ELEC-E8107 - Stochastic models, estimation and control

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Exercises Session 4: Solution

Exercise 1

A nonlinear system dynamic model of a robot moving on the plane is given by the following equation.

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta t cos(\theta_k) \\ 0 & 1 & 0 & \Delta t sin(\theta_k) \\ 0 & 0 & 1 & \frac{\Delta t}{L} tan(\phi) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ v_k \end{bmatrix} + q_k \tag{1}$$

Where v is the speed of the vehicle, θ is the heading and q_k is the process noise vector with covariance matrix Q_k. This covariance matrix can be assumed to be a diagonal matrix. The parameter ϕ is the steering angle and is considered a known input to the system. The constant parameter L is the distance between the front and back wheel of the robot. Here we assume $L = 15cm$.

Only the positions x and y of the robot are measured. The measurement noise is assumed Gaussian with zero mean and has 0.5 meters standard deviation. The measurement noise of x-axis and y-axis are assumed independent.

1. Write the measurement equation for the system.
2. Compute the Jacobian matrix for the state transition model.
3. Implement a first order EKF using the measurement and input data provided in the files *measurement_data.mat* and *input_data.mat*.

4. A true data for the state is given in the file *True_trajectory.mat*. Plot the true trajectory (coordinate x versus coordinate y plot) and your estimated trajectory on the same figure to appreciate the accuracy of your estimation. Adjust the values of the covariance matrix Q to have a good estimation.

5. plot the true and estimated speed, and the true and estimated heading angle.

Solution Exercise 1

1. The measurement equation:
The measurement equation relate the actual measurement to the system state. For this system, the measurement equation is given by:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ v_k \end{bmatrix} + r_k$$

With r_k being the measurement noise.

2. The Jacobian matrix for the state transition model:
The Jacobian matrix is obtained by taking the partial derivative of the nominal expression (disregarding the noise) of x_{k+1}, y_{k+1}, θ_{k+1}, and v_{k+1} with respect to the state vector. The Jacobian matrix is given by:

$$F = \begin{bmatrix} 1 & 0 & -\Delta t v_k sin(\theta_k) & \Delta t cos(\theta k) \\ 0 & 1 & \Delta t v_k cos(\theta_k) & \Delta t sin(\theta_k) \\ 0 & 0 & 1 & \frac{\Delta t}{L} tan(\Phi) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. A first order EKF:
See the provided Matlab script for the implementation of the EKF.

4. Plot of trajectories:

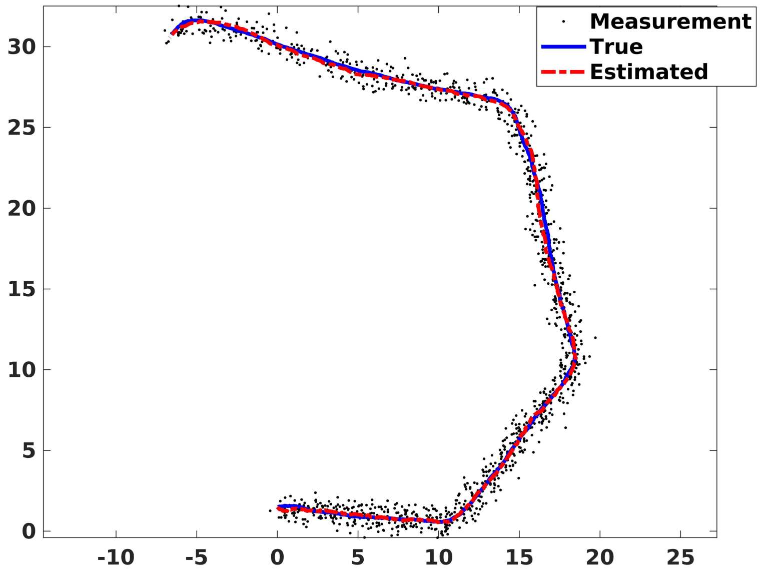

Figure 1: Comparison of EKF estimate with the true trajectory and the measurement

