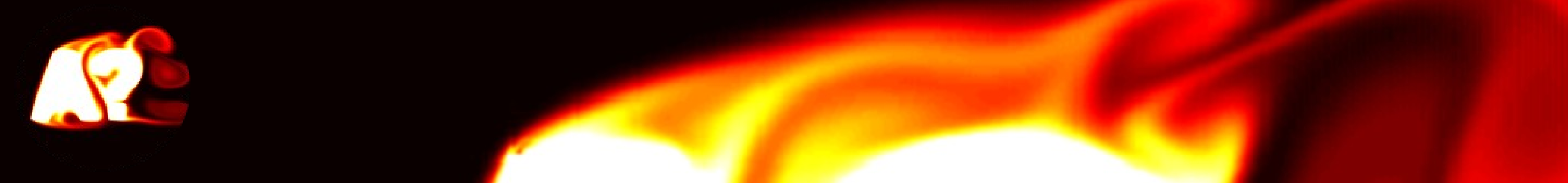
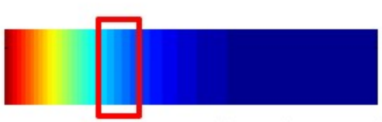


EEN-1020 Heat transfer

Week 5: Heat transfer via natural convection

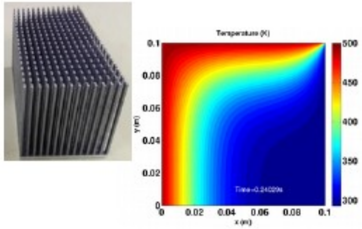
Prof. Ville Vuorinen
Nov. 22nd- 23rd 2022
Aalto University, School of Engineering

Week 1: Energy conservation, heat equation, conduction Fourier/Newton

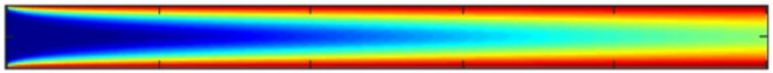


$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta x}\left(k\frac{\partial T(x+\Delta x/2,t)}{\partial x} - k\frac{\partial T(x-\Delta x/2,t)}{\partial x}\right)$$

Week 2: Fin theory, conduction, intro to convection

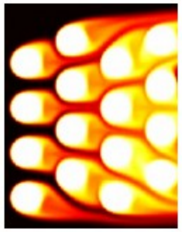


Week 3: convective heat transfer – internal flow (channel)



Week 4: convective heat transfer – external flow (fin systems)

Week 5: natural convection, boiling, correlations



On the heat transfer course, we have “5 friends” i.e. 5 main principles that are used to explain heat transfer phenomena

1) Energy conservation: “J/s thinking”
2) Fourier’s law
3) Newton’s cooling law
4) Energy transport equation – convection/diffusion equation
5) Momentum transport equation – Navier-Stokes equation

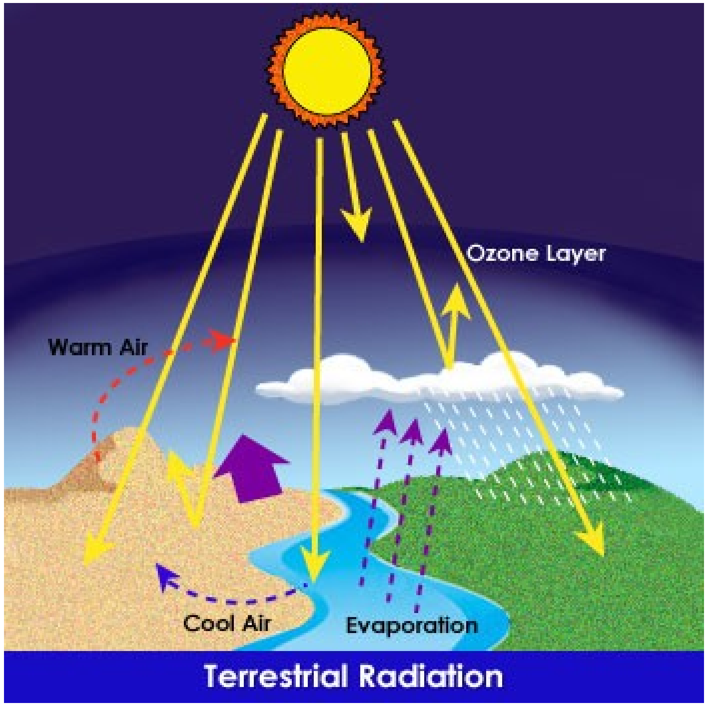
Lecture 5.1 Theory: Natural convection (free convection)

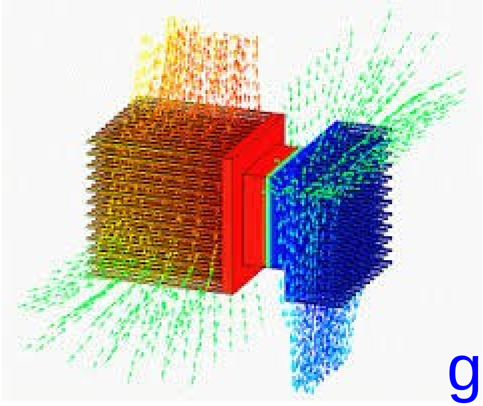
ILO 5: Student can choose Nusselt number correlation equations for different situations including natural convection.

