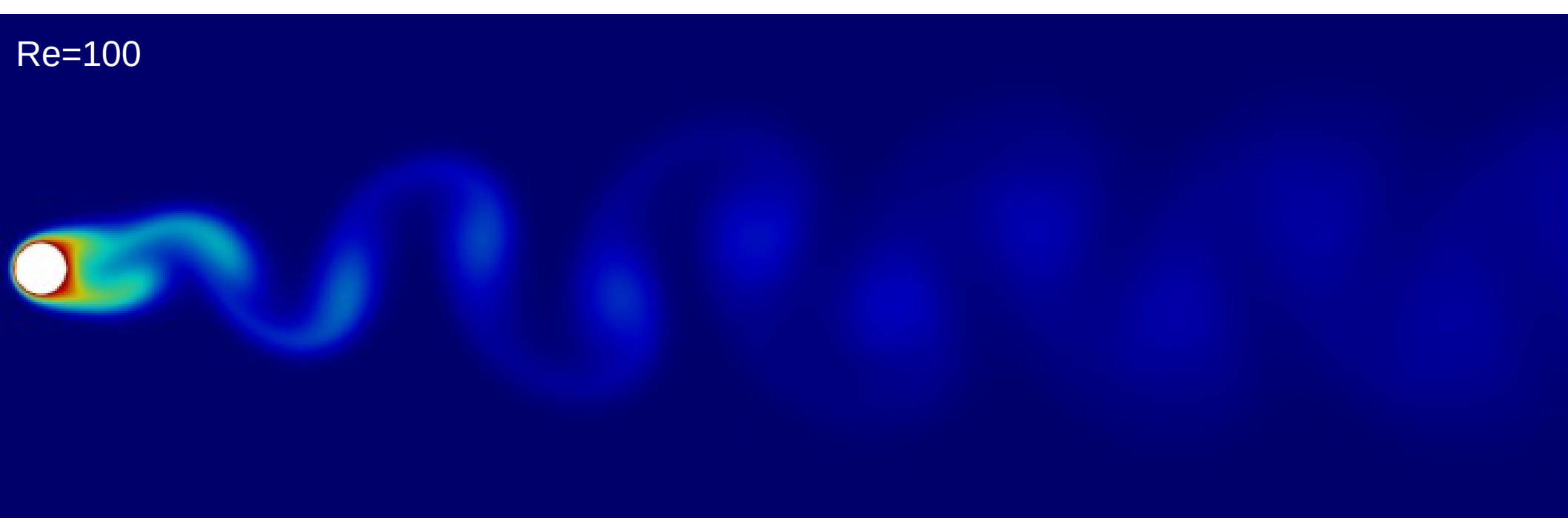
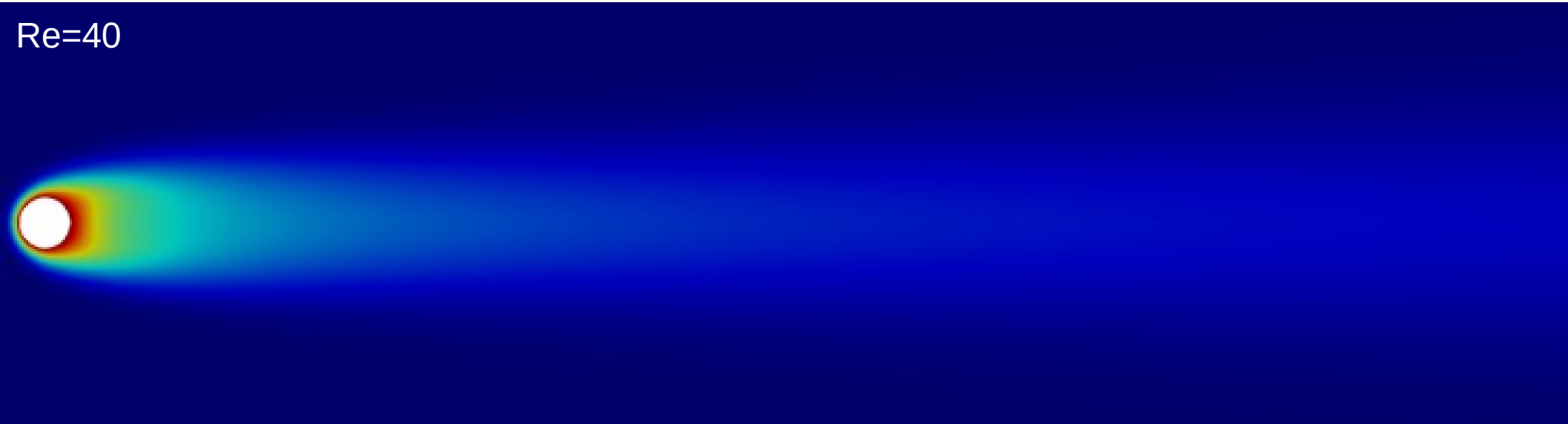
Heat transfer takes place on the surface of the cylinder






Two examples of instantaneous temperature fields from CFD simulation. **Note:** low  $Re \rightarrow$  thicker thermal boundary layer.

Courtesy of: Gizem Ersavas Isitman





# Energy balance (J/s thinking) for gas flow when the gas is heated at power $P$ (W). “Control volume” thinking.

$$Q_{\text{OUT}} - Q_{\text{IN}} = c_p \dot{m} \Delta T_{\text{ave}} = P$$

Mass flow rate of the gas (kg/s):

