Aalto University
School of Electrical Engineering

Probabilistic Robotics

Bayes Filter Implementations

Particle Filter

ELEC-E8107 Stochastic models estimation and control

Arto Visala

Particle Filters

- Represent belief by random samples
- Estimation of non-Gaussian, nonlinear processes

- Monte Carlo filter, Survival of the fittest, Condensation, Bootstrap filter, **Particle filter**

- Filtering: [Rubin, 88], [Gordon et al., 93], [Kitagawa 96]
- Computer vision: [Isard and Blake 96, 98]
- Dynamic Bayesian Networks: [Kanazawa et al., 95]d

Particle Filter algorithm 1/3

Algorithm Particle_filter($\mathcal{X}_{t-1}, u_t, z_t$):

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $m = 1$ *to* M *do*

  sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

  $w_t^{[m]} = p(z_t \mid x_t^{[m]})$

  $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m = 1$ *to* M *do*

  draw i *with probability* $\propto w_t^{[i]}$

  add $x_t^{[i]}$ *to* $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

Particle Filter algorithm 2/3

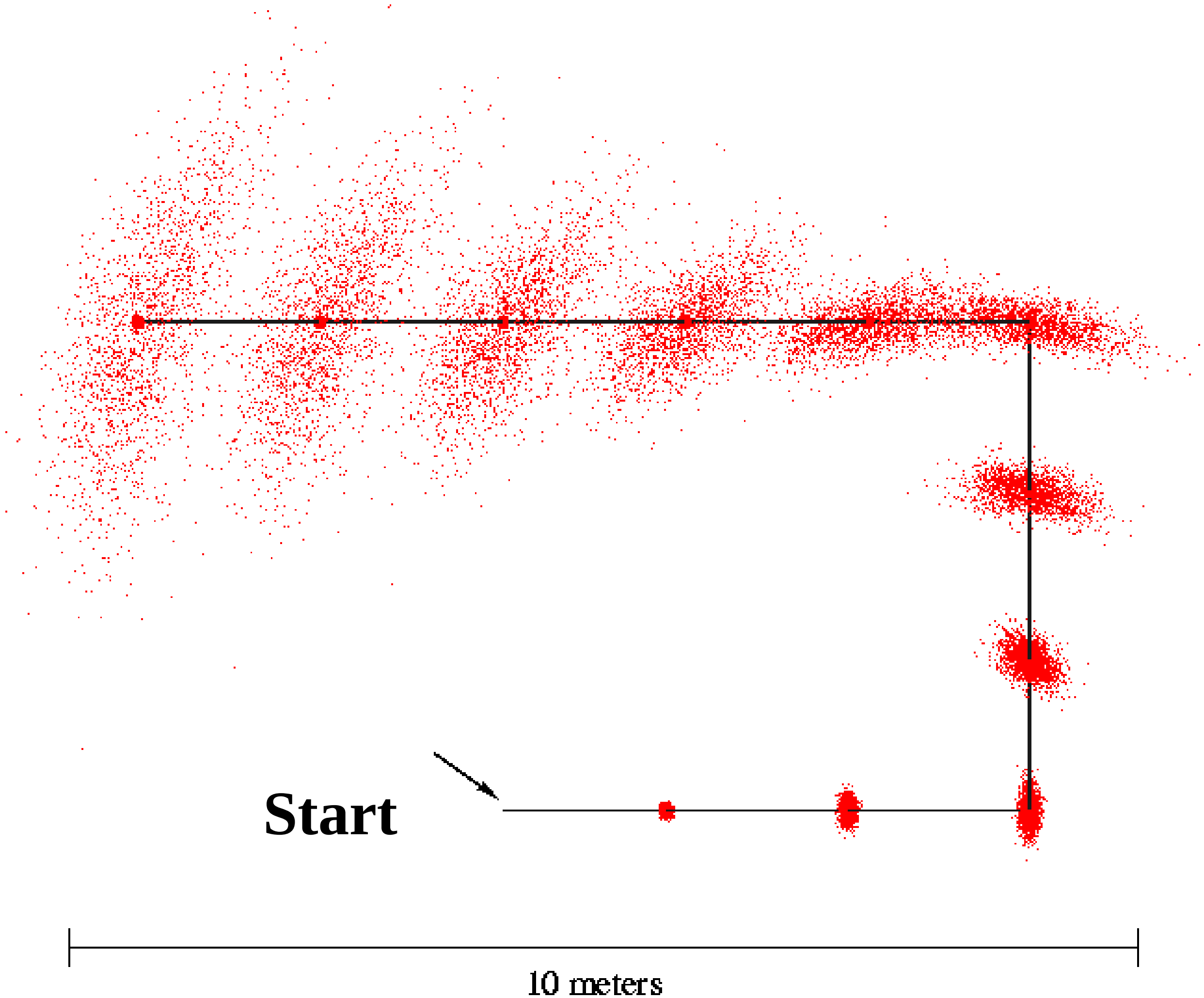
1. Line 4 generates a hypothetical state $x_t^{[m]}$ for time t based on the particle $x_{t-1}^{[m]}$ and the control u_t. The resulting sample is indexed by m, indicating that it is generated from the m-th particle in $\mathcal{X}_{t-1}$. This step involves sampling from the state transition distribution $p(x_t \mid u_t, x_{t-1})$. To implement this step, one needs to be able to sample from this distribution. The set of particles obtained after M iterations is the filter's representation of $\overline{bel}(x_t)$.

2. Line 5 calculates for each particle $x_t^{[m]}$ the so-called *importance factor*, denoted $w_t^{[m]}$. Importance factors are used to incorporate the measurement z_t into the particle set. The importance, thus, is the probability of the measurement z_t under the particle $x_t^{[m]}$, given by $w_t^{[m]} = p(z_t \mid x_t^{[m]})$. If we interpret $w_t^{[m]}$ as the *weight* of a particle, the set of weighted particles represents (in approximation) the Bayes filter posterior $bel(x_t)$.

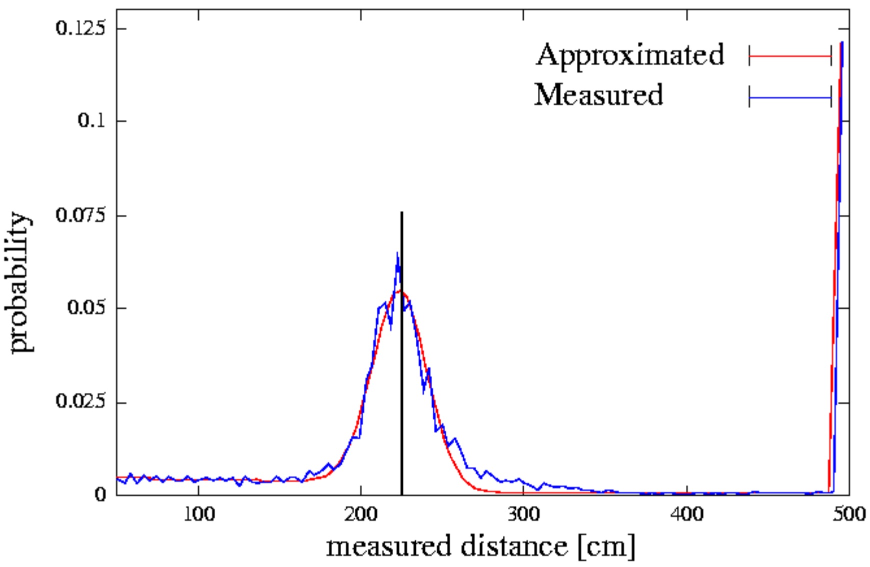
Particle Filter algorithm 3/3

3. The real "trick" of the particle filter algorithm occurs in lines 8 through 11 in Table 4.3. These lines implemented what is known as *resampling* or *importance sampling*. The algorithm draws with replacement M particles from the temporary set $\bar{\mathcal{X}}_t$. The probability of drawing each particle is given by its importance weight. Resampling transforms a particle set of M particles into another particle set of the same size. By incorporating the importance weights into the resampling process, the distribution of the particles change: Whereas before the resampling step, they were distributed according to $\overline{bel}(x_t)$, after the resampling they are distributed (approximately) according to the posterior $bel(x_t) = \eta\, p(z_t \mid x_t^{[m]})\overline{bel}(x_t)$. In fact, the resulting sample set usually possesses many duplicates, since particles are drawn with replacement. More important are the particles *not* contained in $\mathcal{X}_t$: Those tend to be the particles with lower importance weights.

