Advanced Microeconomics 2
Helsinki GSE
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Problem Set 3, Due December 7, 2022

1. Consider the following exchange economy.

   (a) Suppose that the three consumers have utility functions:

   $$u_1(x_{11}, x_{12}) = \ln(x_{11}) + x_{12},$$

   $$u_2(x_{21}, x_{22}) = x_{21} + \ln(x_{22}),$$

   $$u_3(x_{31}, x_{32}) = 2x_{31} + x_{32},$$

   and all consumers consume strictly positive quantities of both goods? Determine the equilibrium price vector with $p_1 = 1$.

   (b) Find a competitive equilibrium allocation for the economy where the agents have utility functions as above and the initial endowments are: $\omega_1 = (0, 3)$, $\omega_2 = (2, 1), \omega_3 = (3, 3)$.

2. Consider the constant returns to scale Cobb-Douglas production technology, where output q is produced from m inputs z_j for $j \in \{1, ..., m\}$ according to:

   $$q = z_1^{\beta_1} z_2^{\beta_2} \dots z_m^{\beta_m},$$

   where $\beta_j > 0$ for all j, and $\sum_{j=1}^{m} \beta_j = 1$. Show that for positive production of q in a competitive equilibrium at output price p and input prices w_j for $j \in \{1, ..., m\}$, the prices must satisfy:

   $$p = (\frac{w_1}{\beta_1})^{\beta_1} (\frac{w_2}{\beta_2})^{\beta_2} \dots (\frac{w_m}{\beta_m})^{\beta_m}.$$

3. Consider a two-commodity productive economy with N individuals, each with the same utility function $u_i(x_i, y_i) = x_i y_i$. They are owners to a single competitive profit maximizing firm that can produce output y_F from input x_F according to $y_F = x_F^{\alpha}$, where $0 < \alpha < 1$. Each individual has an endowment of 1 unit of x (think of this as time that can be spent on leisure in consumption or work for the company). The firm's profit is distributed to the individuals as dividends according to their ownership shares $\theta_i \geq 0$ with $\sum_i \theta_i = 1$.

(a) Normalize the price of x to 1 and let p denote the price of y. Solve the firm's profit maximization problem to get an expression for optimal profit $\pi(p)$ and for optimal output $y_F(p)$ at output price p.

(b) Formulate and solve the consumerr problem of finding optimal $x_i(p), y_i(p)$ (taking into account the endowment and the dividend) at output price p.

(c) Find the equilibrium price p^* by equating the aggregate demand $\sum_i y_i(p)$ to the optimal firm supply $y_F(p)$.

4. Consider an economy with two altruistic agents $1, 2$ and two goods x, y. Normalize the total supply of each good to 1 and assume that each agent derives material utility $u_i(x_i, y_i)$ from consumption. Their total satisfaction depends on the material welfare of both agents so that $U_i(x_1, x_2, y_1, y_2) = v_i(u_1(x_1, y_1), u_2(x_2, y_2))$. Assume that U_i is strictly increasing in both components for $i \in \{1, 2\}$.

(a) Define Pareto-efficiency with respect to total satisfaction and show that all Pareto-efficient allocations are Pareto efficient for non-altruistic economies as well (i.e. the standard ones where each agent cares about her own welfare only).

(b) Suppose the agents have an initial endowment and that they can buy and sell the two goods in a competitive market. Note that they cannot affect the other agent's consumption choices. Define the competitive equilibrium with respect to total satisfaction preferences. Are the two welfare theorems valid for this altruistic economy?

5. Consider a matching model where agents in $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2, y_3\}$, and $Z = \{z_1, z_2, z_3\}$ are to be matched in triples (x_i, y_j, z_k). Assume that all agents have strict preferences over pairs of agents from the other groups. A ranking function $r : X \times Y \times Z \to \{1, ..., 9\}^3$ represents the preferences in the following way.

$$r(x_1, y_2, z_2) = (r_{x_1}(y_2, z_2), r_{y_2}(x_1, z_2), r_{z_2}(x_1, y_2) = (2, 3, 6),$$

means that x_1 ranks $(y_2.z_2)$ as her second best option, y_2 ranks (x_1, z_2) as her third best option and z_2 ranks (x_1, y_2) as her sixth best option (and similarly for all possible triples).

(a) How many different matchings are there?

(b) Consider preferences over the matchings that satisfy the following:

r_{x_1} :

	z_1	z_2	z_3
y_1	3		1
y_2			2
y_3			

, r_{x_2} :

	z_1	z_2	z_3
y_1			
y_2		2	1
y_3			3

, r_{x_3} :

	z_1	z_2	z_3
y_1			
y_2			
y_3			1

.

r_{y_1} :

	z_1	z_2	z_3
x_1	1		
x_2			
x_3			

, r_{y_2} :

	z_1	z_2	z_3
x_1			2
x_2		3	1
x_3			

, r_{y_3} :

	z_1	z_2	z_3
x_1			
x_2			1
x_3			2

.

r_{z_1} :

	y_1	y_2	y_3
x_1	1		
x_2			
x_3			

, r_{z_2} :

	y_1	y_2	y_3
x_1			
x_2		1	
x_3			

, r_{z_3} :

	y_1	y_2	y_3
x_1		3	1
x_2			2
x_3			4

.

The empty spaces in the tables can be filled with arbitrary rankings (consistent with the existing ones though). A matching into triples is blocked by (x', y', z') if x' prefers (y', z'), and y' prefers (x', z') and z' prefers (x', y') to their partners in the matching. A Matching is stable if it is not blocked by any triple. Show that with these preferences, stable matchings into triples do not exist.

Hint: Show that x_1 must get at least her third best option in any stable matching and in fact must be matched with (y_1, z_1). Show next that y_2 must be matched with (x_2, z_2). Finally show that $\{(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ is not stable.