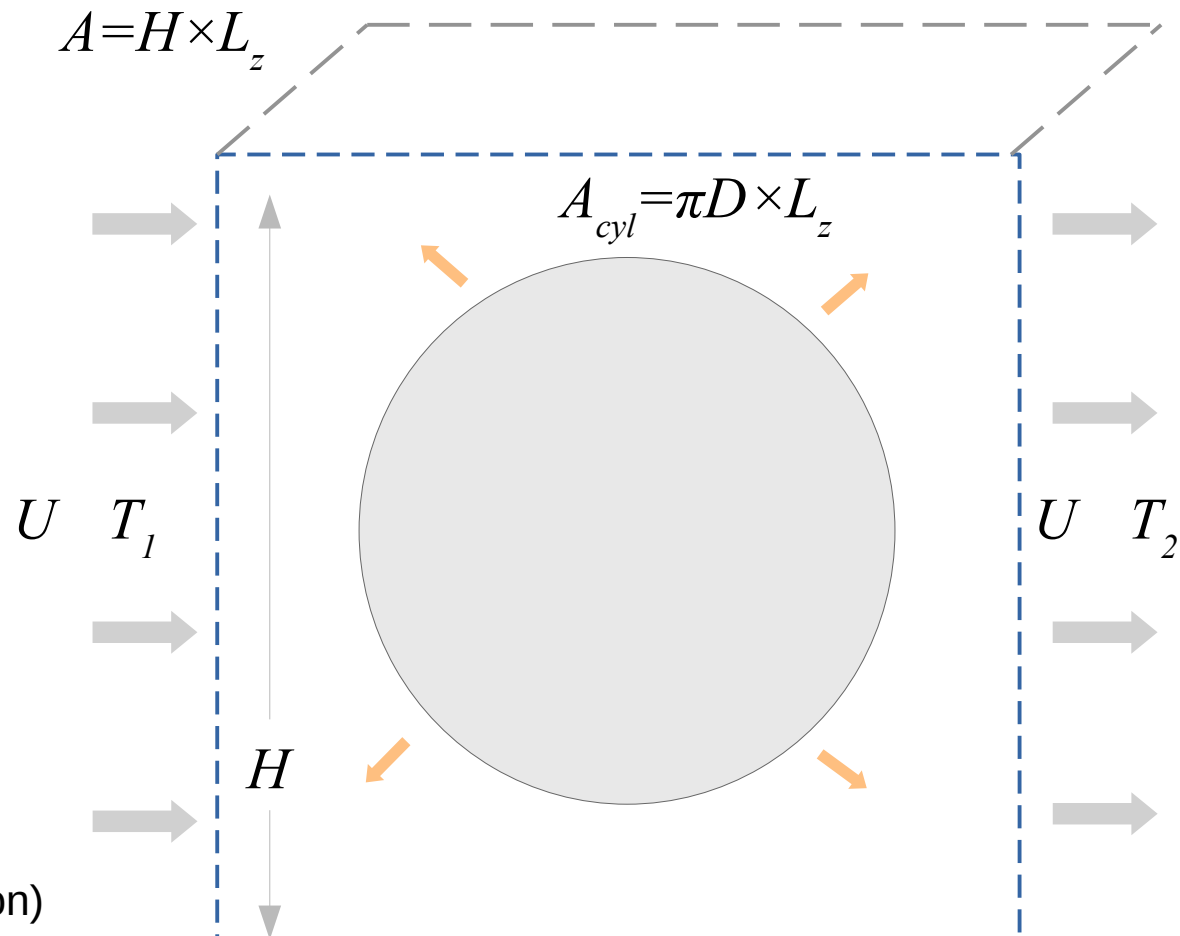
$$\dot{m} = \rho U A$$

Average temperature change:

$$\Delta T_{\text{ave}} = T_2 - T_1$$

$A$ =cross-sectional area (normal to flow direction)  
of control volume

$U$ =flow average velocity

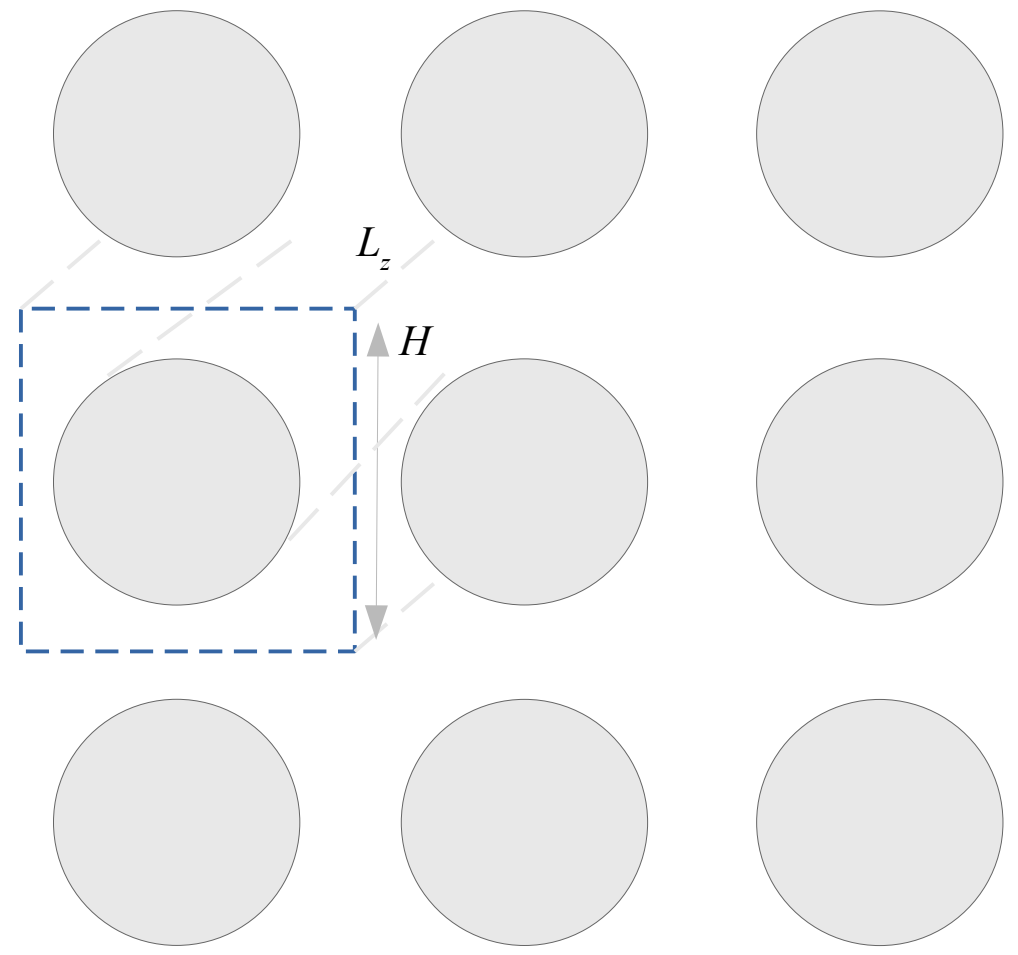
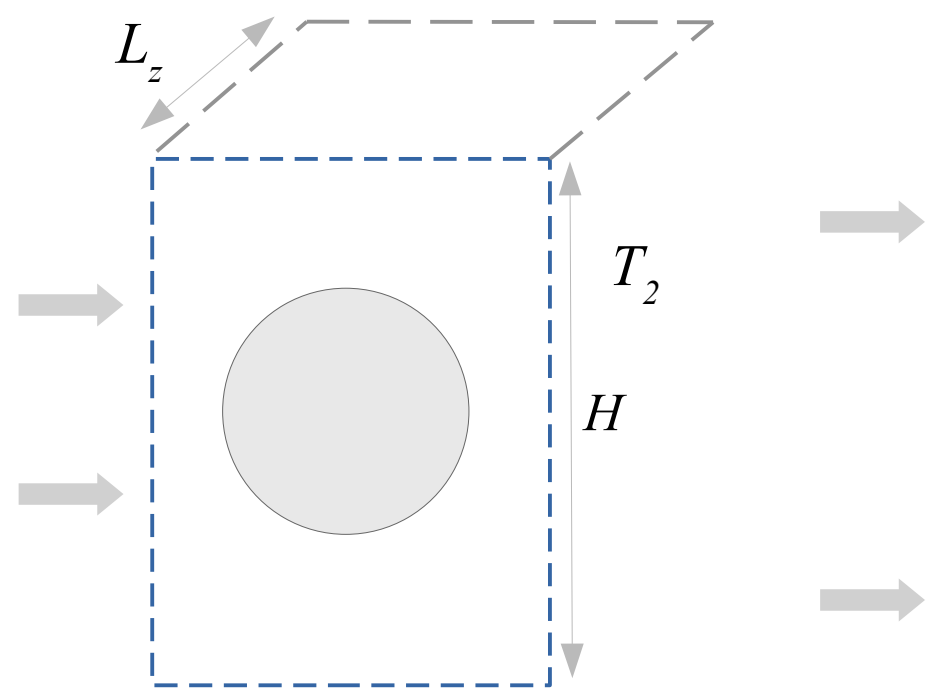




# Note: Choosing H has been a common source of confusion.

**Free cylinder:**  
 $T_2$  depends obviously on the chosen height of the control volume volume (same energy goes to smaller/larger volume).  $L_z$  cancels out (think 1m).