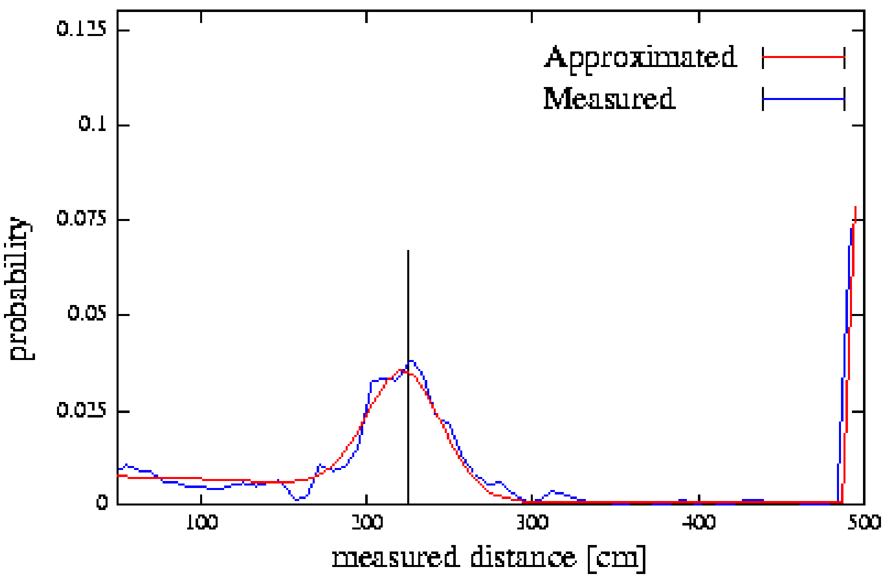
Motion Model Reminder



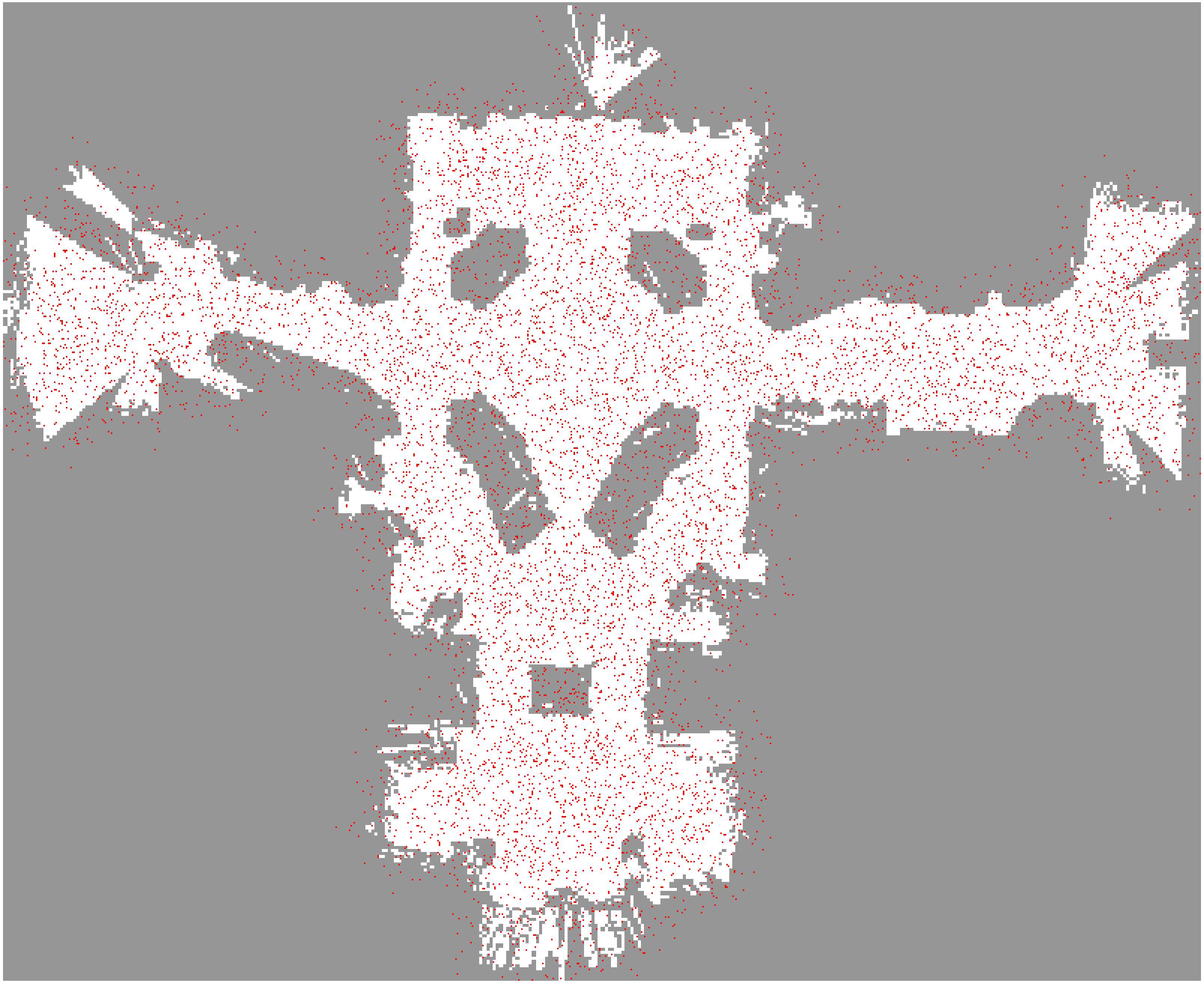
Proximity Sensor Model Reminder

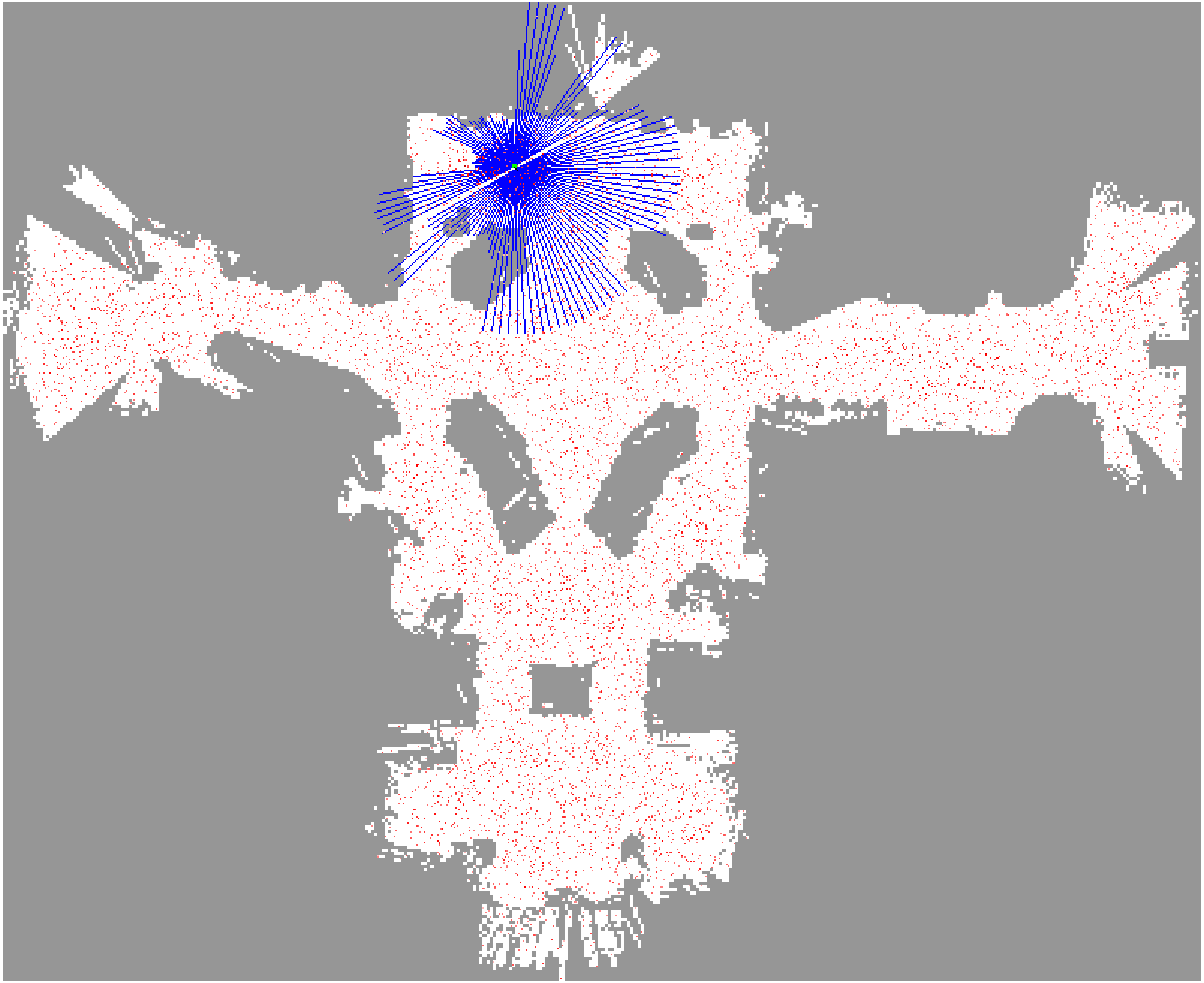