Warm fluids are lighter than cold fluids → ability to rise against gravity → buoyancy

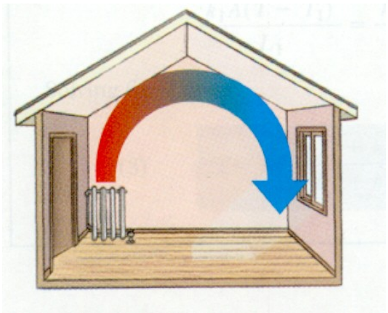
Topics covered

- Governing equations in natural convection
- Non-dimensional numbers
- Stable vs unstable configurations
- Boundary layers in natural convection



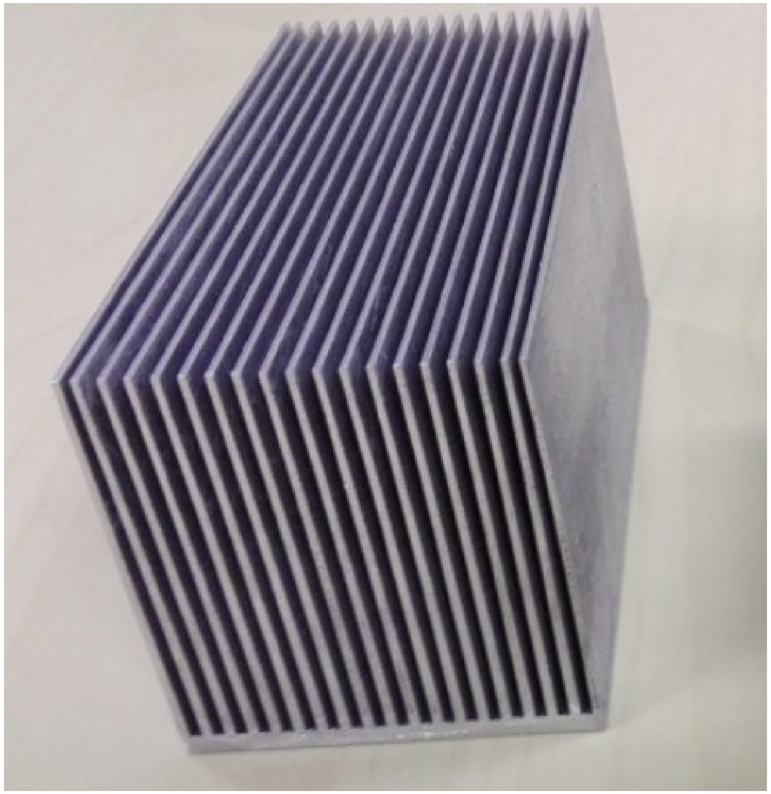


gravity



2018: we **switched off the fan** in the class room demo system
≈ 24W fixed and uniform heating power from the base plate.
→ The physical heat transfer mechanisms are natural convection and radiation.

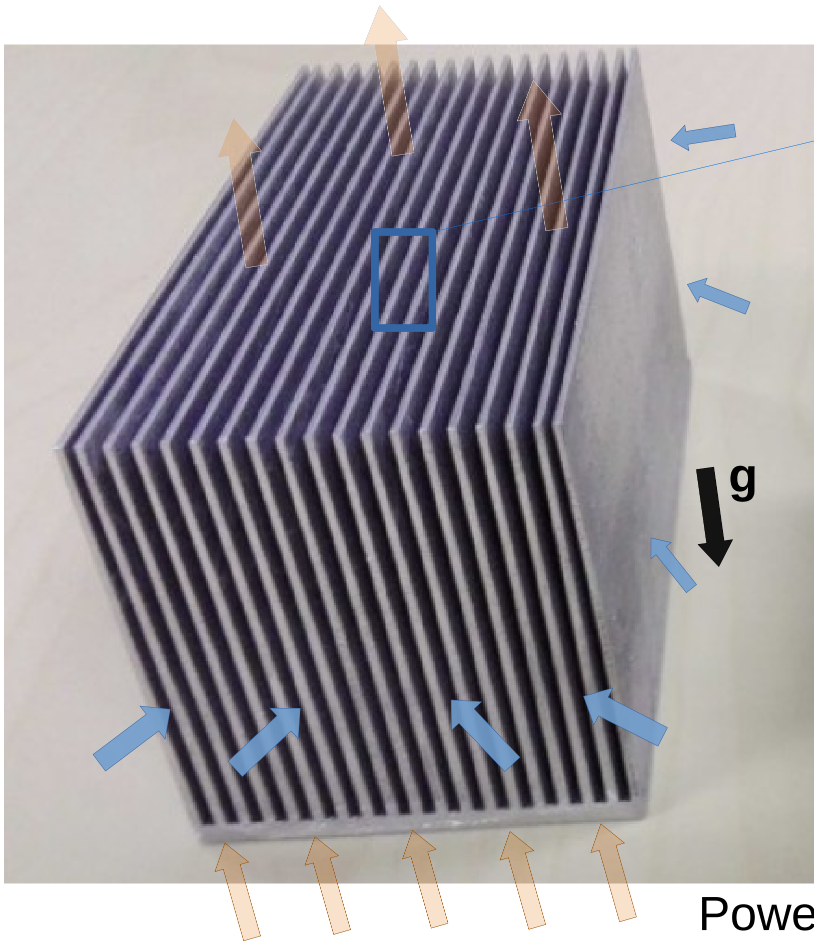
Consequence of switching off the fan:
the heat exchanger became extremely hot!