**Cylinder bed:**  
The natural height of control volume relates to the symmetry of the problem which enables estimation of temperatures inside and after pipe heat exchangers.

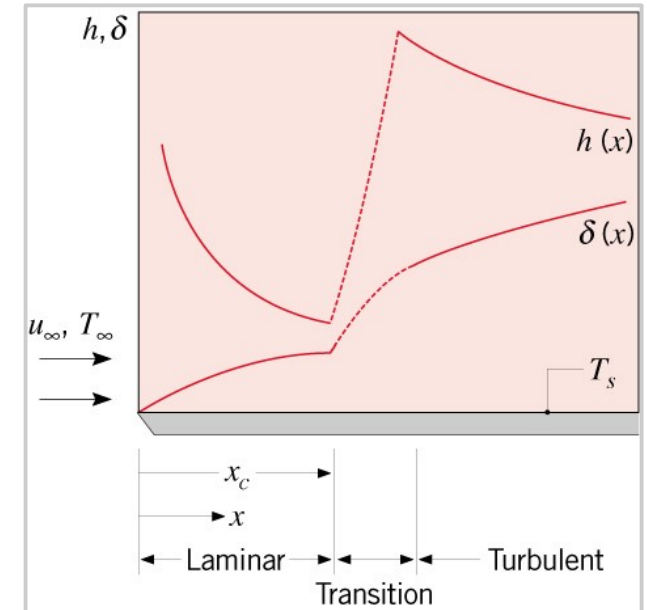
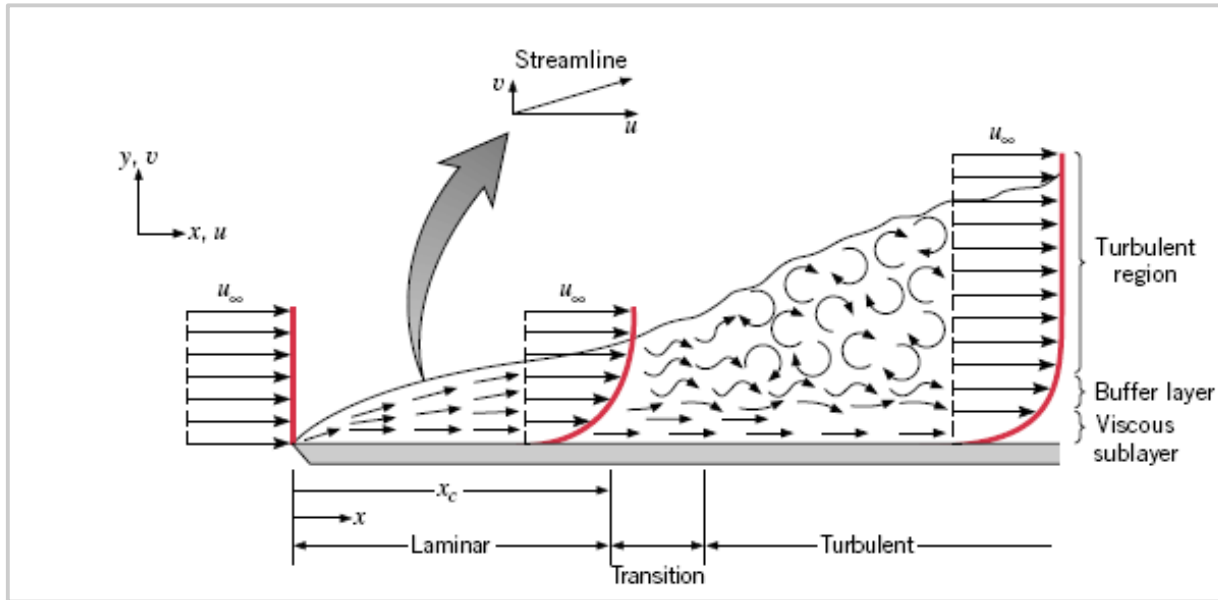


$$A = H \times L_z$$
$$A_{cyl} = \pi D \times L_z$$



**External flows:** Many turbulent flow systems pose laminar to turbulence transition region before turbulence has developed. Heat transfer differs between different regions → needs to be taken into account. Example: free boundary layer.

Here: “Free” refers to “free of the opposite wall”



$$Re_c = \frac{\rho U_\infty x_c}{\mu}$$

Critical Reynolds number at the onset of laminar to turbulence transition.

$$10^5 < Re_c < 3 \cdot 10^6$$

$$U/U_\infty = 0.99$$

$$\delta/\delta_T = Pr^{1/3}$$

Thickness ratio between viscous/thermal BL's depends on  $Pr$ .

$$Nu_x = h_x x / k = 0.332 Re_x^{1/2} Pr^{1/3}$$

Local Nusselt number scaling depends on  $Re_x$  and  $Pr$ .

$$\delta = \frac{5.0}{\left(\frac{U_\infty}{\nu x}\right)^{1/2}} = \frac{5x}{Re_x^{1/2}}$$

Laminar boundary layer thickness grows along the plate.





## Purpose of empirical heat transfer correlations:

For a broad range of Reynolds numbers and Prandtl numbers, express average heat transfer coefficient  $h$  in a non-dimensional form called Nusselt number:  $Nu = Nu(Re, Pr)$



# Different average Nusselt number correlations for heated cylinders have been developed

$$\overline{Nu}_D = \frac{\bar{h} D}{k}$$

Hilpert correlation (see Table 7.2)

$$\overline{Nu}_D = C Re_D^m Pr^{1/3}$$

Churchill & Bernstein correlation (broad applicability)