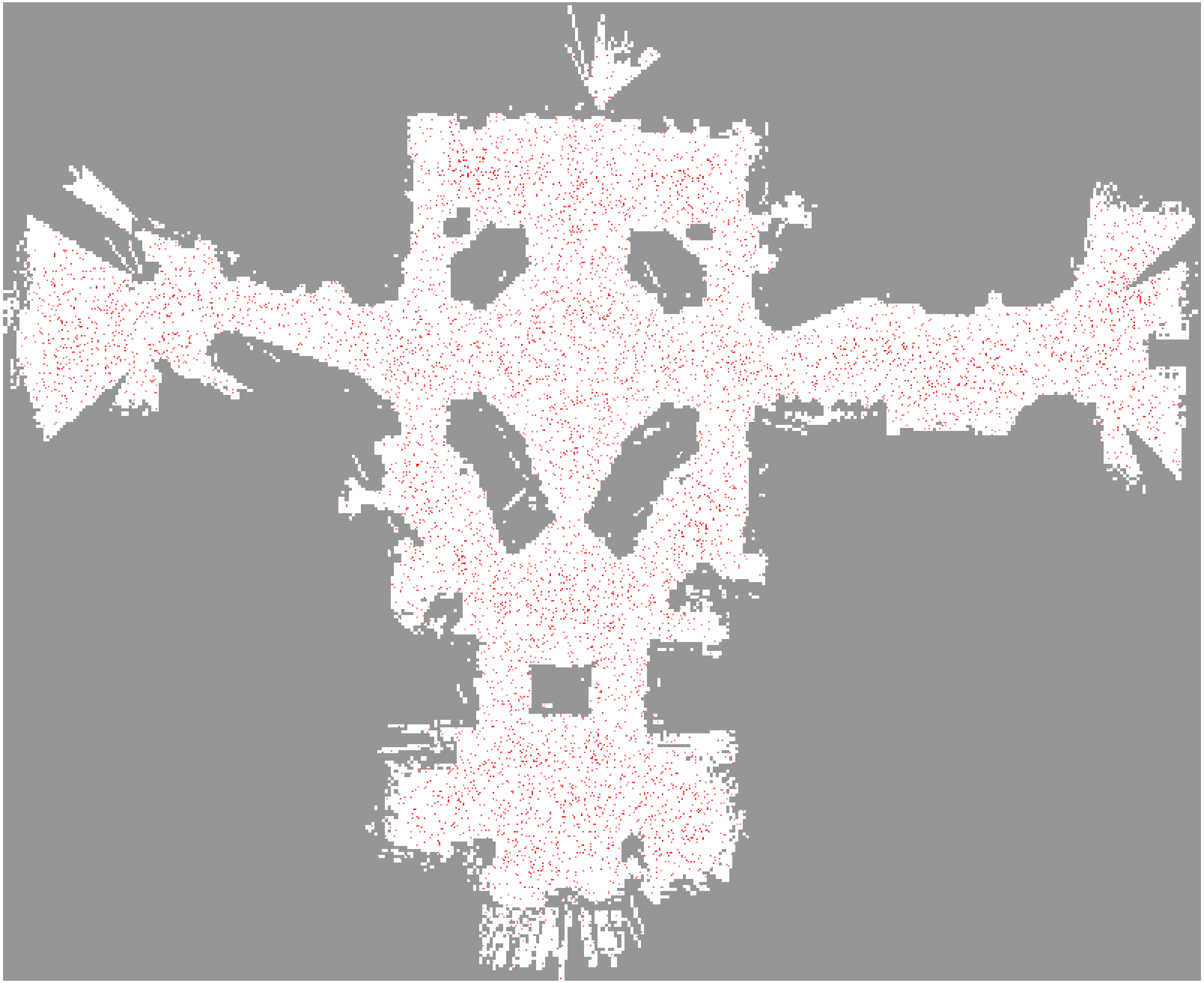
Laser sensor

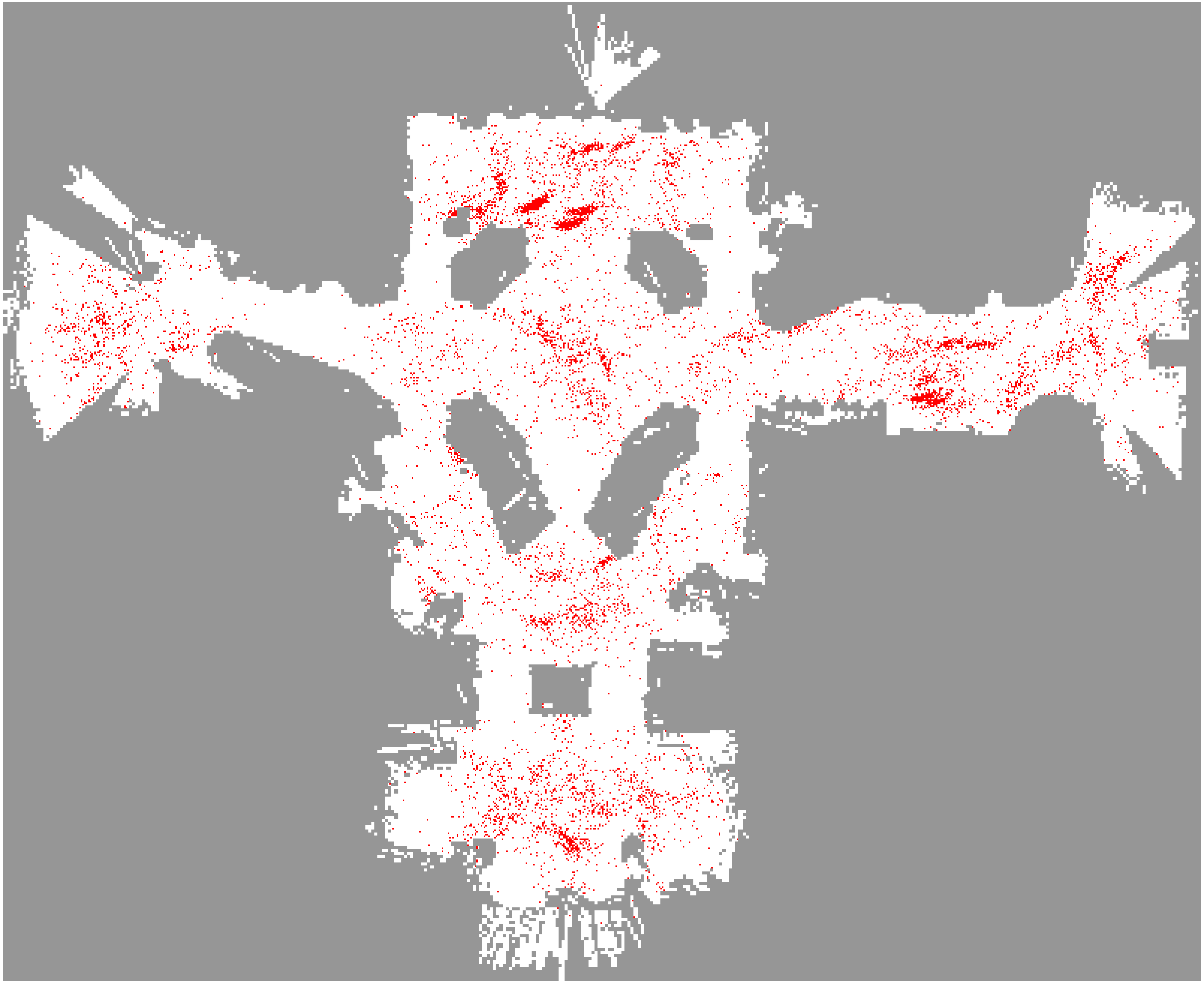


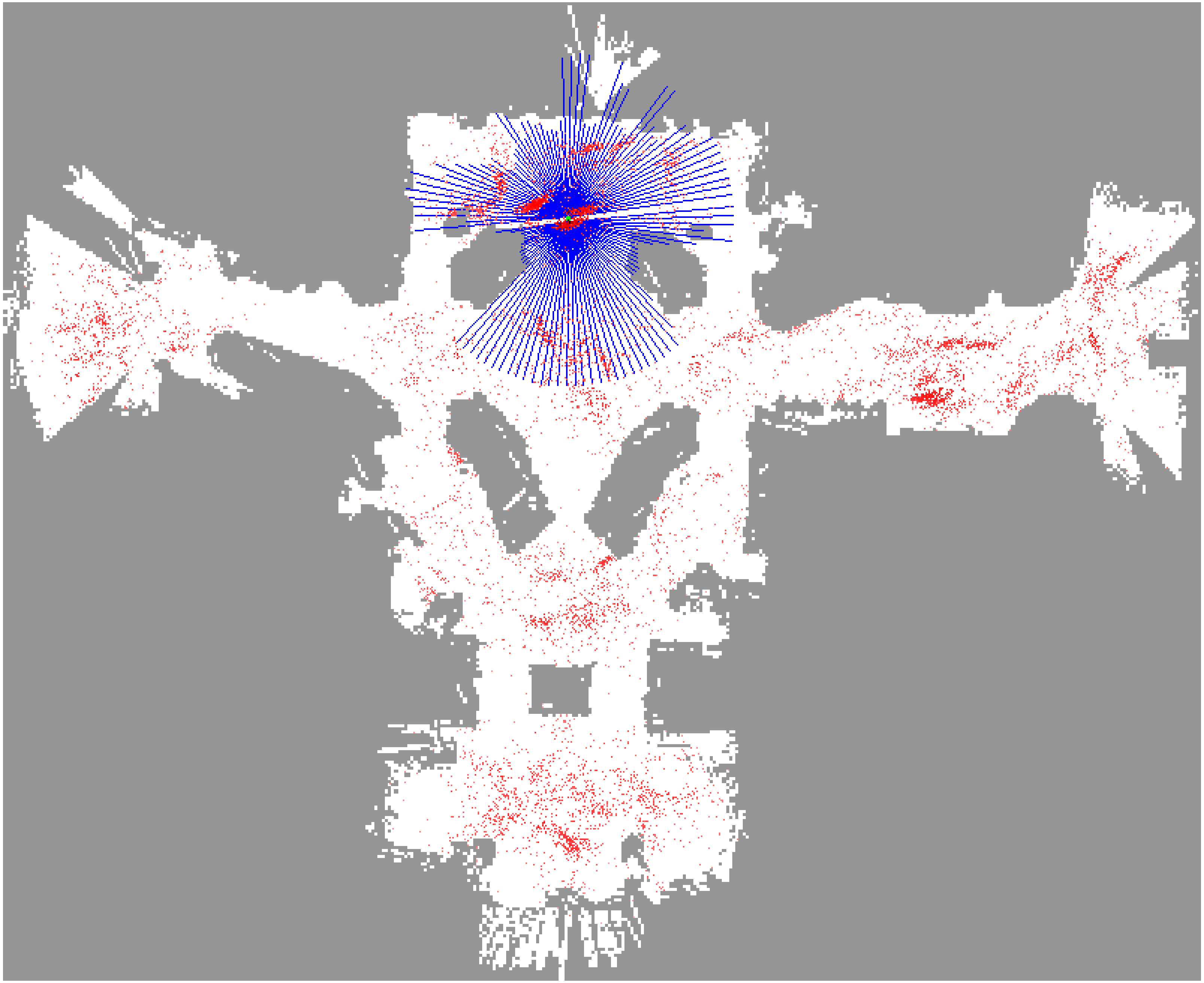
Sonar sensor