5. Plot of estimated and true speed and heading

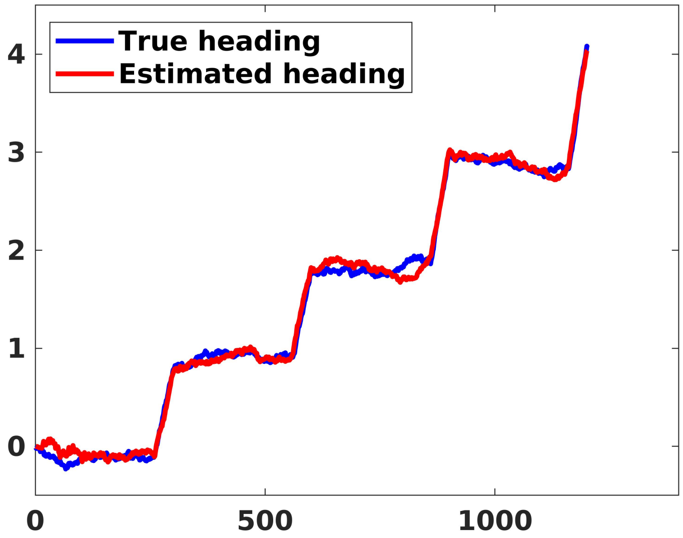

Figure 2: Comparison of EKF estimate with the true heading

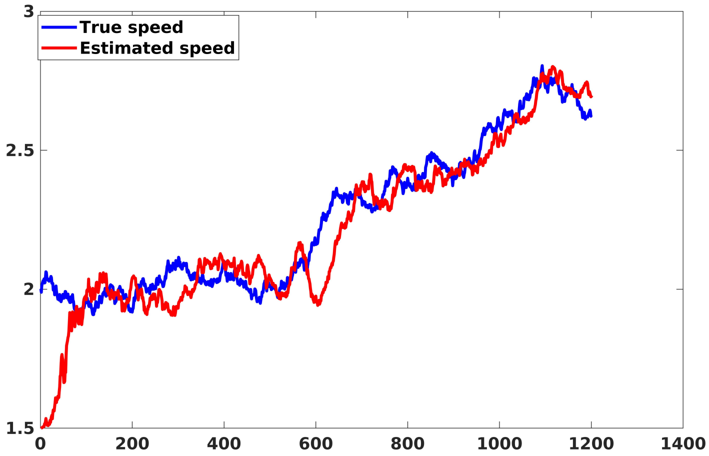

Figure 3: Comparison of EKF estimate with the true speed

Exercise 2

Implement the bootstrap particle filter to estimate the state of the system given in the exercise 1 above.

Solution Exercise 2

The bootstrap particle filter is one of the simplest (basic) form of the particle filter method. It uses the dynamic model as the importance distribution. See the provided Matlab script for the implementation.