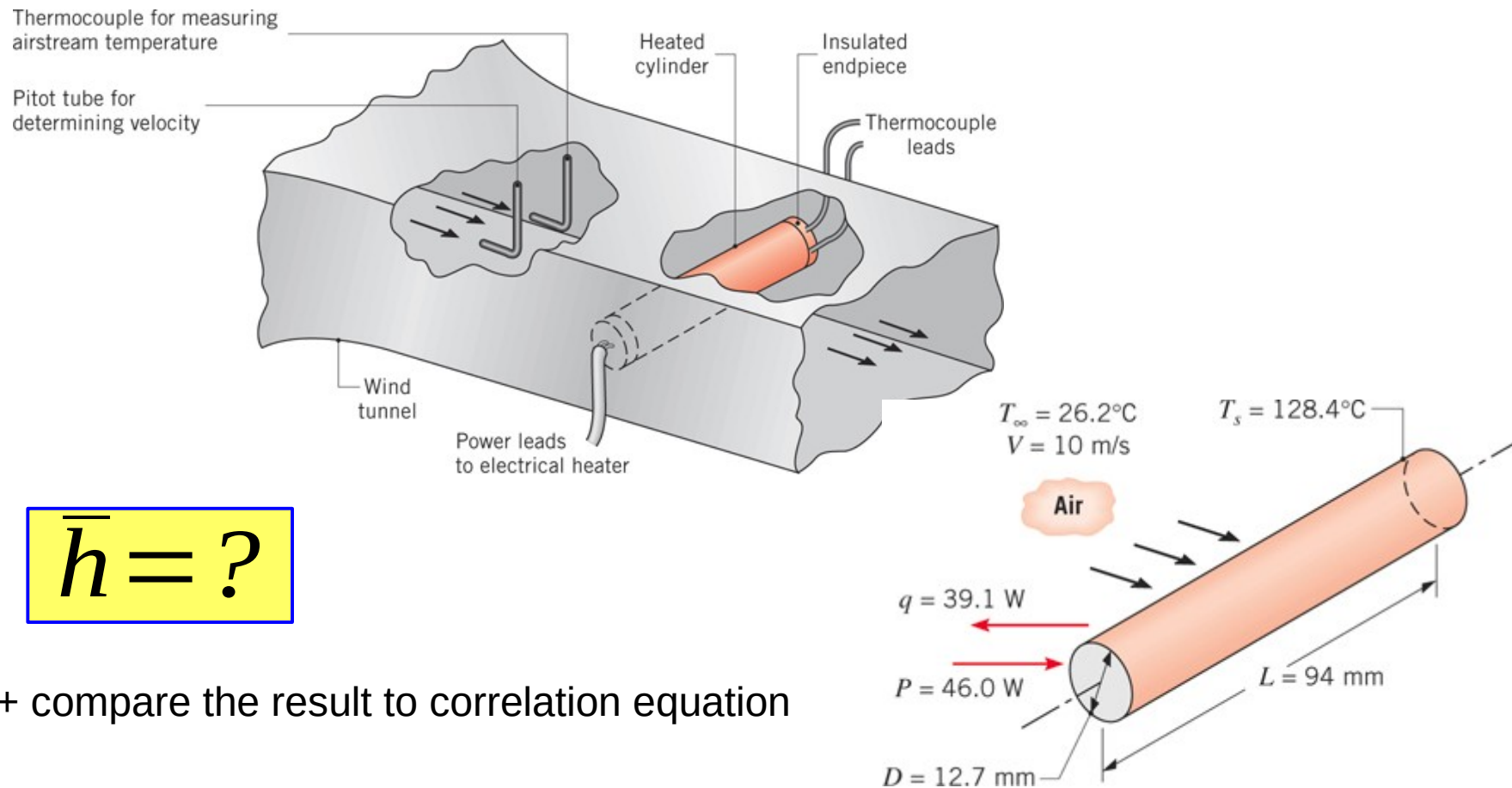
$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} [1 + (Re_D/282000)^{5/8}]^{4/5}$$

Zukauskas correlation (broad applicability, see Table 7.4)

$$\overline{Nu}_D = C Re_D^m Pr^{1/3} \left( \frac{Pr}{Pr_s} \right)^{1/4}$$



# Example 7.4: Heated cylinder in cross-flow



$$\bar{h} = ?$$

+ compare the result to correlation equation



First, we get the convection coefficient directly.

$$\bar{h} = \frac{q}{A(T_s - T_\infty)} = 102 \text{ W/m}^2 \cdot \text{K}$$

Zukauskas relation:

$$\bar{Nu}_D = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$$

$$Re_D = 7992 \rightarrow C = 0.26, m = 0.6, Pr = 0.7 < 10 \rightarrow n = 0.37$$

Table 7.2

Utilization of Zukauskas relation gives:

$$\bar{Nu}_D = 50.5$$

$$\bar{h} = \bar{Nu}_D k / D = 105 \text{ W/m}^2 \cdot \text{K}$$

$$\bar{h} = 105 \text{ W/m}^2 \cdot \text{K} \rightarrow \text{close to first observed value}$$

$$\bar{h} = 102 \text{ W/m}^2 \cdot \text{K}$$

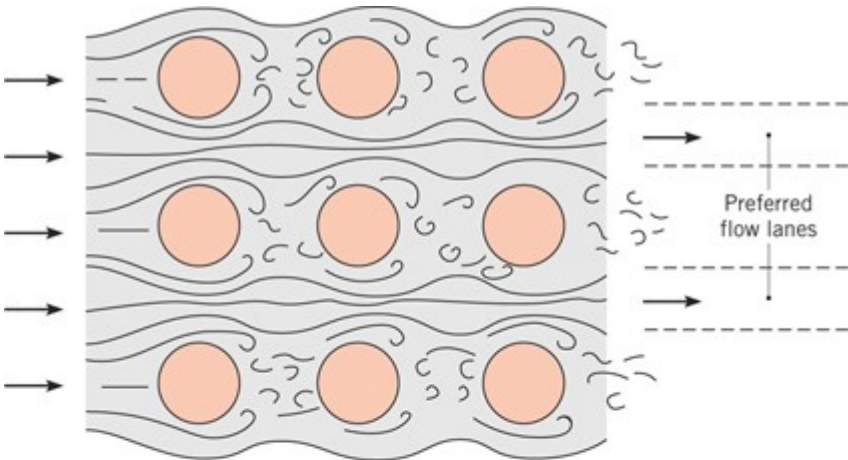


## Lecture 4.2 Numerical approach: 2d heat transfer over a fin bed using Matlab

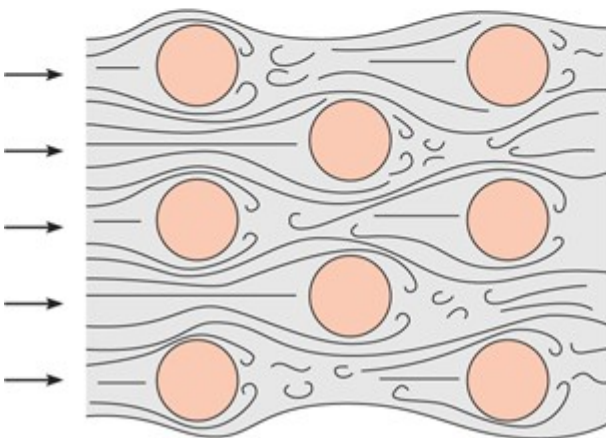
**ILO 4:** Student can formulate energy balance for external flow heat transfer systems and use basic correlations. The student can confirm the analysis using generated/provided simulation data.