For a very slow physical cooling mechanism:
$dT/dt \approx q/mc_p$ → e.g. 0.25K/s = 15K/min.

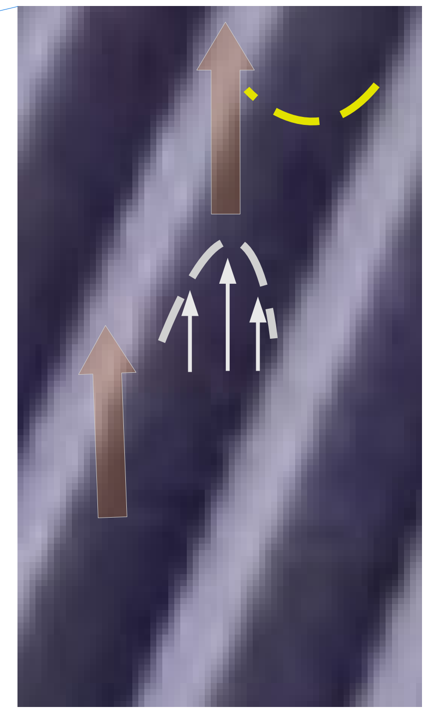
Heated air starts rising upwards and cool air enters through the sides. Heat transfers from the hot fin surfaces to the gas.

Warm air exits by rising upwards: T_{hot}



Cool air enters: T_{∞}

Power (J/s)



Airflow Temperature

Common wall boundary conditions:

Type 1: T_s = known

Type 2: q_s = known

Airflow velocity

Wall boundary condition:

No-slip:
U=V=W=0m/s

Energy balance (J/s thinking) for gas flow when the gas is heated at power P (W). "Control volume" thinking.

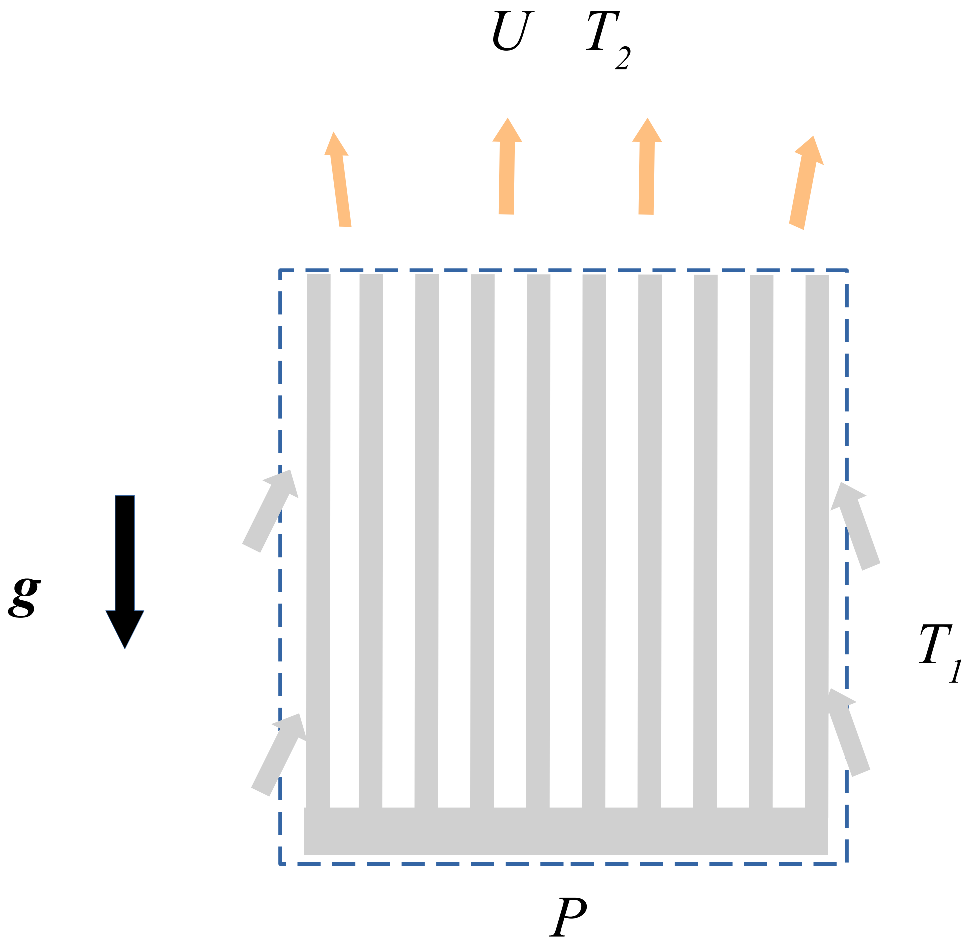
$$Q_{\mathrm{OUT}} - Q_{\mathrm{IN}} = c_p \dot{m} \Delta T_{ave} = P$$