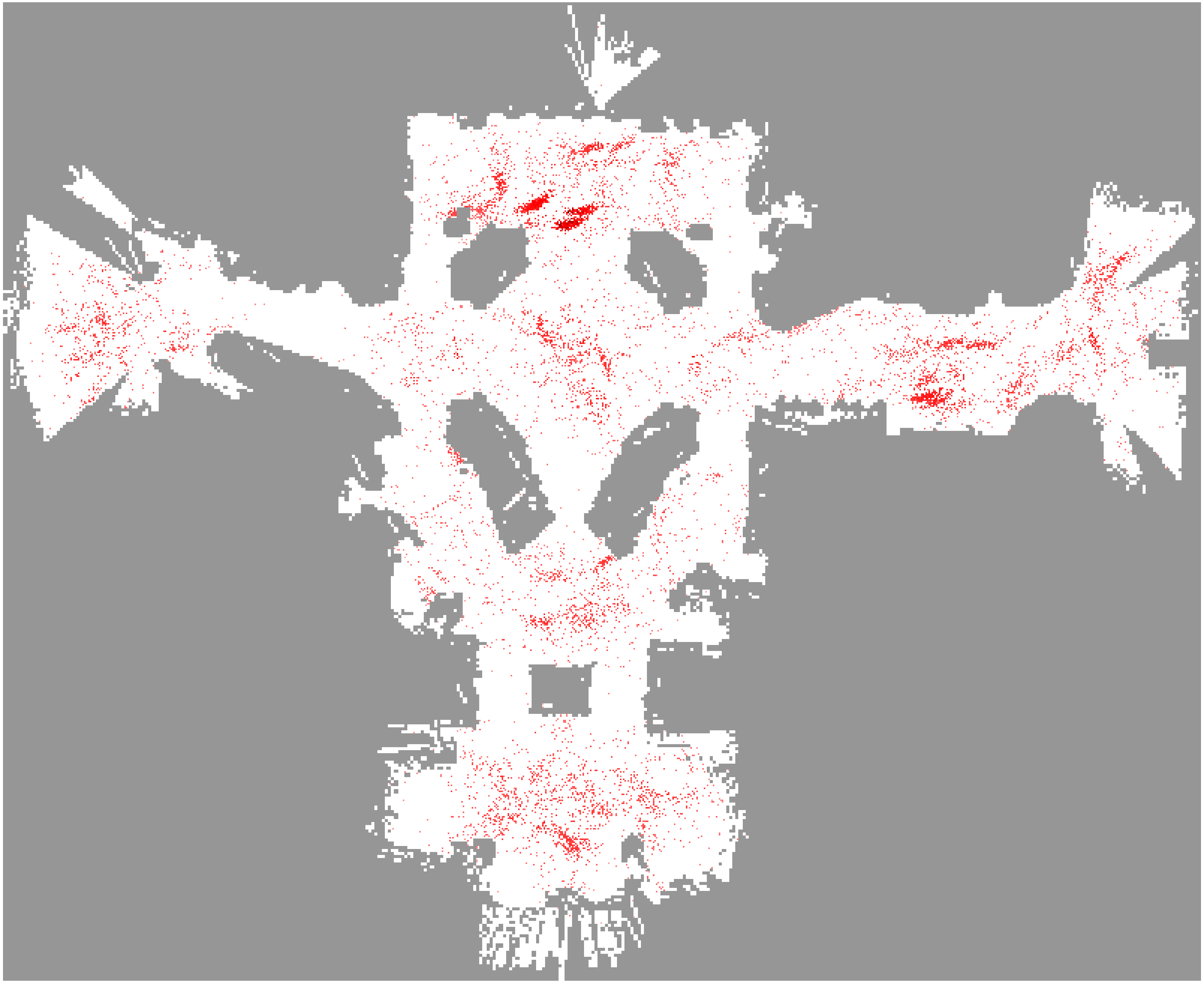


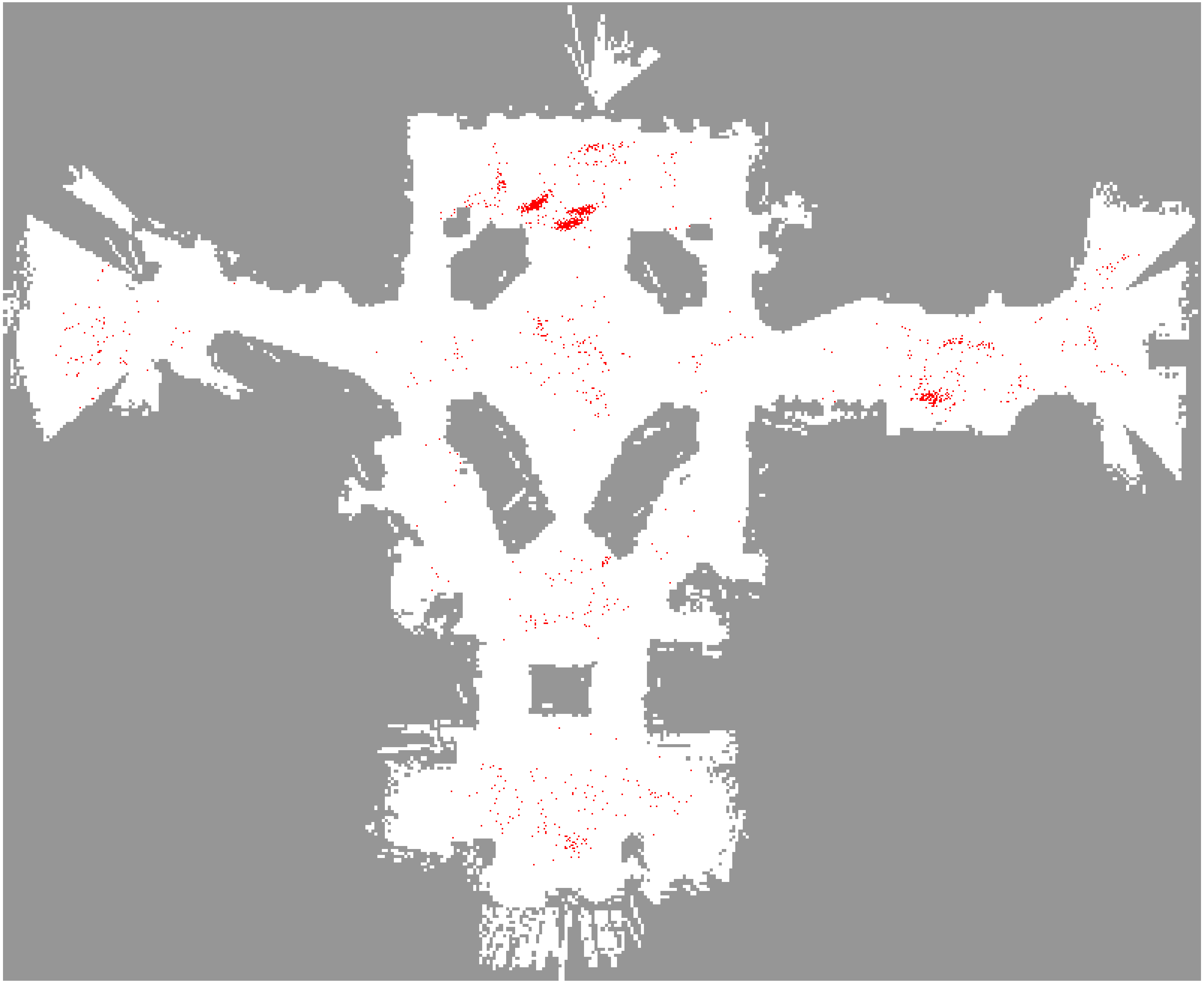


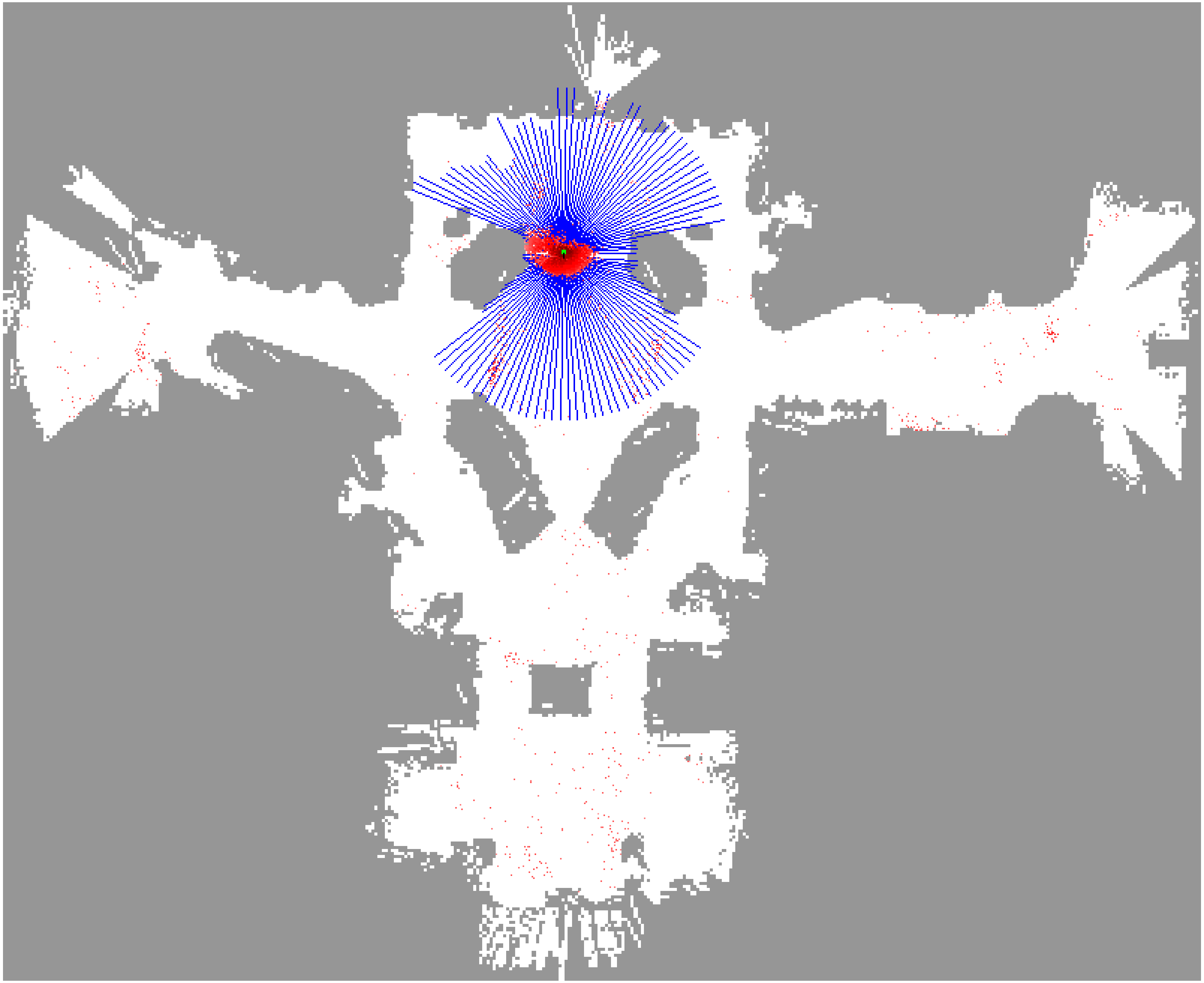


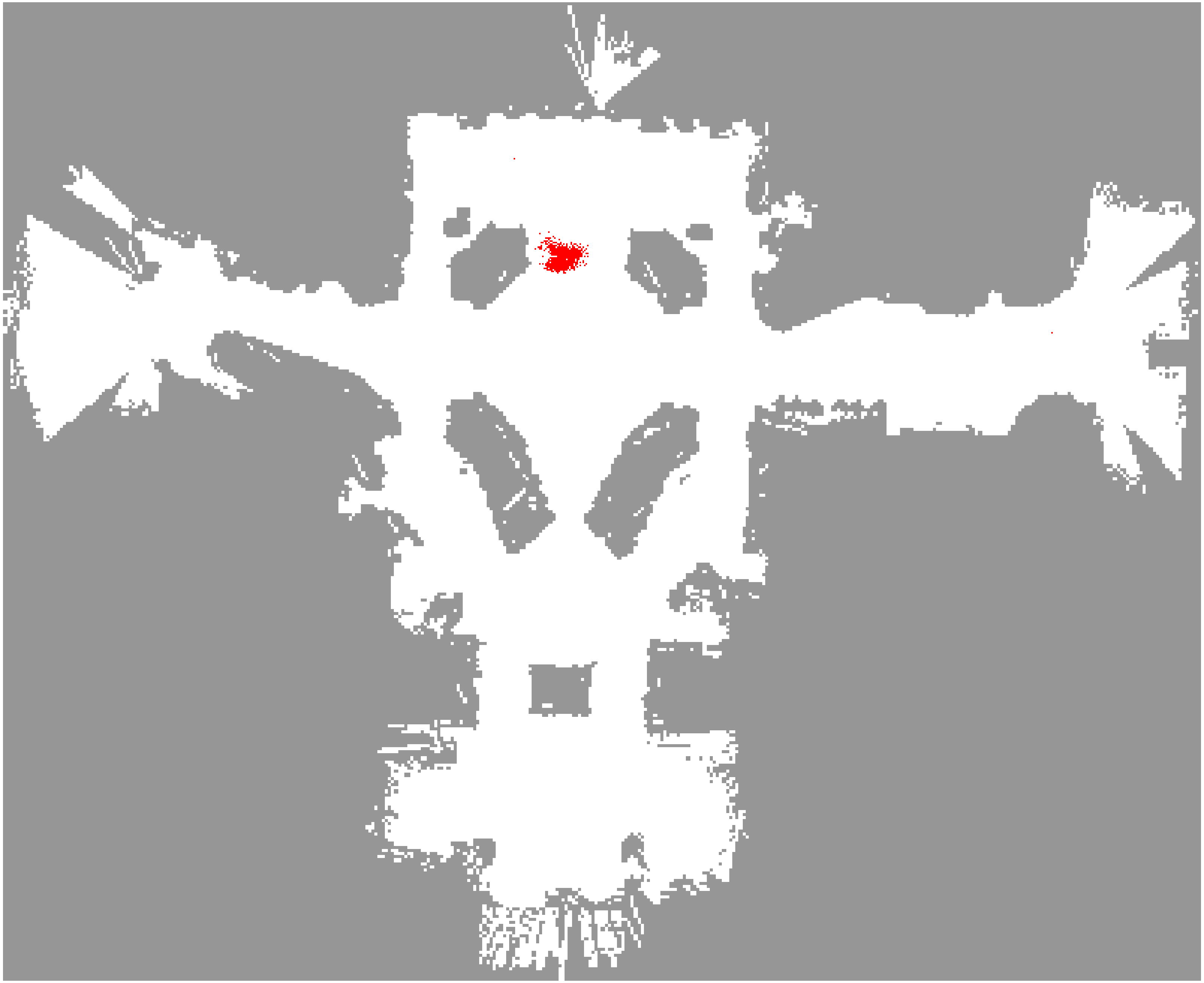


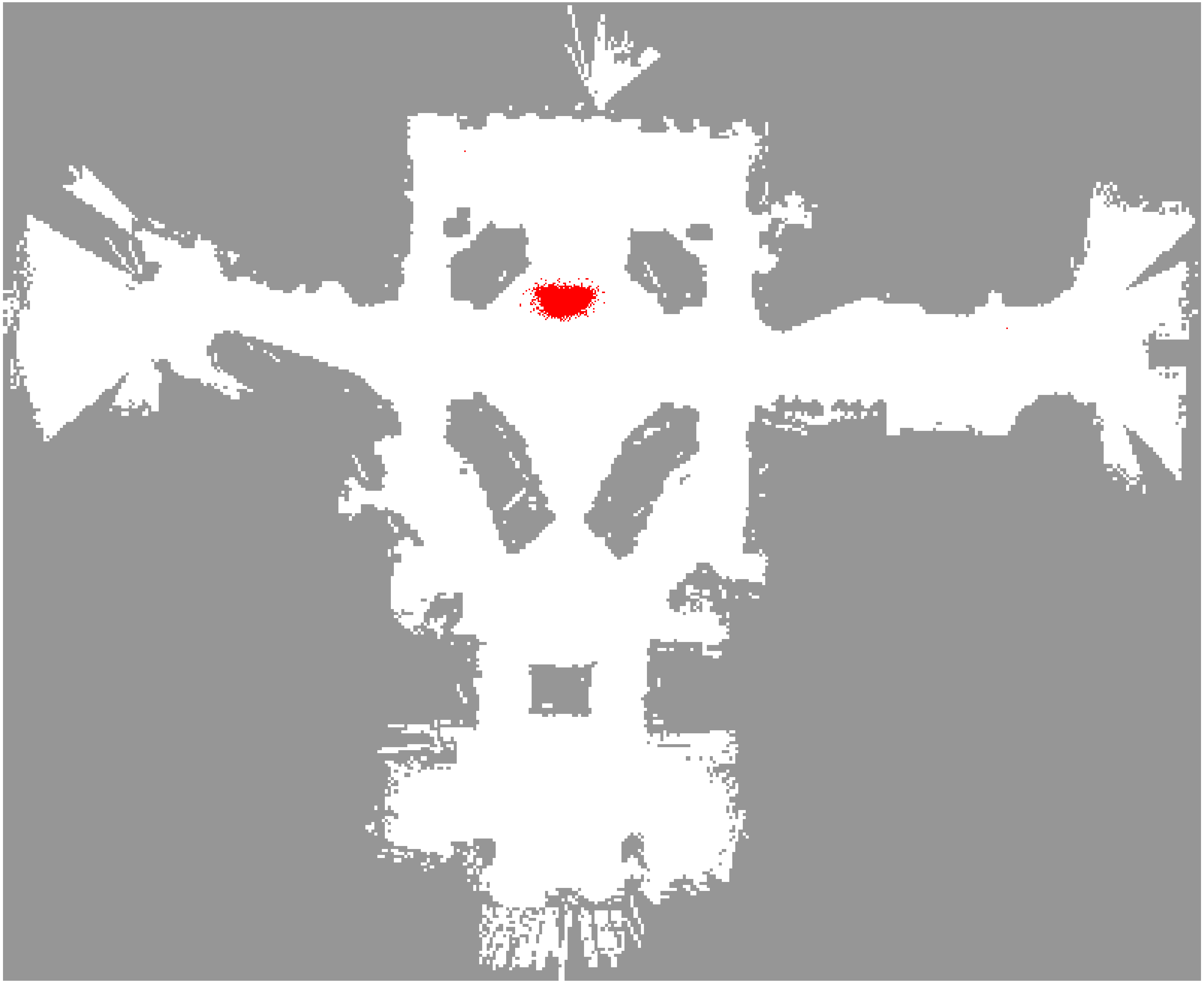