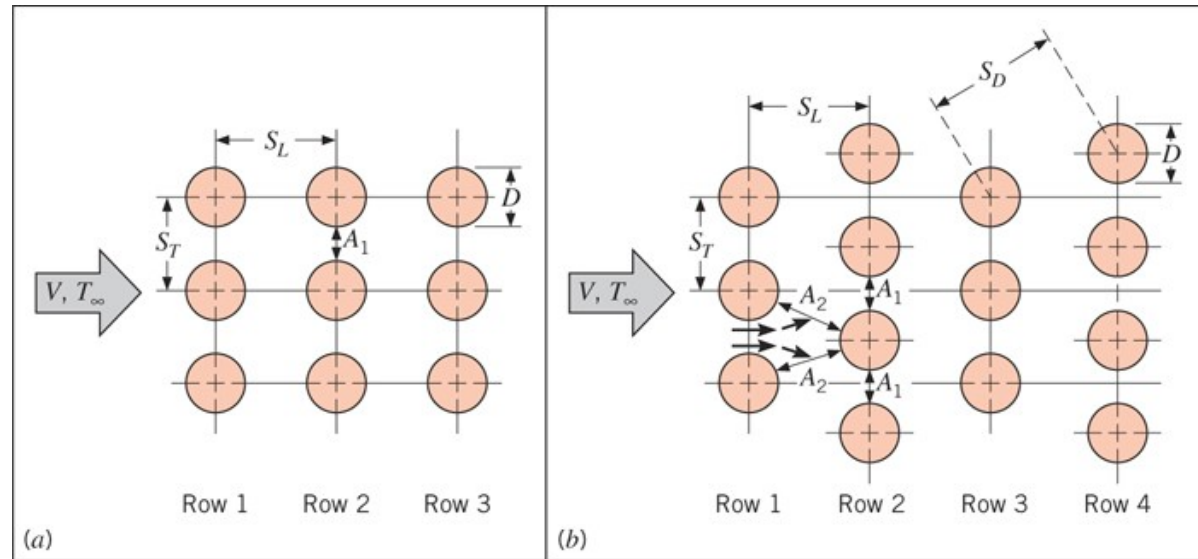
# Fin arrays



(a)



(b)



(a)

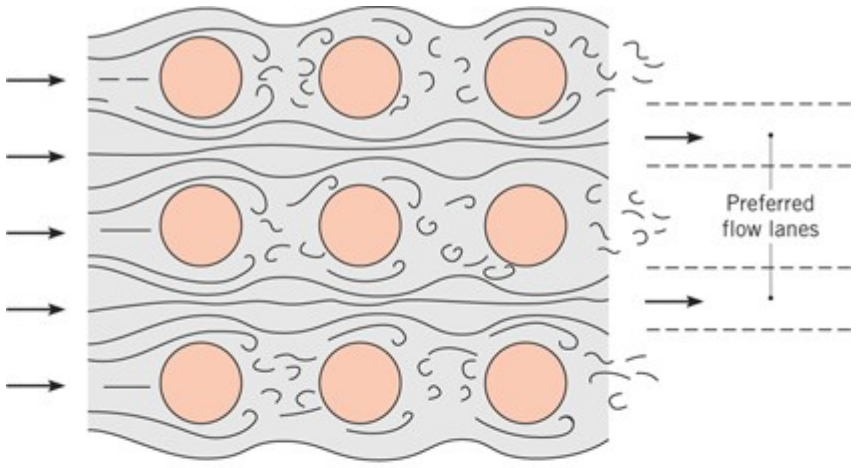
(b)

$S_T = \text{transverse pitch}$

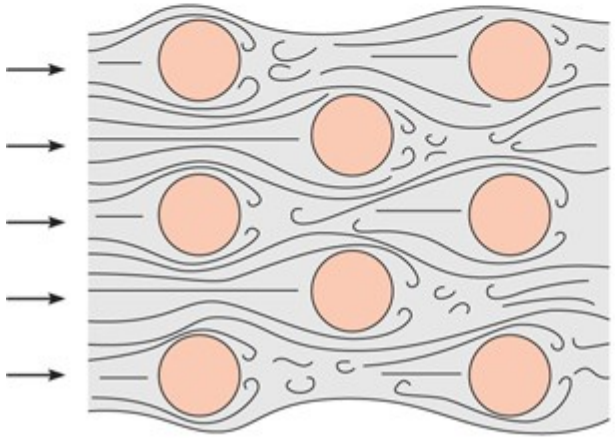
$S_L = \text{longitudinal pitch}$



# Aligned vs staggered configurations



(a)



(b)

Temperature



Temperature

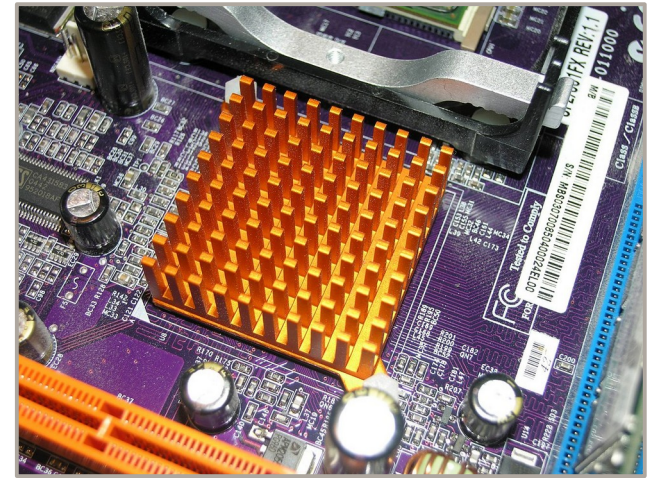


What do you think which one would pose better heat transfer ?



## HW4: Pin fin configuration

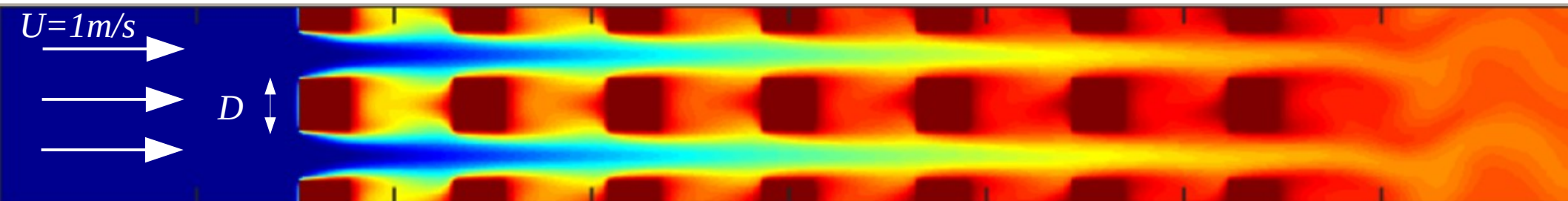
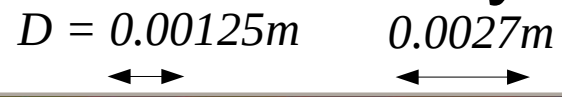
In this session we will look mostly into heat transfer in a pin fin bed. Could occur in electronics cooling or air conditioning system.



<https://pl.wikipedia.org/wiki/Plik:Heatsinkrods.jpg>

### Other assumptions during the session:

- 1) pin fin walls at  $T_{\text{wall}} = +29.6$  deg C
- 2) inflow temperature is  $T_{\text{left}} = +22.6$  deg C
- 3) velocity field is fully developed and laminar and mean inflow velocity is close to 1m/s