Mass flow rate of the gas (kg/s) into the system from sides = exiting mass flow rate from the top:

$$\dot{m} = \rho U_{top} A_{top} = \rho U_s A_s$$

Average temperature change:

$$\Delta T_{ave} = T_2 - T_1$$



Why a drink can cools? Which orientation offers faster cooling: horizontal vs vertical? Why?

Note: radiation neglected on this course.

Newton

$$\boxed{q = hA_s(T_s - T_\infty)}$$

What parameters affect h and Nu ?

Figure: CFD simulation of a cooling object and temperature field.
V.Vuorinen (2016)

Governing equations (here 2d) in natural convection using the Boussinesq approximation for the buoyancy force

Navier-Stokes (momentum)

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 U}{\partial x^2}+\nu\frac{\partial^2 U}{\partial y^2}-\boxed{g_x\beta(T-T_\infty)}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\nu\frac{\partial^2 V}{\partial x^2}+\nu\frac{\partial^2 V}{\partial y^2}-\boxed{g_y\beta(T-T_\infty)}$$

Buoyancy force: tries to promote motion into opposite direction from gravity

Convection-diffusion for temperature (energy equation)

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}+\alpha\frac{\partial^2 T}{\partial y^2}$$

Derivation of thermal expansion coefficient for ideal gas ($\beta = 1/T$)

Ideal gas law

$$\boxed{p = \rho R T}$$

Thermal expansion coefficient

$$\boxed{\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial [p/RT]}{\partial T}\right)_p = \frac{p}{\rho R T^2} = 1/T}$$

Note: for other fluids values of the expansion coefficient have been tabulated.

Assumptions in Boussinesq approximation

- Density is assumed to have a well defined mean part and a fluctuation part

$$\boxed{\rho = \rho_o - \left(\frac{\partial \rho}{\partial T}\right)_p \Delta T}$$

or

$$\boxed{\rho = \rho_o - \beta \rho_o \Delta T}$$

Fluctuation part of density contributing to a buoyancy force in the Boussinesq approximation

- Thermodynamic pressure is assumed to be almost constant
(often a very good assumption because speed of sound is typically high in comparison to other velocities*)

- Temperature will then be a function of density

- When temperature of a point in space increases, the density decreases

- It leads to a buoyancy force promoting motion against gravity ("**hot air balloon effect**")

- we can think that pointwise fluctuations of temperature from the mean ($T' = T - T_{ref}$) promote/drive the flow into motion

*__Note__: e.g. in a typical flame pressure is almost constant but density and temperature depend very strongly on position (low density in hot parts).

Important numbers

Grashof number

$$Gr = \frac{g\beta (T_s - T_\infty) L^3}{\nu^2} = \frac{Buoyancy\ force}{Viscous\ force}$$

$L \rightarrow characteristic\ length\ scale\ of\ surface/object$

$\beta \rightarrow thermal\ expansion\ coefficient$

Thermal expansion coefficient

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Rayleigh number (Ra and Gr closely related)

$$Ra = \frac{g\beta (T_s - T_\infty) L^3}{\nu \alpha} = \frac{Buoyancy\ force}{Viscous\ force}$$

$\alpha = thermal\ diffusivity,\ \nu = kinematic\ viscosity$

Differences in Nusselt number correlations

Note: In forced convection the Reynolds and Prandtl numbers were of very high importance. **Nu=Nu(Re,Pr).**