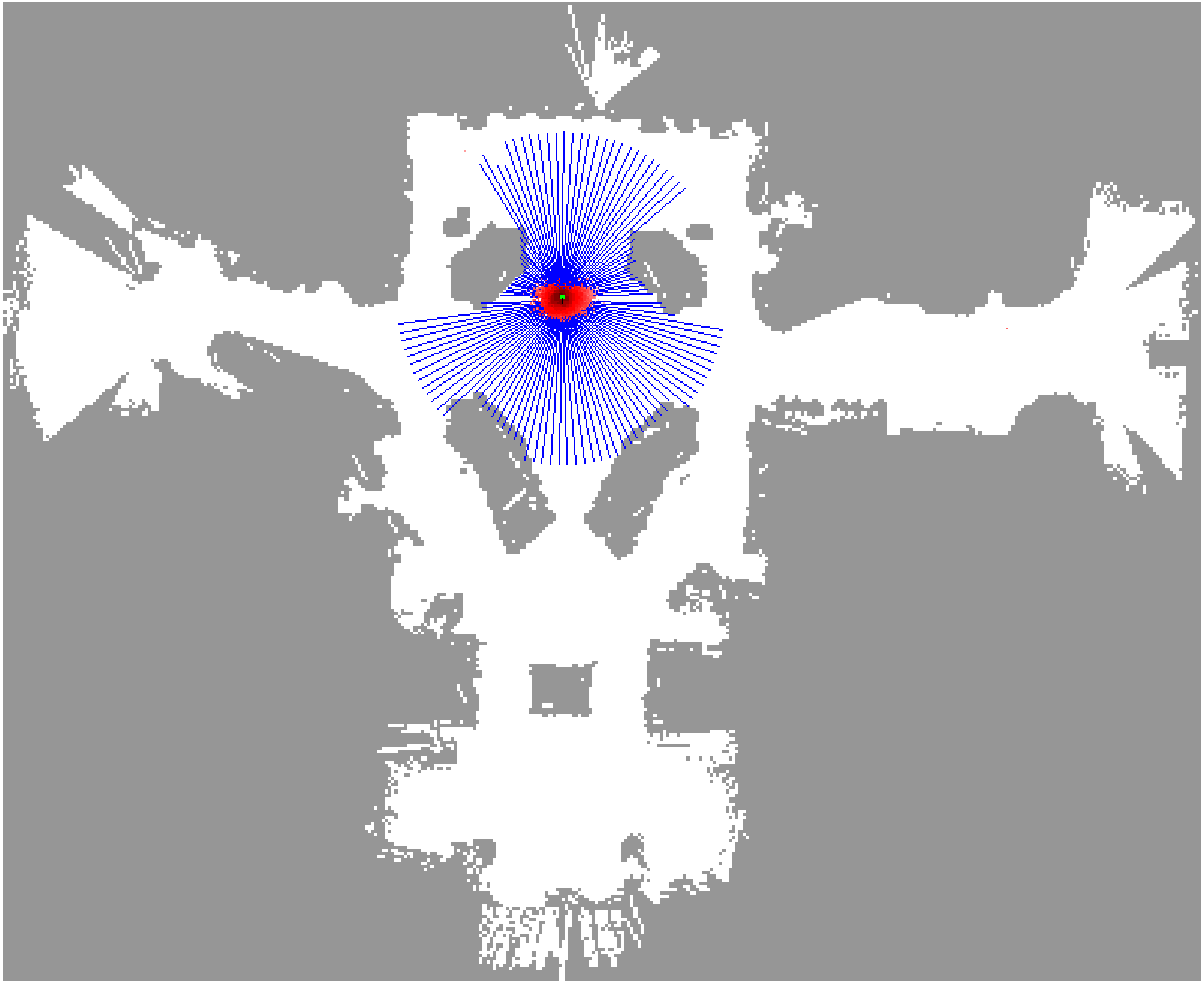


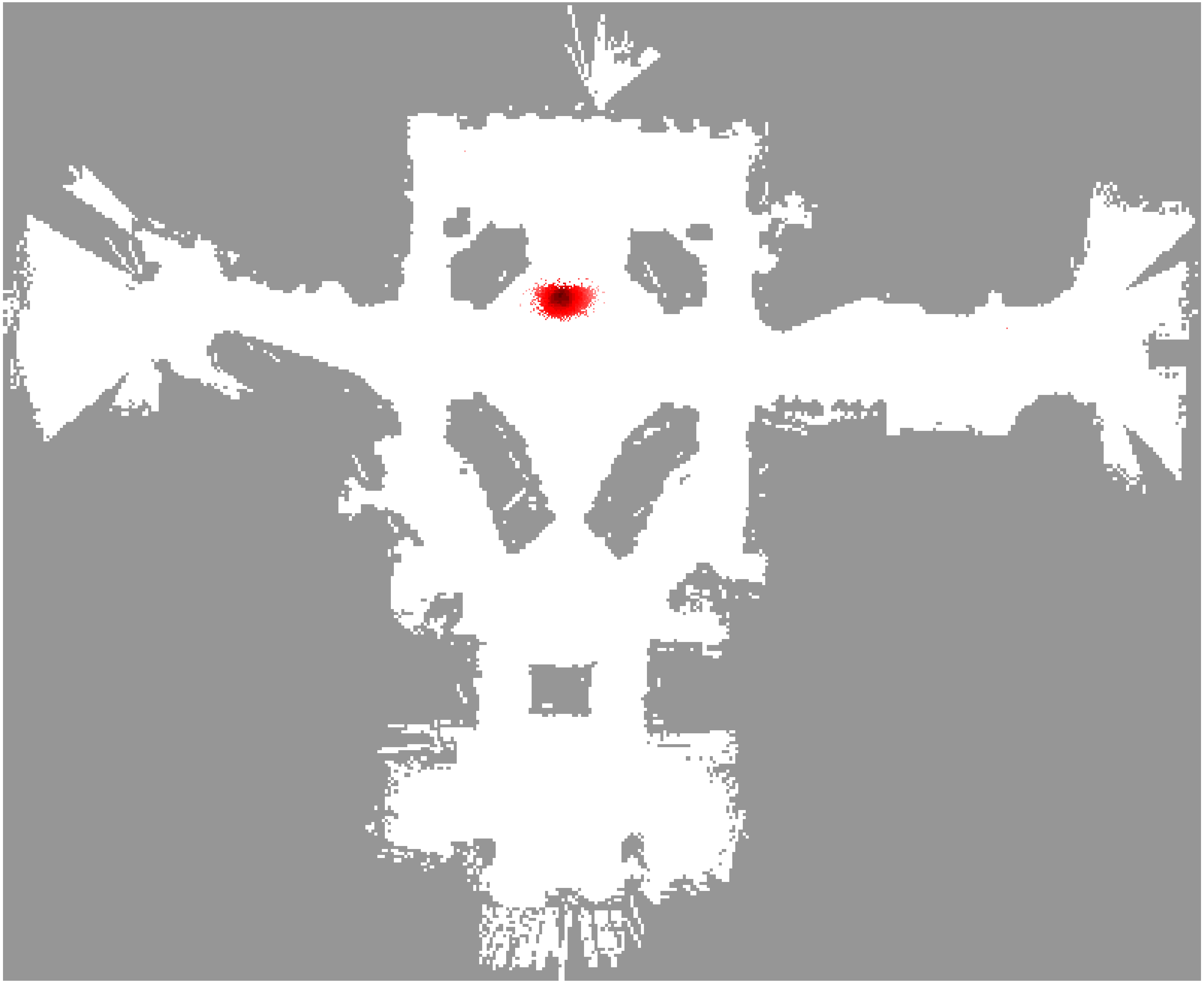


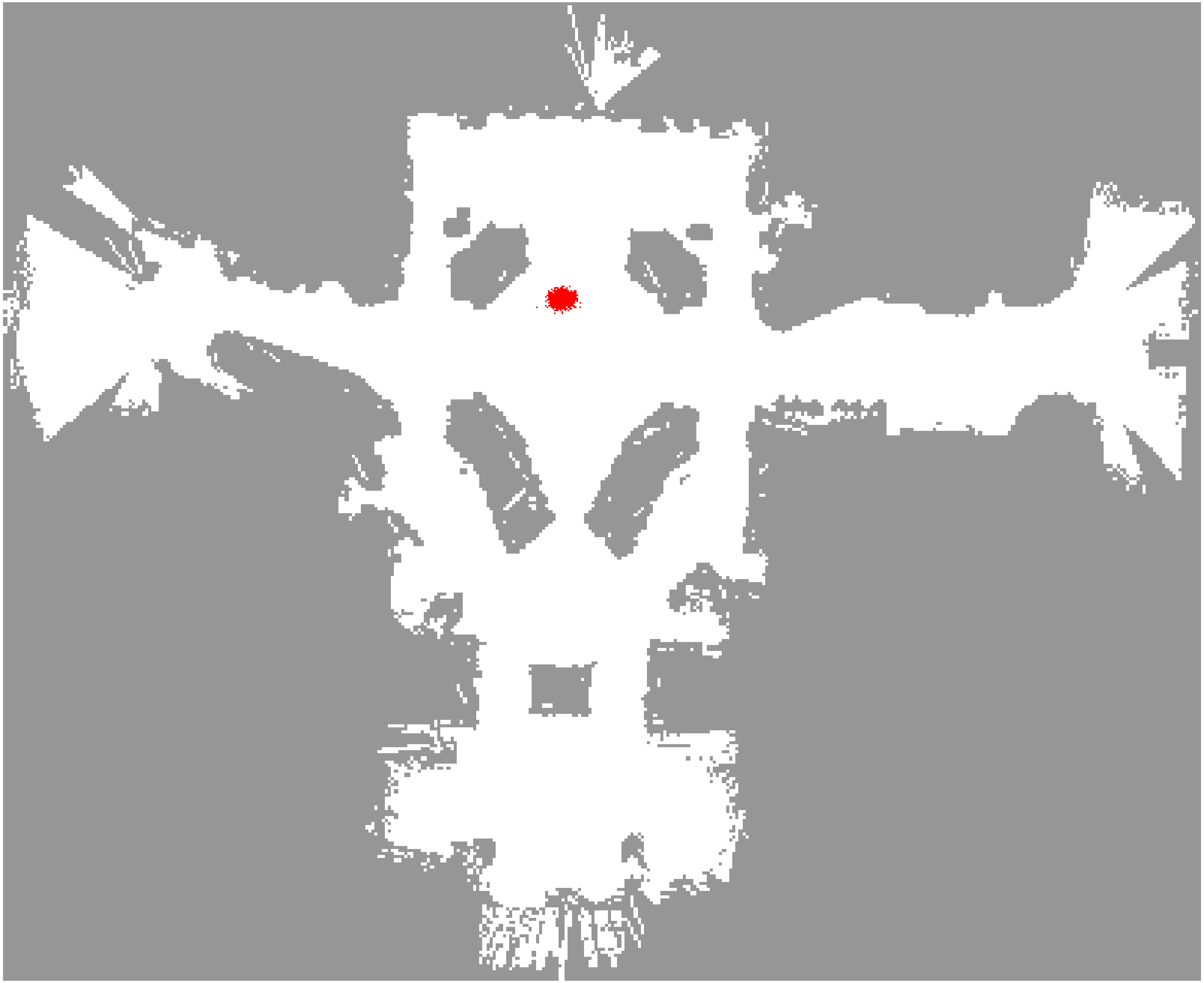


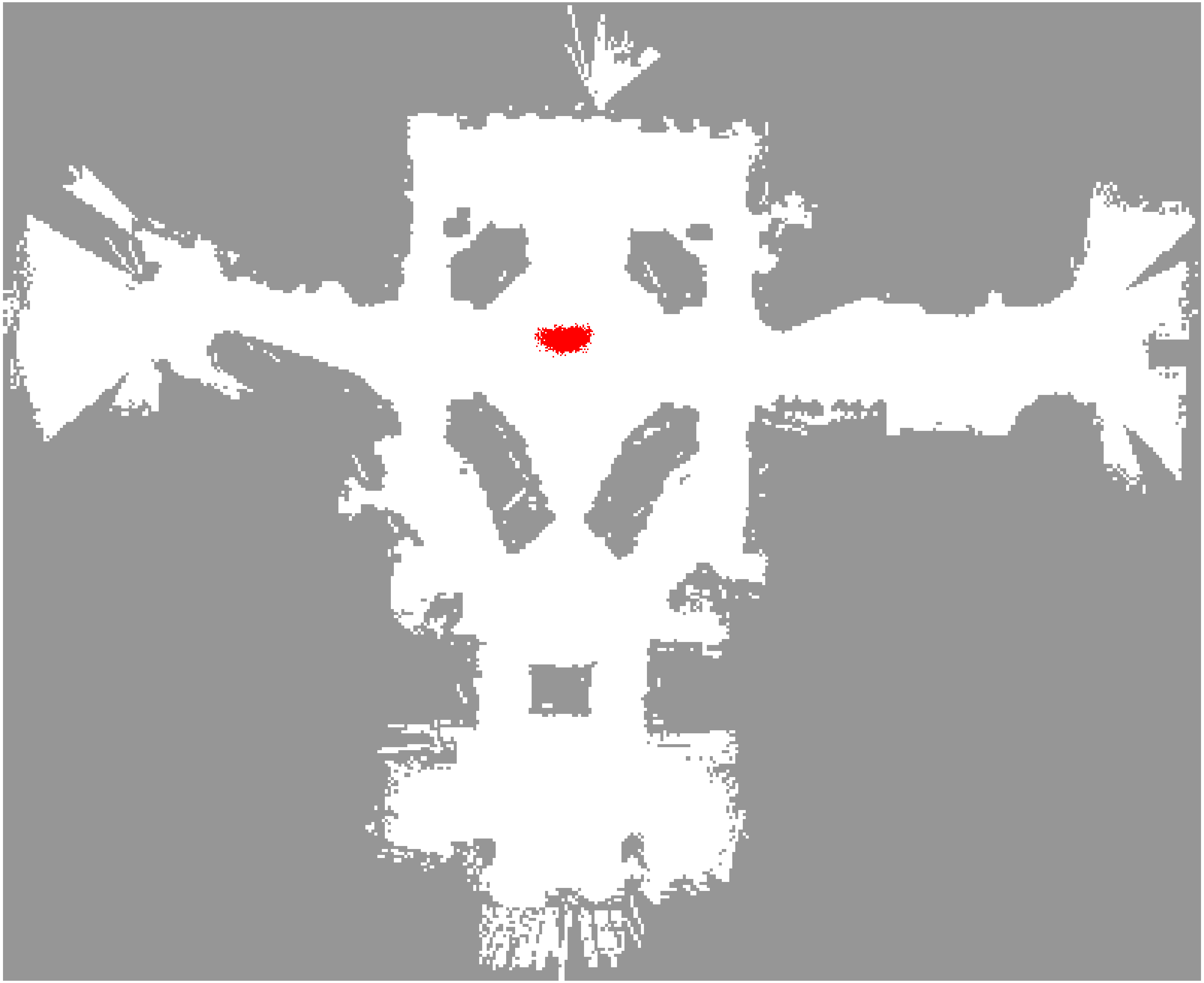