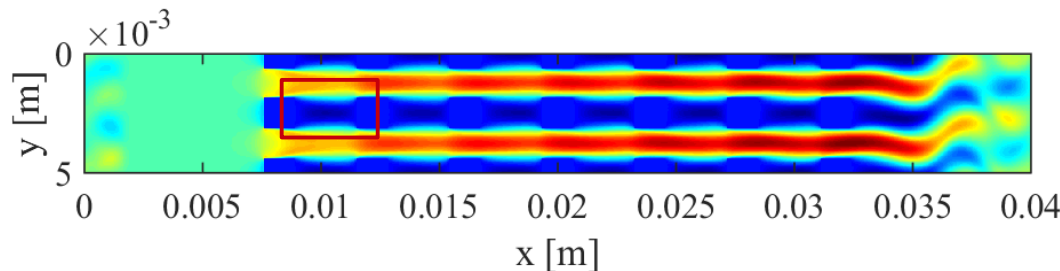
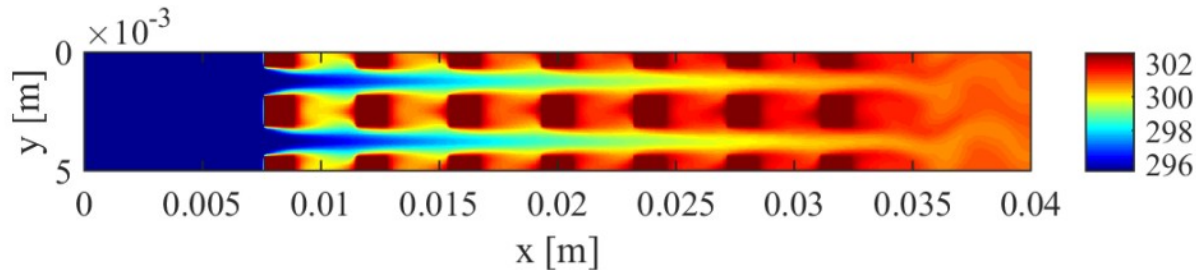


# HW4: Convection-diffusion equation for temperature to estimate Nu

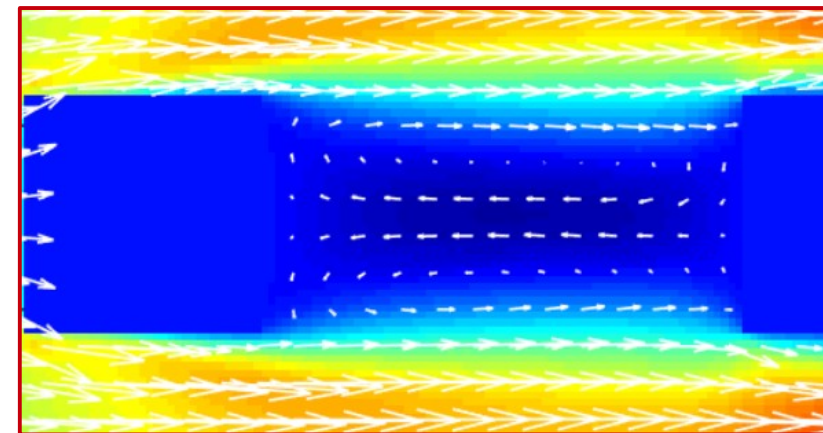
The Matlab session will focus on investigating heat transfer in a 2d fin bed using a provided velocity field which is assumed fixed. Constant wall T is assumed.

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial x^2} + \alpha \frac{\partial^2 T}{\partial y^2}$$

$T=T(x,y)$  in steady state 2d flow with constant wall temperature BC



Zoom to recirculation zone showing reverse flow (every fifth vector shown)



$u=u(x,y)$  and  $v=v(x,y)$  are provided from separate CFD simulation. Assume “frozen flow” and zero velocity on the walls.



## **STEPS 1&2:** Running code and saving the result.

- download the `heat2dfins` code from MyCourses and extract – by right clicking mouse - to new folder `Week4`. The code solves a convection-diffusion eqn for temperature using velocity obtained from another previous CFD simulation.
- run the code by clicking play. You can study either single fin case or multifin case using parameter. Here, let's first practice with a single fin so set the parameter `Single=1` (`Single=2` in the homework).



## STEP 3: postprocessing.

- create a new file called `PostProcess.m` Let us visualize the velocity field  $U(x,y)$  which comes indeed from the previous simulation (is given and assumed fixed herein).

```
a=load('FinsSingle.mat');
```

```
figure(2), clf, box
```

```
% first make image of the x-velocity field U(x,y)
```

```
imagesc([min(min(a.X)) max(max(a.X))],[min(min(a.Y)) max(max(a.Y))], a.U)
```

```
axis equal, axis tight, colormap jet, hold on, colorbar
```

```
print -dpng Velocity2dfins
```



**STEP 4:** use the Hilpert correlation (for cylindrical fins) to estimate Nusselt number under these conditions for this single rectangular fin. You need also the Reynolds and Prandtl numbers.

**STEP 5:** can you get similar order of magnitude from the **simulation result** for Nusselt number from **the single fin** case ?

$$\text{Nu}_{ave} = \text{Nu} = \frac{hD}{k} = C \text{Re}_D^m \text{Pr}^{1/3}$$

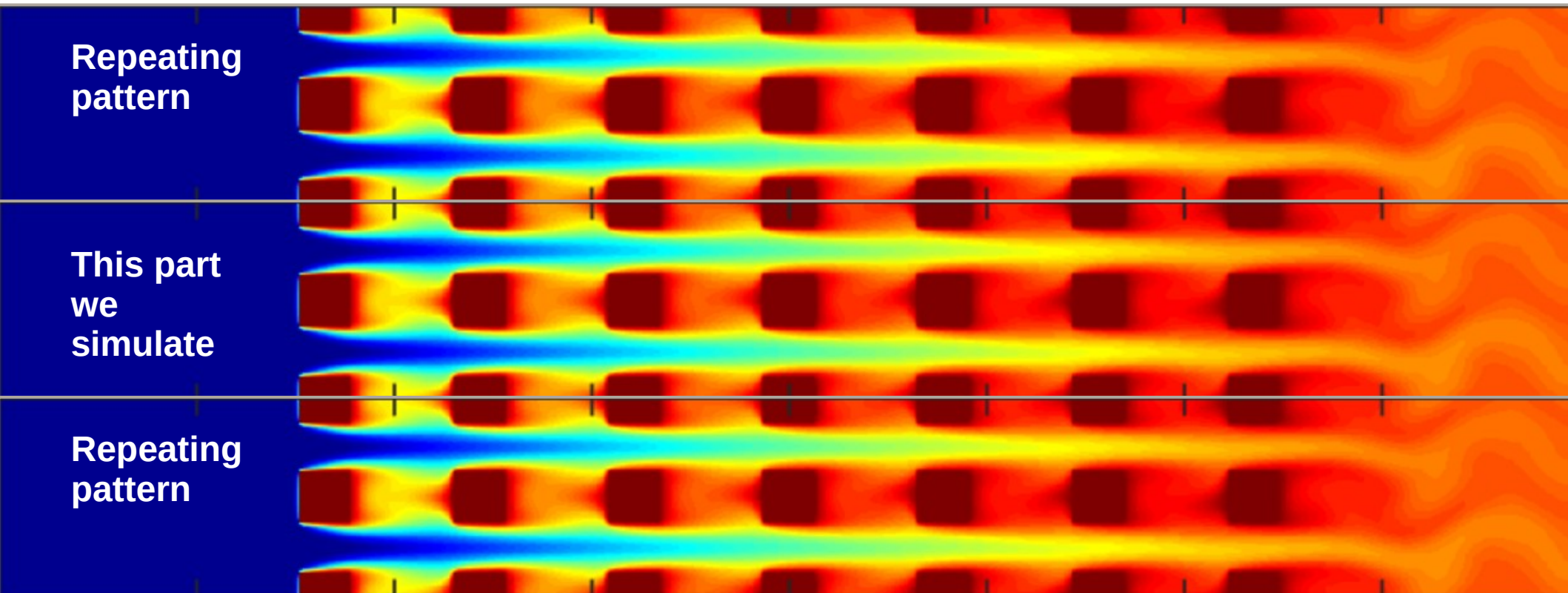


We can think we are in fact modeling an “infinite”, repeating pattern of fins ( $T_s = \text{const.}$ )