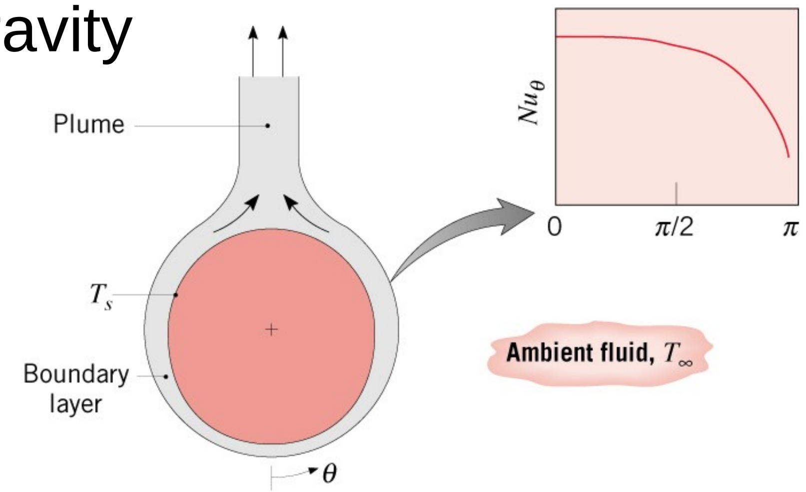
Note: In natural convection the Rayleigh (and/or Grashof) number is typically the key driving parameter. **Nu=Nu(Ra,Pr)** or **Nu=Nu(Gr,Pr)**.

Note: here we do not discuss the mixed convection case.

Nusselt number correlation for a horizontal cylinder

Horizontal cylinder

gravity



$$\boxed{Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{Buoyancy\ force}{Viscous\ force}}$$

$$\boxed{Ra_D < 10^{12}}$$

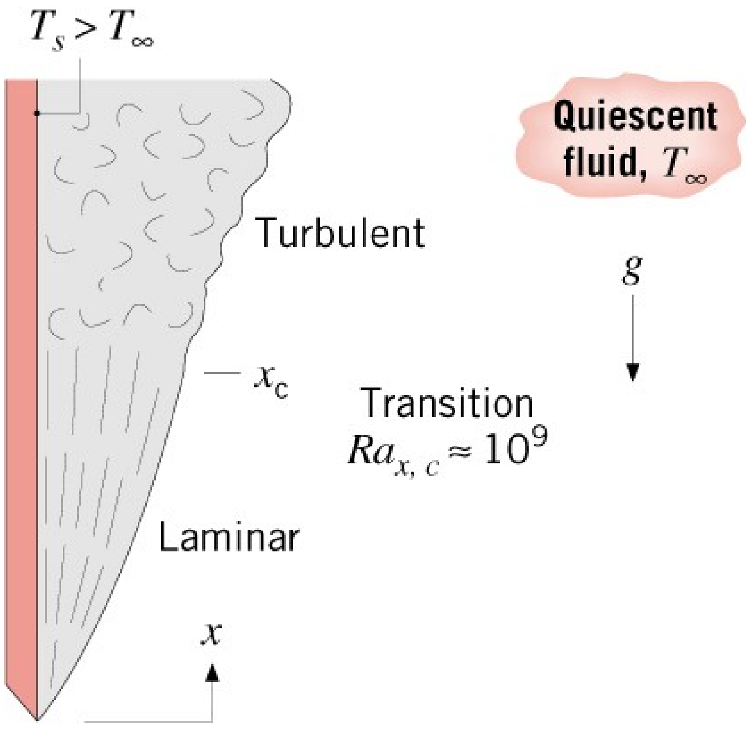
$$\boxed{\bar{Nu}_D = 0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}}$$

$$\boxed{What\ happens\ when\ Ra_D \rightarrow 0\ ?}$$

$$\boxed{\bar{Nu}_D = \frac{\bar{h}D}{k}}$$

Nusselt number correlation for a vertical plate

NC creates flow against gravity → near-wall boundary layers → possibility for laminar to turbulence transition → critical Rayleigh number



Critical Rayleigh number where flow becomes turbulent at $x=x_c$

$$\boxed{Ra_c = 10^9}$$

Example: Vertical plate average Nusselt number for laminar conditions ($x<x_c$)

$$\bar{Nu} = 0.68 + \frac{0.670\, Ra_L^{1/4}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{4/9}}$$

Example: Vertical plate average Nusselt number for all conditions (see a few slides ahead)

$$\bar{Nu}_L = \left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(0.492/Pr\right)^{9/16}\right]^{8/27}}\right]^2$$

https://www.youtube.com/watch?v=6ney_Vx00zU

Which way does a can cool faster in the fridge: horizontally or vertically ?

