Advanced Microeconomics 2
Helsinki GSE
Juuso Välimäki

Problem Set 2, Due November 30, 2022

1. Consider the college admission problem where n students apply to k colleges. Students have preferences over the colleges and you may assume that their preference depends only on the college that admits them (i.e. their preferences do not depend on their peers). The colleges rank the students according to the student grades in a single exam that all applicants must take (the system for STEM subjects in e.g. Turkey and Iran).

   College $j \in \{1, ..., k\}$ has capacity $\overline{x}_j$. A matching of students to colleges is a function $\mu : \{1, ..., n\} \to \{1, ..., k\}$ such that the inverse image $\mu^{-1}(j)$ of j has no more than $\overline{x}_j$ elements for all j. A matching μ is stable if for all $i \in \{1, ..., n\}$ and all $j \in \{1, ..., k\}$ such that $j \succ_i \mu(i)$ implies that $i' \succ_j i$ for all i' such that $\mu(i') = j$.

   (a) Formulate a variant of the deferred acceptance algorithm for this problem and show that the resulting match is stable.

   (b) Compare the resulting match to the serial dictator algorithm where the applicants are allowed to choose amongst the remaining vacancies in the descending order of their exam grades.

2. Two firms are hiring workers $i \in \{1, 2, 3\}$. Each firm has a capacity constraint of two workers and all workers are equally good to both firms. Hiring two. workers dominates hiring one worker and hiring one worker is better than not hiring anybody. Suppose each worker prefers having a coworker to working alone and working alone to not working at all. Suppose further that $i + 1$ is the favorite co-worker of i (with the understanding that 1 is the favorite of 3).

   (a) What are the Pareto-efficient allocations (working arrangements) for this economy?

   (b) An allocation is stable if no firm can propose an alternative allocation where the firm and the workers that it hires are better off than at the original. What are the stable allocations in this economy?

3. Consider the following exchange economy with two agents and three goods (real Edgeworth Box). Agent 1 has linear preferences represented by the utility function

$$u_1(x_{11}, x_{12}, x_{13}) = \alpha_1 x_{11} + \alpha_2 x_{12} + \alpha_3 x_{13},$$

and agent 2 has utility function

$$u_2(x_{21}, x_{22}, x_{23}) = \beta_1 x_{21} + \beta_2 x_{22} + \beta_3 x_{23}.$$

(a) Assume that the total resources in the economy are: $\overline{x_1} = 1, \overline{x_2} = 2, \overline{x_3} = 3$ and that $\frac{\alpha_l}{\alpha_{l'}} \neq \frac{\beta_l}{\beta_{l'}}$ for all l, l'. What are the Pareto-efficient allocations for this society. (Hint: you may want to recall the characterization of Pareto-efficient points from Lecture 2 as the solution to a weighted utilitarian maximization problem).

(b) Consider consumption in a discrete infinite-horizon generalization of the above linear model where x_{it} denotes the consumption of agent i in period t and:

$$u_1(\boldsymbol{x}_1) = \sum_{t=0}^{\infty} \alpha^t x_{1t} \text{ and } u_2(\boldsymbol{x}_2) = \sum_{t=0}^{\infty} \beta^t x_2 t.$$

Assume that $1 > \alpha > \beta > 0$ and characterize the Pareto-efficient allocations when $\overline{x_t} = 1$ for all t.

4. Consider the following exchange economy with two agents and two goods. The preferences of Agent 1 are represented by utility function

$$u_1(x_1, y_1) = x_1 + y_1$$

and the preferences of agent 2 are represented by utility function

$$u_2(x_2, y_2) = \sqrt{x_2 y_2}$$

Let the initial endowments be $\omega_{1x} = 1$, $\omega_{1y} = 1$, $\omega_{2x} = 2$, $\omega_{2y} = 3$.

(a) Find the competitive equilibria of this economy.

(b) Find "Pareto weights" (λ_1, λ_2) such that competitive equilibrium allocation a^* solves maximization problem

$$\max_{a \in \mathcal{A}} \sum_{i=1}^{n} \lambda_i u_i(a). \tag{1}$$

where $\mathcal{A}$ is the set of all (feasible) allocations.

(c) Consider an otherwise similar economy but now with the following utility function for Agent 1: $u_1(x_1, y_1) = 2x_1 + 2y_1$. Find again the competitive equilibria and Pareto weights (λ_1, λ_2) such that the competitive equilibrium allocation solves maximization problem (1).

5. Assume that the agents in an exchange economy have the same differentiable, strictly increasing and strictly quasi-concave utility function that is homogenous of degree 1 and that the aggregate endowment of all goods is positive. The agents may have different initial endowments.

   Assume that $u(x_1, ..., x_L) \to -\infty$ if $x_l \to 0$ for some l (so you can assume interior optimal demands).

   (a) Find the Pareto-efficient allocations for this economy.

   (b) Find the Walrasian equilibria for this economy.

6. Consider the exchange economy with two agents $i \in \{1, 2\}$ and two goods, x, y . Agent 1 has a utility function

$$u_1(x_1, y_1) = \sqrt{1 + x_1} + \sqrt{1 + y_1},$$

   and agent 2 has utility function where the goods are complements

$$u_2(x_2, y_2) = \min\{x_2, y_2\}.$$

   (a) Do the individual demands satisfy the four properties i) -iv) in Theorem 3.1. of the lecture notes (continuity in $\boldsymbol{p}$ for $\boldsymbol{p} \in \Delta^o$, homogeneity of degree 0 in prices, Walras' law and the boundary condition)?

   (b) Let the initial endowments be $\omega_{1x} = 1, \omega_{1y} = 0, \omega_{2,x} = 3, \omega_{2y} = 2$. Draw the Edgeworth rectangle for this economy and compute its Pareto-efficient allocations.

   (c) Show that this economy has no competitive equilibria.