Row 1

Row 2

Row 7



**Question:** How to use Newton's cooling law **correctly** and **consistently** in this situation ?

Is  $h$  constant ? What is  $T_m$  ?

$$q = hA(T_s - T_m)$$



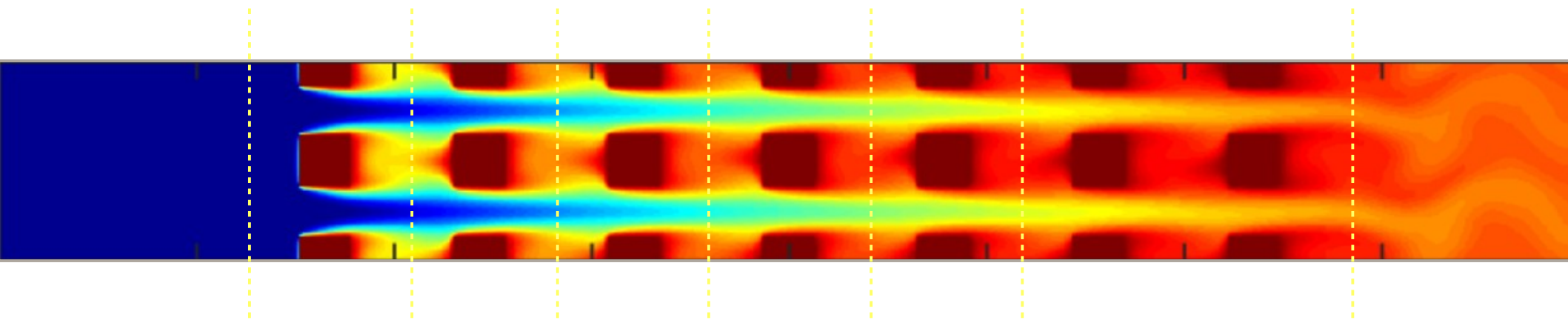
Things we are interested in:

$$\Delta T, h, T_m(x)$$

$$q = h_n A_{fin} (T_s - T_n) = c_p \rho U A_{in} (T_{n+1} - T_n)$$

$$T_o = T_{m,0} \quad T_1 = T_{m,1} \quad T_2 = T_{m,2}$$

$$T_{m,7}$$



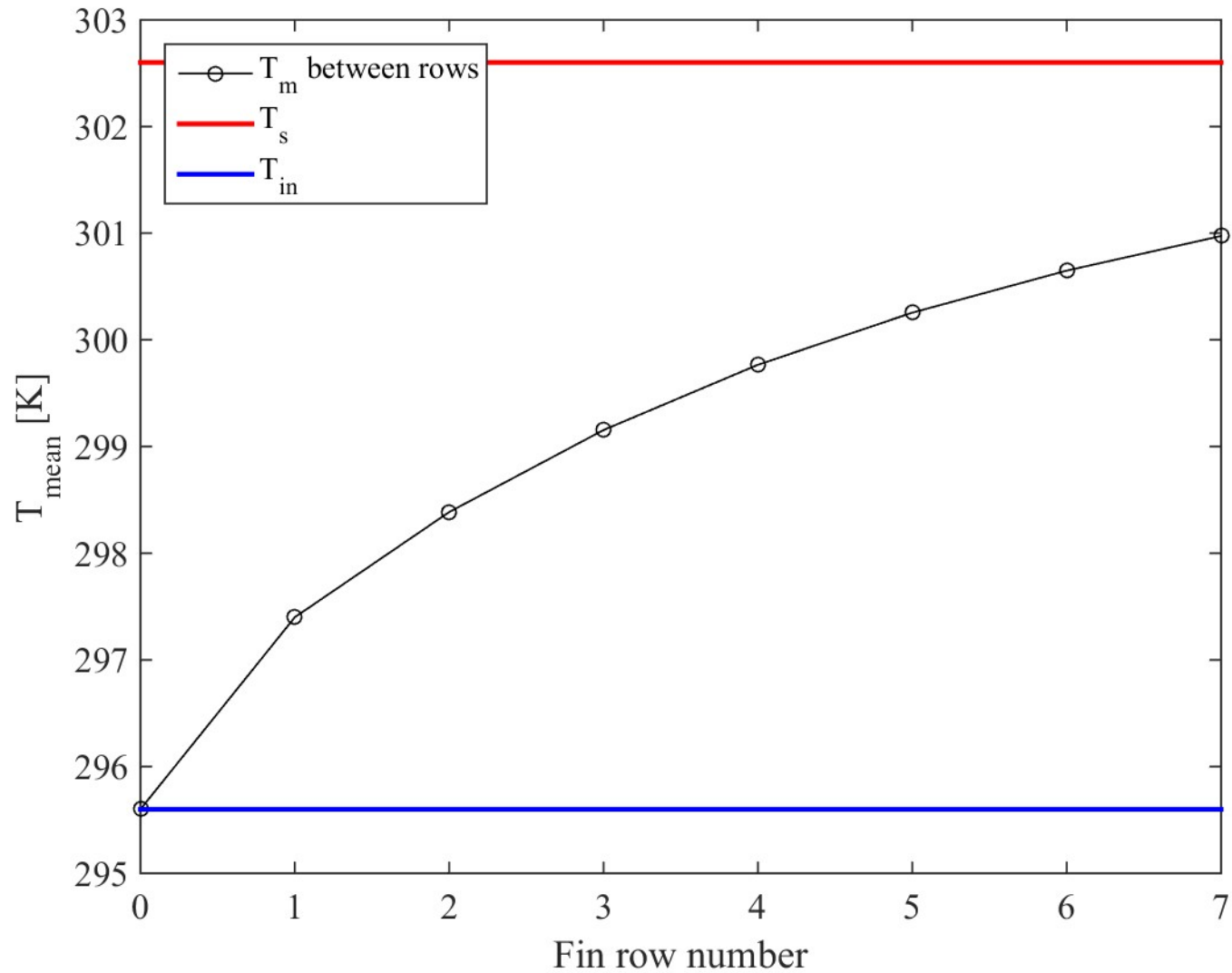
**Note:**

- we do not know if  $h$  is constant row-wise.
- we do not know if temperature jump per row is constant

```
%Calculation of mean temperature and velocity  
Tm = sum(T(iny,inx).*U)./(sum(U)); Um = mean(U);
```