Rayleigh number (general length scale L)

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{Buoyancy\ force}{Viscous\ force}$$

D = 0.06m, L = 0.17m

L

D

Horizontal cylinder:

$$\overline{Nu}_D = 0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}$$

Vertical cylinder:

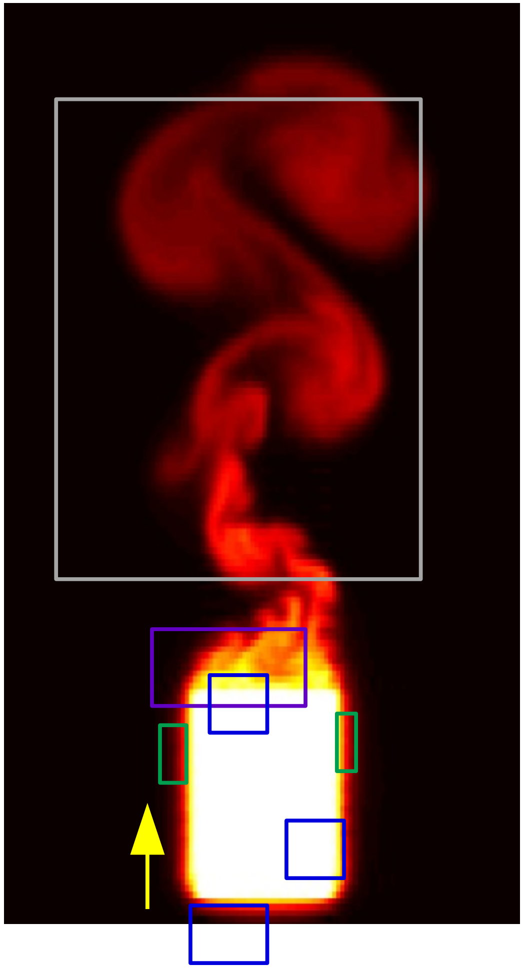
$$\overline{Nu}_L = \left[0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2$$

Some physical steps how heat transfer away from a can in natural convection

Step 1: Conduction from the wall to the fluid and conduction in the thermal boundary layer (TBL).

Step 2: Heated fluid starts rising upwards already when conducting in the TBL

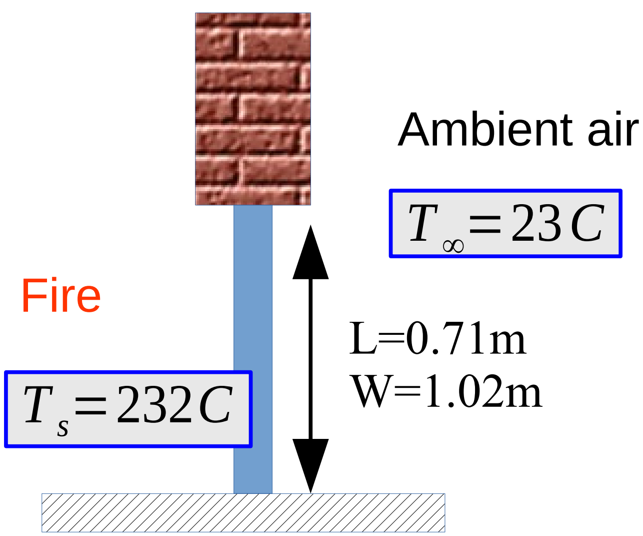
Step 3: Accelerated flow forms **viscous and thermal** boundary layers around the can.



Step 5: fluid rises constantly and the hot air is "**self-transported**" away from the object in a plume which poses fluid dynamical structures (e.g. vortices, turbulence)

Step 4: fluid motion becomes 3d and turbulence starts to transport heat from the top surface

Example 9.2: Estimate convective heat rate for glass window of a fireplace – relevance HW5



First, estimate Ra for air rising along glass window:

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = 1.813\cdot 10^9 > Ra_c$$

Use the correlation valid at all conditions ($Ra>Ra_c$):

$$\bar{Nu}_L = \left[0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right]^2 = 147$$

Estimate heat transfer coefficient:

$$\bar{h} = \frac{\bar{Nu}_L\,k}{L} = 7.0\,W/m^2K$$

Heat rate from Newton's law of cooling:

$$q = \bar{h}\,A_s(T_s - T_\infty) = 1060\,W$$

Note: radiative heat transfer would be essential here:

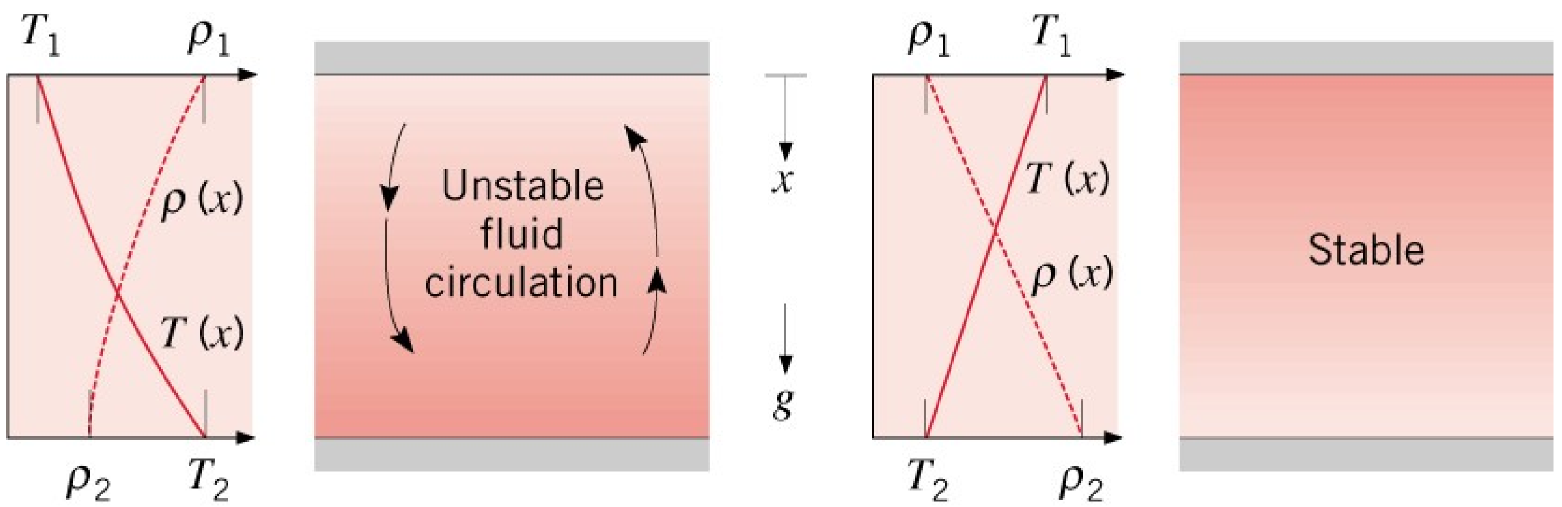
$$q_{rad} = \epsilon A_s \sigma (T_s^4 - T_\infty^4) = 2355\,W$$

Flow in confinements

Unstable vs stable configurations

cold

hot



hot

cold

$$\frac{dT}{dx} > 0, \frac{d\rho}{dx} < 0$$

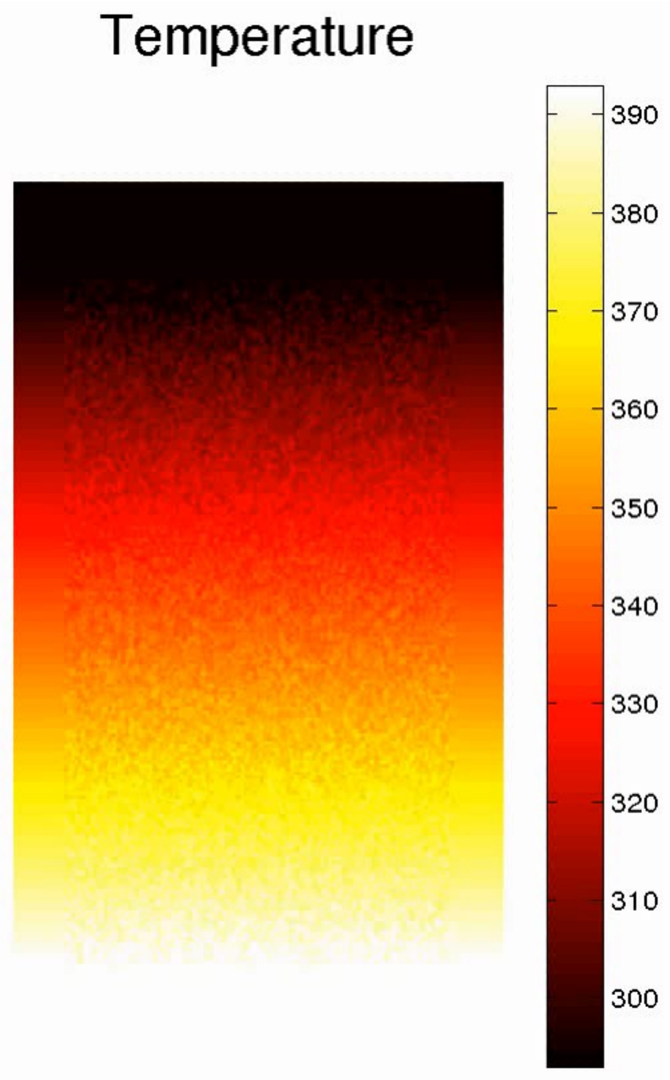
(a)

$$\frac{dT}{dx} < 0, \frac{d\rho}{dx} > 0$$

(b)