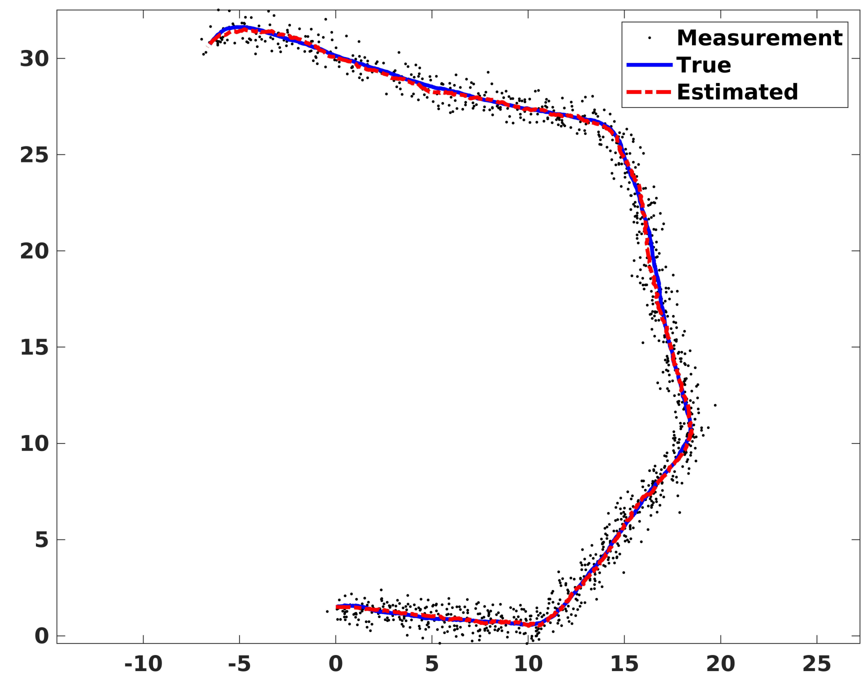

Figure 4: Comparison of PF estimate with the true trajectory and the measurement

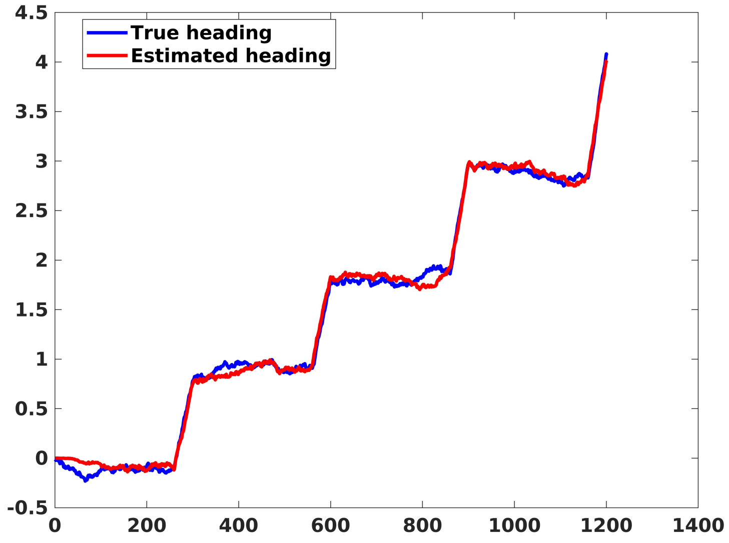

Figure 5: Comparison of PF estimate with the true heading

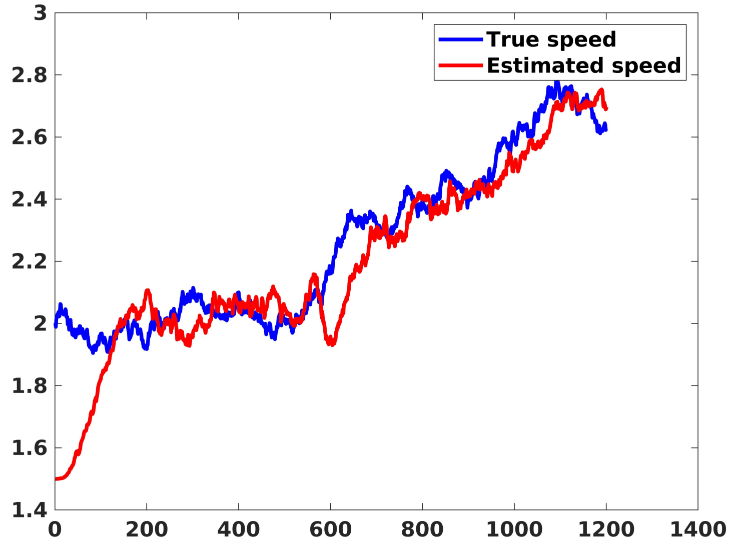

Figure 6: Comparison of PF estimate with the true speed