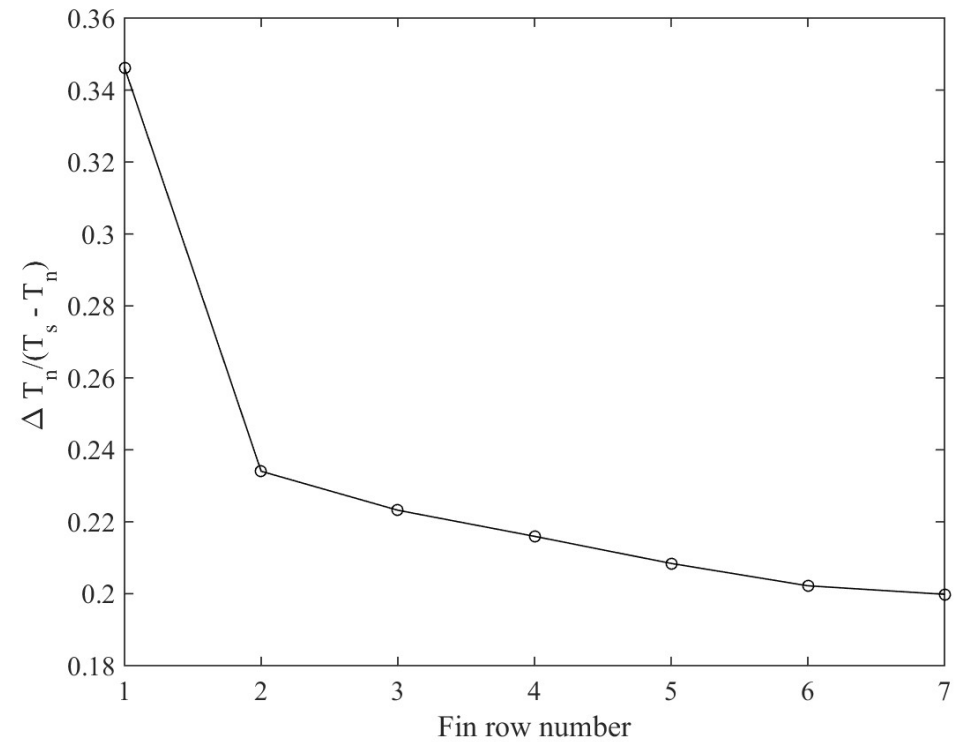
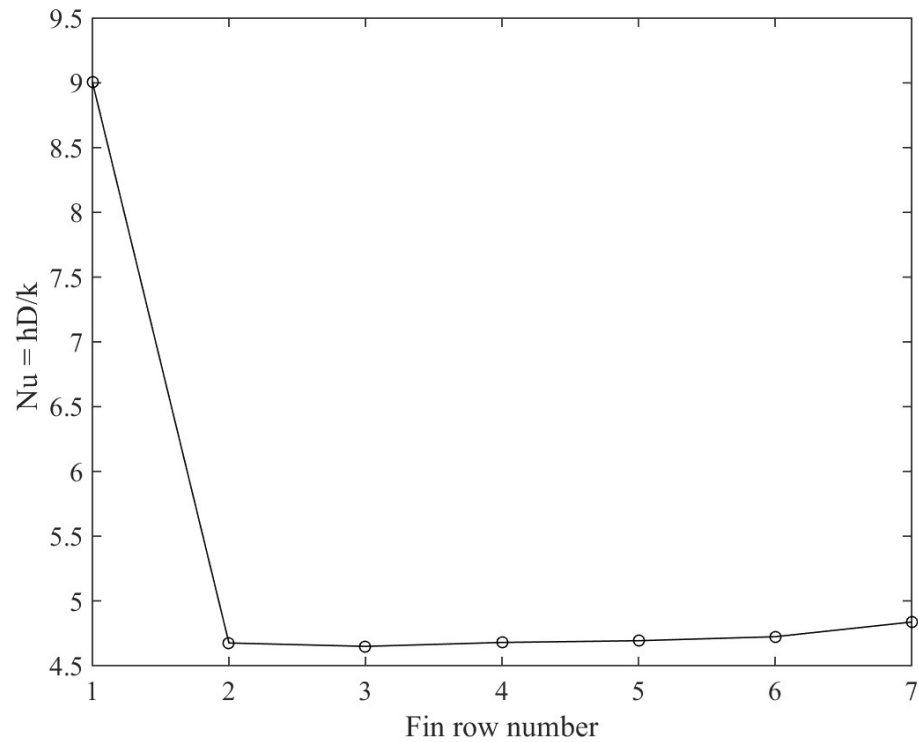


# Mean temperature after $n$ fin rows





# Relative temperature jump for fin array and Nusselt number

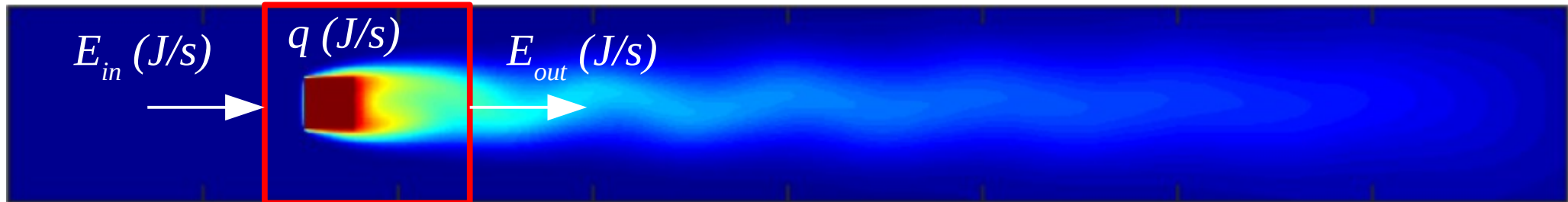






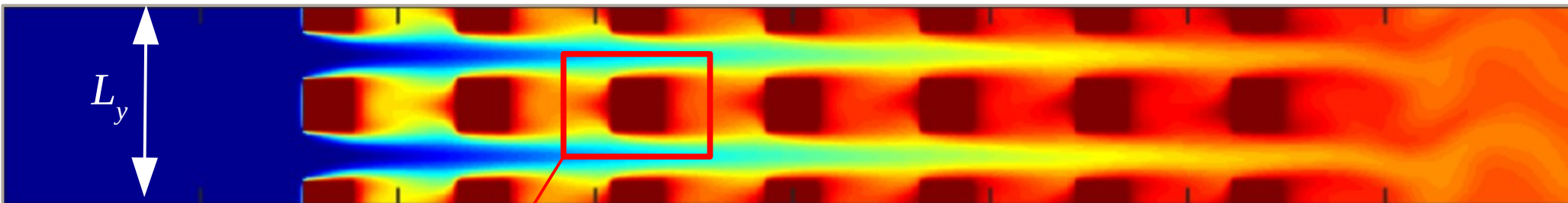
## Control volume for energy conservation analysis:

In HW4 one needs to evaluate Nusselt numbers for single fin and multiple fin cases

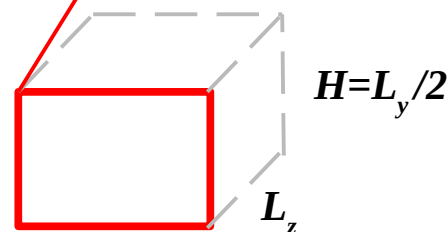


$$A_{in} = L_y \cdot L_z \text{ and } A_{fin} = 4 DL_z$$

$$q = h_n A_{fin} (T_s - T_n) = c_p \rho U A_{in} (T_{n+1} - T_n)$$



$$A_{in} = \frac{L_y}{2} \cdot L_z \text{ and } A_{fin} = 4 DL_z$$



Chosen control volume here