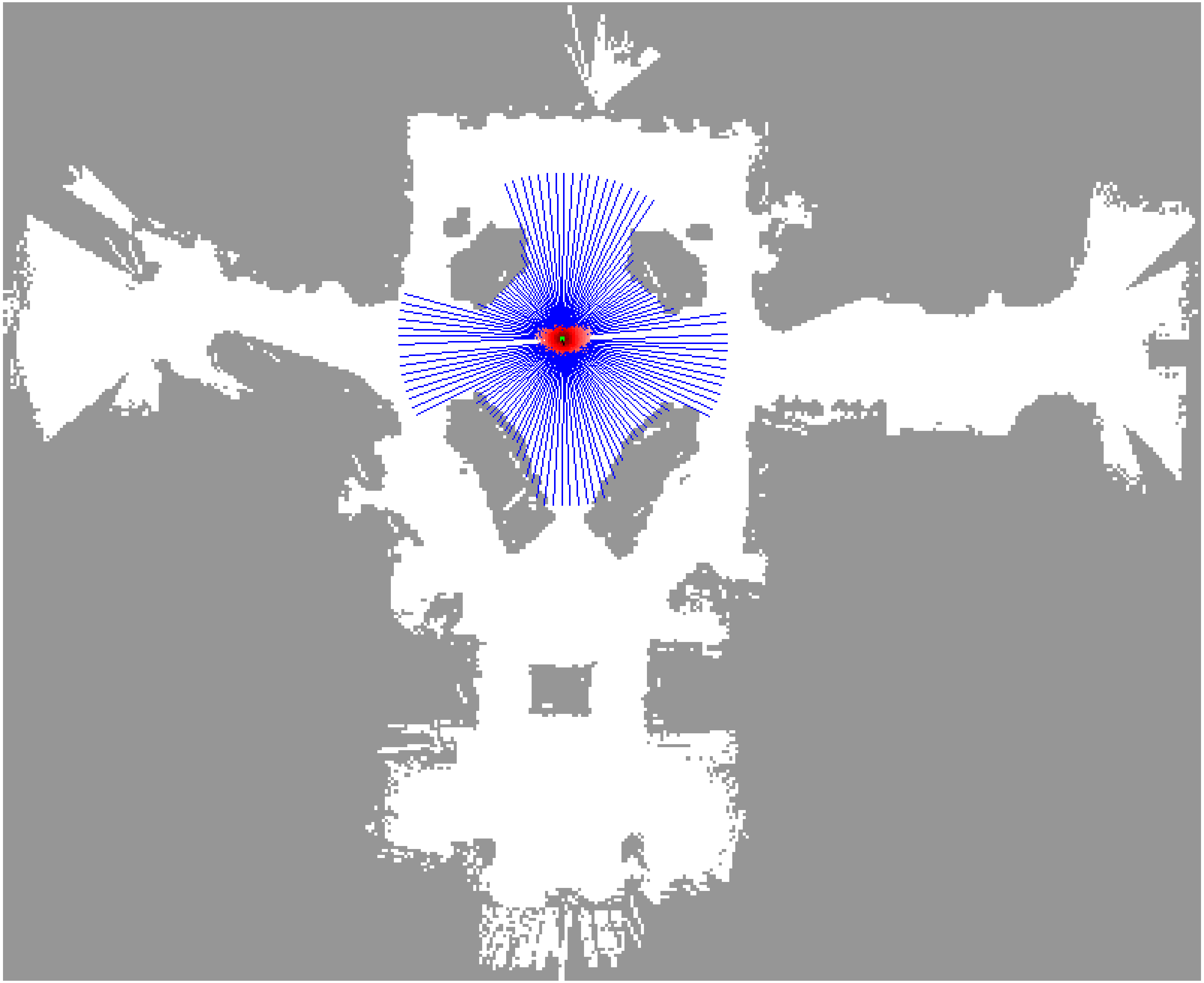


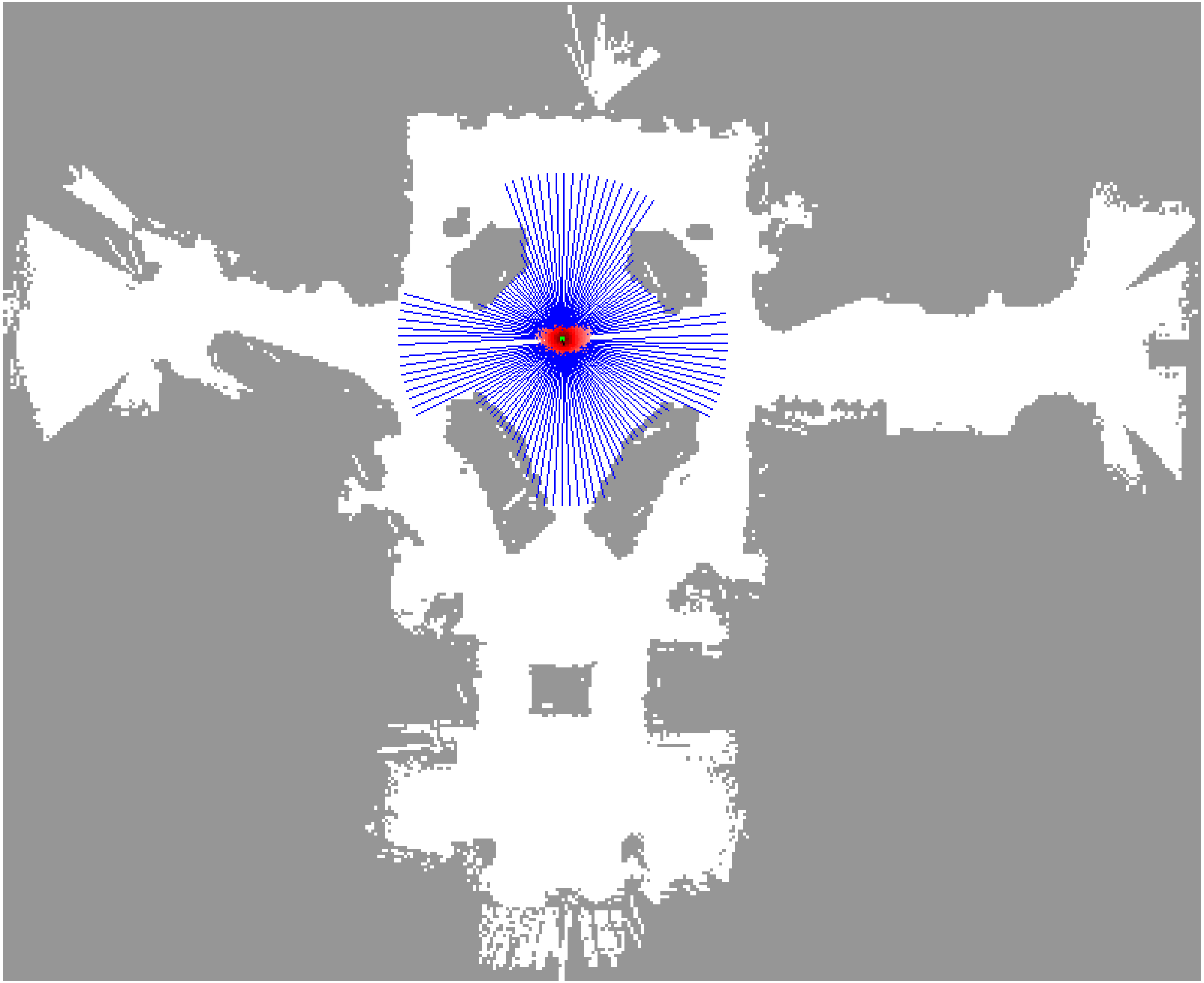












Summary

- Particle filters are an implementation of recursive Bayesian filtering
- They represent the posterior by a set of weighted samples.
- In the context of localization, the particles are propagated according to the motion model.
- They are then weighted according to the likelihood of the observations.
- In a re-sampling step, new particles are drawn with a probability proportional to the likelihood of the observation.