MS-EV0004 Vertex operator algebras
(Periods II and III)

- Lecturer: Shinji Koshida (shinji.koshida@aalto.fi)
- Lectures
  - Fridays 12—14
  - M3 (this room)
- Exercises
  - every two weeks on Fridays 14—16 (First session -> 4.11)
  - M3 (during period II -> M2 during period III)

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- Credits: 5
- Grading: pass/fail
- There is no exam. Grading is based on contribution to exercises.
- To pass, do either
  - give a presentation solving one problem during the session, or
  - submit a solution to one problem after the session

  for each exercise sheet.

Why VOA?

VOA = Vertex operator algebra

Conformal field theory (CFT) in two dimensions (Physics!)

Quantum field theory with large symmetry (= very nice QFT)

$$\psi(z)\varphi(w) \sim (z-w)^h \eta(w) + \cdots$$

VOA explains what they are, what this equation means.

Why VOA?

VOA = Vertex operator algebra

Monstrous Moonshine

j-function (from modular forms)

$$j(\tau) = q^{-1} + 744 + 196\,884\,q + 21\,493\,760\,q^2 + 864\,299\,970\,q^3 + \cdots$$

Coefficients seem to be related to …

dimensions of irreducible representations of $\mathbb{M}$: Monster finite group

VOA explains why this happens (Borchards 1992 -> Fields Medal 1998).

Course plan

1. Lie algebras and representation
2. Formal calculus
3. Definitions of vertex algebra
4. Construction of examples (**First goal!**)
5. Vertex operator algebras
6. ??

References

- James Lepowsky and Haisheng Li, *Introduction to Vertex Operator Algebras and Their Representations*, https://doi.org/10.1007/978-0-8176-8186-9.
- Victor G. Kac, *Vertex Algebras for Beginners*, American Mathematical Soc., 1998.
- Edward Frenkel and David Ben-Zvi, *Vertex Algebras and Algebraic Curves*, American Mathematical Soc., 2004.
- James Humphreys, *Introduction to Lie algebras and Representation Theory*, GTM