gravity

Case: Enclosed, tight water-filled kettle on the stove

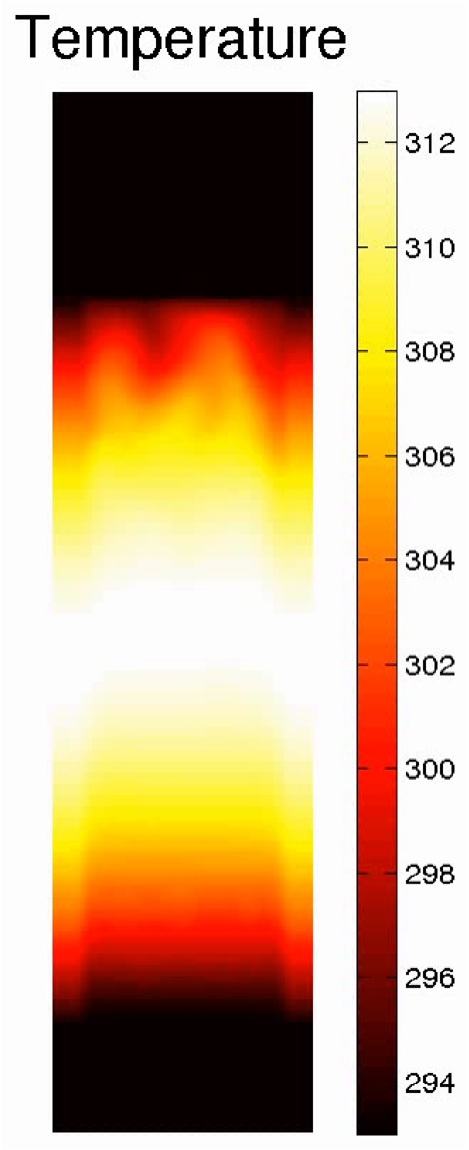


Case: enclosed “kettle” on the stove with space-dependent heating at the walls (linearly decreasing towards the top).

Question 1: Does the schematic on stable vs unstable configuration explain what happens here?

Question 2: Does a steady state solution exist when time $\rightarrow$ infinity?

Case: Enclosed furnace with space-dependent wall heating



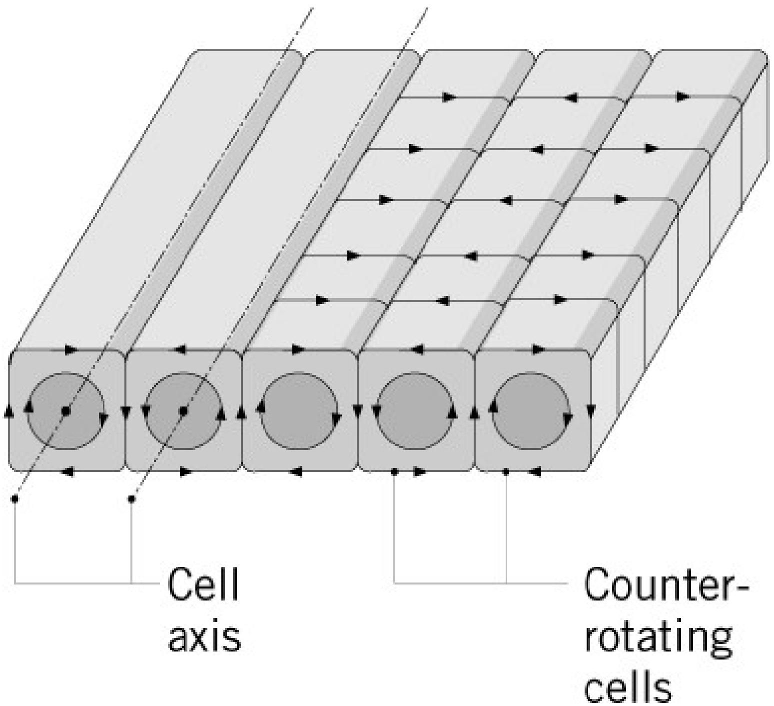
Recall some previous slides:
Stable vs unstable configuration

Case: enclosed “furnace” with space-dependent heating at the walls (cold at top and bottom parts, hot in the center).

Question: Does the schematic picture from the previous slide (stable vs unstable configuration) explain what happens?

Enclosed cavities, heating from below

https://www.youtube.com/watch?v=OM0l2YPVMf8
https://www.youtube.com/watch?v=jFI5KaAqfXI



$$\boxed{Ra < Ra_c = 1708}$$

$$\boxed{\overline{Nu}_L = \frac{\bar{h}L}{k} = 1}$$

Case 2: Thermally unstable but regular cell patterns

$$\boxed{1708 < Ra_L < 5 \cdot 10^4}$$

Case 3: Flow is turbulent

$$\boxed{3 \cdot 10^5 < Ra_L < 7 \cdot 10^9}$$

https://www.youtube.com/watch?v=gSTNxS96fRg&t=56s